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Brief paper

An improved stability condition for Kalman filtering with bounded Markovian packet losses[☆]

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ABSTRACT

In this paper, we consider the peak-covariance stability of Kalman filtering subject to packet losses. The length of consecutive packet losses is governed by a time-homogeneous finite-state Markov chain. We establish a sufficient condition for peak-covariance stability and show that this stability check can be recast as a linear matrix inequality (LMI) feasibility problem. Compared with the literature, the stability condition given in this paper is invariant with respect to similarity state transformations; moreover, our condition is proved to be less conservative than the existing results. Numerical examples are provided to demonstrate the effectiveness of our result.

1. Introduction

Networked control systems are closed-loop systems, wherein sensors, controllers and actuators are interconnected through a communication network. In the last decade, advances of modern control, micro-electronics, wireless communication and networking technologies have given birth to a considerable number of networked control applications.

In networked control systems, state estimation such as using a Kalman filter is necessary whenever precise measurement of the system state cannot be obtained. When a Kalman filter is running subject to intermittent observations, the stability of the estimation error is affected by not only the system dynamics but also by the statistics of the packet loss process. The stability of Kalman filtering with packet drops has been intensively studied in the literature. In Sinopoli et al. (2004), Plarre and Bullo (2009), Mo and Sinopol (2010), Shi, Epstein, and Murray (2010) and Kar, Sinopoli, and Moura (2012), independently and identically distributed (i.i.d.) Bernoulli packet losses have been considered. Some other research works assume the packet drops, due to the Gilbert–Elliott channel (Gilbert, 1960; Elliott, 1963), are governed by a time-homogeneous Markov chain. Huang and Dey (2007) introduced the notion of peak covariance, which describes an upper envelope of the sequence of error covariance matrices for the case of an unstable scalar system. They focused on its stability with Markovian packet losses and gave a sufficient stability condition. The stability condition was further improved in Xie and Xie (2007) and Xie and Xie (2008). In Wu, Shi, Anderson, and Johansson (0000), the authors proved that the peak-covariance stability implies mean-square stability for general random packet drop processes, if the system matrix has no defective eigenvalues on the unit circle. In addition to the peak-covariance stability, the mean-square stability was considered for some classes of linear systems in Mo and Sinopoli (2012), You, Fu, and Xie (2011), and weak convergence of the estimation error covariance was studied in Xie (2012).

In the aforementioned packet loss models, the length of consecutive packet losses can be infinitely large. In contrast, some works also considered bounded packet loss model, whereby the length of consecutive packet losses is restricted to be less than a finite integer. A real example of bounded packet losses is the WirelessHART (Wireless Highway Addressable Remote Transducer) protocol, the state-of-the-art wireless communication solution for process automation applications. In WirelessHART, there are two types of time slots: one is the dedicated time slot allocated to a specific field device for time-division multiple-access (TDMA) based transmission and the other is the shared time slot for contention-based

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communication. A contiguous group of time slots during a constant period of time forms a superframe, within which every node is guaranteed at least one time slot for data communication. Various networked control problems with bounded packet loss models have been studied, e.g., Wu and Chen (2007) and Xiong and Lam (2007); while the stability of Kalman filtering for this kind of models was rarely discussed. In Xiao, Xie, and Fu (2009), the authors gave a first attempt to the stability issue related to the Kalman filtering with bounded Markovian losses. They provided a sufficient condition for peak-covariance stability, the stability notion studied in Huang and Dey (2007), Xie and Xie (2007) and Xie and Xie (2008). Their result has established a connection between peak-covariance stability, the dynamics of the underlying system and the probability transition matrix of the underlying packet-loss process. In this paper, we consider the same problem as in Xiao et al. (2009) and improve the stability condition thereof. The main contributions of this work are summarized as follows:

(1) We present a sufficient condition for peak-covariance stability of the Kalman filtering subjected to bounded Markovian packet losses (Theorem 1). Different from that of Xiao et al. (2009), this stability check can be recast as a linear matrix inequality (LMI) feasibility problem (Proposition 1).
(2) We compare the proposed condition with that of Xiao et al. (2009). We show both theoretically and numerically that the proposed stability condition is invariant with respect to similarity state transformations, while the one given in Xiao et al. (2009) may generate opposite conclusions under different similarity transformations. Moreover, the analysis also suggests that our condition is less conservative than the former one.

The remaining part of the paper is organized as follows. Section 2 presents the mathematical models of the system and packet losses, and introduces the preliminaries of Kalman filtering. Section 3 provides the main results. Comparison with Xiao et al. (2009) and numerical examples are presented in Section 4. Some concluding remarks are drawn in the end.

Notations. $\mathbb{N}$ is the set of positive integers and $\mathbb{C}$ is the set of complex numbers. $\mathbb{S}_+^n$ is the set of n by n positive semi-definite matrices over the field $\mathbb{C}$. For a matrix $X \in \mathbb{C}^{n\times n}$, $\sigma(X)$ denotes the spectrum of X, i.e., $\sigma(X) = \{\lambda : \det(\lambda I - X) = 0\}$, and $\rho(X)$ denotes the spectrum radius of X, X^*, X' and $\overline{X}$ are the Hermitian conjugate, transpose and complex conjugate of X, respectively. $\|\cdot\|$ means the L_2-norm on $\mathbb{C}^n$ or the matrix norm induced by L_2-norm. The symbol $\otimes$ represents the Kronecker product operator of two matrices. For any matrices A, B, C with compatible dimensions, we have $\mathrm{vec}(ABC) = (C' \otimes A)\mathrm{vec}(B)$, where $\mathrm{vec}(\cdot)$ is the vectorization of a matrix. Moreover, the indicator function of a subset $\mathcal{A} \subset \Omega$ is a function $\mathbf{1}_{\mathcal{A}} : \Omega \to \{0, 1\}$ where $\mathbf{1}_{\mathcal{A}}(\omega) = 1$ if $\omega \in \mathcal{A}$, otherwise $\mathbf{1}_{\mathcal{A}}(\omega) = 0$. The symbol $\mathbb{E}[\cdot]$ (resp., $\mathbb{E}[\cdot|\cdot]$) represents the expectation (resp., conditional expectation) of a random variable.

2. Problem setup

2.1. System model

Consider the following discrete-time LTI system:

$$x_{k+1} = Ax_k + w_k, \tag{1a}$$

$$y_k = Cx_k + v_k, \tag{1b}$$

where $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times n}$, $x_k \in \mathbb{R}^n$ is the process state vector, $y_k \in \mathbb{R}^m$ is the observation vector, $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are zero-mean Gaussian random vectors with $\mathbb{E}[w_k w_j{}'] = \delta_{kj} Q$ ($Q \geq 0$), $\mathbb{E}[v_k v_j{}'] = \delta_{kj} R$ ($R > 0$), $\mathbb{E}[w_k v_j{}'] = 0\ \forall j, k$. Note that δ_{kj} is the Kronecker delta function with $\delta_{kj} = 1$ if $k = j$ and $\delta_{kj} = 0$ otherwise. The initial state x_0 is a zero-mean Gaussian random vector that is uncorrelated to w_k and v_k, with covariance $\Sigma_0 \geq 0$. It can be seen that, by applying a similarity transformation, the unstable and stable modes of the LTI system can be decoupled. An open-loop prediction of the stable mode always has a bounded estimation error covariance, therefore, this mode does not play any key role in the problem considered below. Without loss of generality, all eigenvalues of A are assumed to have magnitudes not less than 1. We also assume that (A, C) is observable and $(A, Q^{1/2})$ is controllable. We introduce the definition of the observability index of (A, C), which is taken from Antsaklis and Michel (2006).

Definition 1. The observability index I_o is defined as the smallest integer such that $[C', A'C', \ldots, (A^{\mathrm{I}_o-1})'C']'$ has rank n. If $\mathrm{I}_o = 1$, the system (A, C) is called one-step observable.

2.2. Bounded Markovian packet-loss process

In this paper, we consider the estimation scheme, where the raw measurements $\{y_k\}_{k\in\mathbb{N}}$ of the sensor are transmitted to the estimator over an erasure communication channel: packets may be randomly dropped or successively received by the estimator. Denote by a random variable $\gamma_k \in \{0, 1\}$ whether or not y_k is received at time k. If $\gamma_k = 1$, it indicates that y_k arrives error-free at the estimator; otherwise $\gamma_k = 0$. Whether γ_k equals 0 or 1 is assumed to have been known by the estimator before time $k + 1$. In order to introduce the packet loss model, we further define a sequence of stopping times, the time instants at which packets are received by the estimator:

$$t_1 \triangleq \min\{k : k \in \mathbb{N}, \gamma_k = 1\},$$

$$t_2 \triangleq \min\{k : k > t_1, \gamma_k = 1\},$$

$$\vdots \tag{2}$$

$$t_j \triangleq \min\{k : k > t_{j-1}, \gamma_k = 1\}, \tag{3}$$

where we assume $t_0 = 0$ by convention. The packet-loss process, τ_j, is defined as

$$\tau_j \triangleq t_j - t_{j-1} - 1.$$

As for the model of packet losses, we assume that the packet-loss process $\{\tau_j\}_{j\in\mathbb{N}}$ is modeled by a time-homogeneous ergodic Markov chain, where $\mathcal{S} = \{0, \ldots, s\}$ is the finite-state space of the Markov chain with s being the maximum length of consecutive lost packets allowed. Here the Markov chain is characterized by a known transition probability matrix $\mathbf{\Pi} \triangleq [\pi_{ij}]_{i,j\in\mathcal{S}}$ in which

$$\pi_{ij} \triangleq \mathbb{P}(\tau_{k+1} = j | \tau_k = i) \geq 0. \tag{4}$$

Denote the initial distribution as $p \triangleq [p_0, \ldots, p_s]$, where $p_j = \mathbb{P}(\tau_1 = j)$.

2.3. Kalman filtering with packet losses

Sinopoli et al. (2004) shows that, when performed with intermittent observations, the optimal linear estimator is a modified Kalman filter. The modified Kalman filter is slightly different from the standard one in that only time update is performed in the presence of the lost packet. Define the minimum mean-squared error estimate and the one-step prediction at the estimator respectively as $\hat{x}_{k|k} \triangleq \mathbb{E}[x_k|\gamma_1 y_1, \ldots, \gamma_k y_k]$ and $\hat{x}_{k+1|k} \triangleq \mathbb{E}[x_{k+1}|\gamma_1 y_1, \ldots, \gamma_k y_k]$. Let $P_{k|k}$ and $P_{k+1|k}$ be the corresponding estimation and prediction error covariance matrices, i.e.,

$$P_{k|k} \triangleq \mathbb{E}[(x_k - \hat{x}_{k|k})(\cdot)'|\gamma_1 y_1, \ldots, \gamma_k y_k]$$

$$P_{k+1|k} \triangleq \mathbb{E}[(x_{k+1} - \hat{x}_{k+1|k})(\cdot)'|\gamma_1 y_1, \ldots, \gamma_k y_k].$$

These parameters can be computed recursively by a modified Kalman filter (see Sinopoli et al., 2004 for more details). In particular,

$$P_{k+1|k} = AP_{k|k-1}A' + Q - \gamma_k AP_{k|k-1}C'(CP_{k|k-1}C' + R)^{-1}CP_{k|k-1}A'. \quad (5)$$

To simplify notations, we denote $P_k \triangleq P_{k|k-1}$ for shorthand and define the functions h, g, h^k and g^k: $\mathbb{S}_+^n \to \mathbb{S}_+^n$ as follows:

$$h(X) \triangleq AXA' + Q, \quad (6)$$

$$g(X) \triangleq AXA' + Q - AXC'(CXC' + R)^{-1}CXA', \quad (7)$$

$h^k(X) \triangleq \underbrace{h \circ h \circ \cdots \circ h}_{k \text{ times}}(X)$ and $g^k(X) \triangleq \underbrace{g \circ g \circ \cdots \circ g}_{k \text{ times}}(X)$, where $\circ$ denotes the function composition.

2.4. Problems of interest

To study the stability of Kalman filtering with packet losses, one way is to study the asymptotic behavior of the expected prediction error covariance. In the following we introduce the concept of peak-covariance stability, which is first studied in Huang and Dey (2007). To this end, we need the following auxiliary definitions, which were introduced in Huang and Dey (2007),

$$\alpha_1 \triangleq \min\{k : k \in \mathbb{N}, \gamma_k = 0\},$$
$$\beta_1 \triangleq \min\{k : k > \alpha_1, \gamma_k = 1\}, \quad (8)$$
$$\vdots$$
$$\alpha_j \triangleq \min\{k : k > \beta_{j-1}, \gamma_k = 0\},$$
$$\beta_j \triangleq \min\{k : k > \alpha_j, \gamma_k = 1\}, \quad (9)$$

where $\beta_0 = 0$ by convention. It is straightforward to verify that $\{\alpha_j\}_{j\in\mathbb{N}}$ and $\{\beta_j\}_{j\in\mathbb{N}}$ are two sequence of stopping times (see Durrett, 2010).

Definition 2. The Kalman filtering system with packet losses is said to be peak-covariance stable if $\sup_{j\in\mathbb{N}} \mathbb{E}\|P_{\beta_j}\| < \infty$.

The above definition is made regardless of the initial distribution p, due to the following lemma. For any initial distribution p, let $\mathbb{E}^p[\cdot]$ denote the expectation conditioned on p. Thanks to the following lemma, we can omit the superscript from then on.

Lemma 1. *For any initial distributions, p and p_*, of the packet-loss process $\{\tau_j\}_{j\in\mathbb{N}}$, $\sup_{j\in\mathbb{N}} \mathbb{E}^p\|P_{\beta_j}\| < \infty$ if and only if $\sup_{j\in\mathbb{N}} \mathbb{E}^{p_*}\|P_{\beta_j}\| < \infty$*

Proof. Since $\mathbb{E}^p\|P_{\beta_j}\| = \sum_{i=1}^{s} p_i \mathbb{E}[\|P_{\beta_j}\| \,|\tau_1 = i]$, $\sup_{j\in\mathbb{N}} \mathbb{E}^p\|P_{\beta_j}\| < \infty$ if and only if $\mathbb{E}[\|P_{\beta_j}\| \,|\tau_1 = i] < \infty$ for any $i \in \mathcal{S}$. The result follows using the same argument for $\mathbb{E}^{p_*}\|P_{\beta_j}\|$. □

In the literature, stability of Kalman filtering with Markovian packet losses (driven by a two-state Gilbert–Elliott packet loss model) (Huang & Dey, 2007; Xie & Xie, 2008; You et al., 2011) and with i.i.d. packet losses (Sinopoli et al., 2004; Shi et al., 2010) has been intensively studied. The main problem of this work is to study stability of Kalman filtering at packet reception times subject to bounded Markovian packet-loss process. As the packet loss is modeled differently, the stability also behaves differently. Due to the nonlinearity of the Kalman filter, it seems challenging to find necessary and sufficient stability conditions for a general LTI system. In Section 3, we manage to give a sufficient peak-covariance stability condition for general LTI systems with bounded Markovian packet-loss process. Our result is mainly built on the prior work (Xiao et al., 2009). Compared with the result thereof, ours prevails from at least two aspects. We will discuss in detail later in Section 4.

3. Main result

In the following theorem, we will present a sufficient condition for peak-covariance stability of Kalman filtering with bounded Markovian packet-loss process.

Theorem 1. *Consider the system described in (1a) and (1b), and the bounded Markovian packet-loss process described by a probability transition matrix $\mathbf{\Pi}$ in (4). If there exists $K \triangleq [K^{(1)}, \ldots, K^{(\mathsf{l}_o-1)}]$, where $K^{(i)}$'s are matrices with compatible dimensions, such that $\rho(H_K) < 1$, where*

$$H_K \triangleq \operatorname{diag}\Big(A \otimes A, \ldots, (A \otimes A)^s\Big)\left[\mathbf{P}' \otimes \mathbf{H} + \mathbf{Q}' \otimes \mathbf{K}\right], \quad (10)$$

$\mathbf{P} \triangleq [\pi_{ij}]_{i,j\in\mathcal{S}/\{0\}}$ *and* $\mathbf{Q} \triangleq [\pi_{i0}\pi_{0j}]_{i,j\in\mathcal{S}/\{0\}}$,

$$\mathbf{H} \triangleq \overline{(A + K^{(1)}C)} \otimes (A + K^{(1)}C),$$

$$\mathbf{K} \triangleq \sum_{l=2}^{\mathsf{l}_o-1} (\pi_{00})^{l-2}\,\overline{(A^l + K^{(l)}C^{(l)})} \otimes (A^l + K^{(l)}C^{(l)})$$

with

$$C^{(i)} \triangleq [C', A'C', \ldots, (A')^{i-1}C']'; \quad (11)$$

then the state estimator is peak-covariance stable, i.e., $\sup_{j\in\mathbb{N}} \mathbb{E}\|P_{\beta_j}\| < \infty$.

Before proceeding to the proof, we first give a few supporting definitions and lemmas.

Consider k compositions of g together. We introduce the following lemma.

Lemma 2. *Consider the operator*

$$\phi_i(K^{(i)}, P) \triangleq (A^i + K^{(i)}C^{(i)})X(\cdot)^* + [A^{(i)}\ K^{(i)}]\begin{bmatrix} Q^{(i)} & Q^{(i)}(D^{(i)})' \\ D^{(i)}(Q^{(i)}) & D^{(i)}(Q^{(i)})(D^{(i)})' + R^{(i)} \end{bmatrix}[A^{(i)}\ K^{(i)}]^*,$$

where $i \in \mathbb{N}$, $A^{(i)} \triangleq [A^{i-1}, \ldots, A, I]$, $D^{(i)} = 0$ for $i = 1$ otherwise $D^{(i)} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2} & CA^{i-3} & \cdots & 0 \end{bmatrix}$, $Q^{(i)} \triangleq \operatorname{diag}(\underbrace{Q, \ldots, Q}_{i})$, $R^{(i)} \triangleq \operatorname{diag}(\underbrace{R, \ldots, R}_{i})$, *and $K^{(i)}$'s are of compatible dimensions. For any $X \geq 0$ and $K^{(i)}$, the following statement holds:*

$$g^i(X) \triangleq \min_{K^{(i)}} \phi_i(K^{(i)}, X) \leq \phi_i(K^{(i)}, X).$$

Proof. The result is readily established when setting $B = I$ in Lemmas 2 and 3 in Xiao et al. (2009). For $i = 1$, The result is well known as Lemma 1 in Sinopoli et al. (2004). □

The following lemma is about the nonlinearity of g operator: for $k \geq \mathsf{l}_o + 1$, $g^k(X)$ is uniformly bounded no matter what the positive semidefine matrix X is.

Lemma 3 (Huang & Dey, 2007, Lemma 5)**.** *Assume that (A, C) is observable and $(A, Q^{1/2})$ is controllable. Define*

$$\mathbb{S}_0^n \triangleq \{P : 0 \leq P \leq AP_0A' + Q, \text{ for some } P_0 \geq 0\}.$$

Then there exists a constant $\mathsf{L} > 0$ such that

(i) *for any $X \in \mathbb{S}_0^n$, $g^k(X) \leq \mathsf{L}I$ for all $k \geq \mathsf{l}_o$;*
(ii) *for any $X \in \mathbb{S}_+^n$, $g^{k+1}(X) \leq \mathsf{L}I$ for all $k \geq \mathsf{l}_o$.*

According to the definitions of α_j and β_j, we can further define the sojourn times at the state 1 and 0 respectively as follows:

$$\alpha_j^* \triangleq \alpha_j - \beta_{j-1} \in \mathbb{N},$$
$$\beta_j^* \triangleq \beta_j - \alpha_j \in \{1, \dots, s\}.$$

The distributions of the sojourn times α_j^* and β_j^* are given in the following lemma.

Lemma 4 (Xiao et al., 2009, *lemma 4*). *Denote the joint distribution of α_j^* and β_j^* by*

$$\pi(l) \triangleq \mathbb{P}\left(\alpha_1^* = a_1, \beta_1^* = b_1, \dots, \alpha_l^* = a_l, \beta_l^* = b_l\right),$$

for any $\alpha_j \in \mathbb{N}$ and $\beta_j \in \{1, \dots, s\}$. Then it holds that

$$\pi(1) = \begin{cases} p_{b_1}, & \text{if } a_1 = 1; \\ p_0(\pi_{00})^{a_1-2}\pi_{0b_1}, & \text{if } a_1 \geq 2, \end{cases}$$
$$\pi(l+1) = \begin{cases} \pi_{b_l b_{l+1}}\pi(l), & \text{if } a_{l+1} = 1; \\ \pi_{b_l 0}(\pi_{00})^{a_{l+1}-2}\pi_{0b_{l+1}}\pi(l), & \text{if } a_{l+1} \geq 2. \end{cases}$$

Proof of Theorem 1. Compute $\mathbb{E}[P_{\beta_1}]$ as follows:

$$\begin{aligned}
\mathbb{E}[P_{\beta_1}] &= \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s} P_{\beta_1}\pi(1) \\
&= \sum_{b_1=1}^{s} p_{b_1} h^{b_1} \circ g(\Sigma_0) \\
&\quad + \sum_{a_1=2}^{\mathrm{l_o}}\sum_{b_1=1}^{s} p_0(\pi_{00})^{a_1-2}\pi_{0b_1} h^{b_1} \circ g^{a_1}(\Sigma_0) \\
&\quad + \sum_{a_1=\mathrm{l_o}+1}^{\infty}\sum_{b_1=1}^{s} p_0(\pi_{00})^{a_1-2}\pi_{0b_1} h^{b_1} \circ g^{a_1}(\Sigma_0) \\
&\leq \sum_{b_1=1}^{s} p_{b_1} A^{b_1}(A + K^{(1)}C)\Sigma_0(A + K^{(1)}C)^*(A^{b_1})' \\
&\quad + \sum_{b_1=1}^{s} A^{b_1}\left[\sum_{a_1=2}^{\mathrm{l_o}} p_0(\pi_{00})^{a_1-2}\,\pi_{0b_1} \right. \\
&\quad \left. \times (A^{a_1} + K^{(a_1)}C^{(a_1)})\Sigma_0(\,\cdot\,)^*\right](A^{b_1})' \\
&\quad + \sum_{b_1=1}^{s} p_{b_1} A^{b_1}\left([A\ K^{(1)}]J_1[A\ K^{(1)}]^*\right)(A^{b_1})' \\
&\quad + \sum_{b_1=1}^{s}\sum_{a_1=2}^{\mathrm{l_o}} p_0(\pi_{00})^{a_1-2}\pi_{0b_1} \\
&\quad \times A^{b_1}\left([A^{a_1}\ K^{(a_1)}]J_{a_1}[A^{a_1}\ K^{(a_1)}]^*\right)(A^{b_1})' \\
&\quad + \sum_{b_1=1}^{s}\left(\sum_{a_1=2}^{\mathrm{l_o}} p_0(\pi_{00})^{a_1-2}\pi_{0b_1} + p_{b_1}\right)\sum_{i=0}^{b_1-1} A^i Q(A^i)' \\
&\quad + \sum_{a_1=\mathrm{l_o}+1}^{\infty}\sum_{b_1=1}^{s} p_0(\pi_{00})^{a_1-2}\pi_{0b_1} h^{b_1}(\mathrm{L}I) \\
&\triangleq \Lambda_1 + \Lambda_2 + \Lambda_3 + \Lambda_4 + \Lambda_5 + \Lambda_6,
\end{aligned} \tag{12}$$

where $J_i \triangleq \begin{bmatrix} Q^{(i)} & Q^{(i)}(D^{(i)})' \\ D^{(i)}(Q^{(i)}) & D^{(i)}(Q^{(i)})(D^{(i)})' + R^{(i)} \end{bmatrix}$, the partition of Λ_1 to Λ_6 is based on whether or not $g^{k+1}(X) \leq \mathrm{L}I$ has a uniform bound when $k \geq \mathrm{l_o}$ or $k \leq \mathrm{l_o} - 1$ by Lemma 3 and the distribution of $\pi(1)$ given in Lemma 4, and the inequality is from Lemmas 2 and 3. One can verify that Λ_3, Λ_4, Λ_5 and Λ_6 are all bounded matrices. Then $U \triangleq \Lambda_3 + \Lambda_4 + \Lambda_5 + \Lambda_6$ is also bounded. To facilitate the analysis in the following, we will impose (12) to take equality. Without loss of generality, the conclusions in this paper still hold without imposing equality as (12) renders us an upper bound of $\mathbb{E}[P_{\beta_1}]$. Next we vectorize both sides of (12).

$$\begin{aligned}
\mathbb{E}[\mathrm{vec}(P_{\beta_1})] &= \sum_{b_1=1}^{s}(A \otimes A)^{b_1}\Big[p_{b_1}\overline{(A + K^{(1)}C)} \otimes (A + K^{(1)}C) \\
&\quad + \sum_{a_1=2}^{\mathrm{l_o}} p_0(\pi_{00})^{a_1-2}\pi_{0b_1}\overline{(A^{a_1} + K^{(a_1)}C^{(a_1)})} \\
&\quad \otimes (A^{a_1} + K^{(a_1)}C^{(a_1)})\Big]\mathrm{vec}(\Sigma_0) + \mathrm{vec}(U) \\
&= T\Psi\,\mathrm{vec}(\Sigma_0) + \mathrm{vec}(U),
\end{aligned} \tag{13}$$

where

$$T = [\underbrace{1, \dots, 1}_{s \text{ numbers}}] \otimes I_{n^2 \times n^2}$$

and $\Psi = [\psi_1', \dots, \psi_s']' \in \mathbb{C}^{sn^2 \times n^2}$ with

$$\begin{aligned}
\psi_i = (A \otimes A)^i\Big[&p_1\overline{(A + K^{(1)}C)} \otimes (A + K^{(1)}C) \\
&+ \sum_{a_1=2}^{\mathrm{l_o}} p_0(\pi_{00})^{a_1-2}\pi_{01}\overline{(A^{a_1} + K^{(a_1)}C^{(a_1)})} \\
&\otimes (A^{a_1} + K^{(a_1)}C^{(a_1)})\Big]
\end{aligned}$$

for $i \in \{1, \dots, s\}$.

Similarly, for any $l \geq 1$, $\mathbb{E}[P_{\beta_{l+1}}]$ can be calculated as

$$\begin{aligned}
\mathbb{E}[P_{\beta_{l+1}}] &= \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{a_{l+1}=1}^{\infty}\sum_{b_{l+1}=1}^{s} P_{\beta_{l+1}}\pi(l+1) \\
&= \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{a_{l+1}=\mathrm{l_o}}^{\infty}\sum_{b_{l+1}=1}^{s} P_{\beta_{j+1}}\pi(l+1) \\
&\quad + \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{a_{l+1}=2}^{\mathrm{l_o}-1}\sum_{b_{l+1}=1}^{s} \pi_{b_l 0}(\pi_{00})^{a_{l+1}-2}\pi_{0b_{l+1}} \\
&\quad \cdot h^{b_{l+1}} \circ g^{a_{l+1}}(P_{\beta_l})\pi(l) \\
&\quad + \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{b_l=1}^{s}\sum_{b_{l+1}=1}^{s} \pi_{b_l b_{l+1}} h^{b_{l+1}} \circ g(P_{\beta_l})\pi(l) \\
&\triangleq \Gamma_1 + \Gamma_2 + \Gamma_3.
\end{aligned}$$

Next we will analyze the boundedness of Γ_1, Γ_2 and Γ_3 one by one.

$$\Gamma_1 \leq \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{a_{l+1}=\mathrm{l_o}}^{\infty}\sum_{b_{l+1}=1}^{s} h^{b_{l+1}}(\mathrm{L}I)\pi(l+1) \triangleq W_1, \tag{14}$$

where the inequality is derived from Lemma 3 and W_1 is a bounded matrix.

$$\begin{aligned}
\Gamma_2 &\leq \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{a_{l+1}=2}^{\mathrm{l_o}-1}\sum_{b_{l+1}=1}^{s} \pi_{b_l 0}(\pi_{00})^{a_{l+1}-2}\pi_{0b_{l+1}} h^{b_{l+1}} \\
&\quad \circ \phi_{a_{l+1}}(K^{(a_{l+1})}, P_{\beta_l})\pi(l) \\
&= \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{a_{l+1}=2}^{\mathrm{l_o}-1}\sum_{b_{l+1}=1}^{s} \pi_{b_l 0}(\pi_{00})^{a_{l+1}-2}\pi_{0b_{l+1}} \\
&\quad \times A^{b_{l+1}}(A^{a_{l+1}} + K^{(a_{l+1})}C^{a_{l+1}})P_{\beta_l}(\,\cdot\,)^*(A^{b_{l+1}})'\pi(l) \\
&\quad + \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{a_{l+1}=2}^{\mathrm{l_o}-2}\sum_{b_{l+1}=1}^{s} \pi_{b_l 0}(\pi_{00})^{a_{l+1}-2}\pi_{0b_{l+1}}
\end{aligned}$$

$$
\begin{aligned}
&\times \sum_{i=0}^{b_{l+1}-1} A^i Q (A^i)' \pi(l) \\
&+ \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{a_{l+1}=2}^{\mathrm{l_o}-1}\sum_{b_{l+1}=1}^{s} \pi_{b_l 0}(\pi_{00})^{a_{l+1}-2}\pi_{0 b_{l+1}} \\
&\times A^{b_{l+1}}\left([A^{a_{l+1}}\ K^{(a_{l+1})}]J_{a_{l+1}}[\,\cdot\,]^*\right)(A^{b_{l+1}})'\pi(l) \\
&\triangleq \Gamma_2' + W_2 + W_3.
\end{aligned}
$$

It is straightforward to verify that W_2 and W_3 is bounded.

$$
\begin{aligned}
\Gamma_3 &\le \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{b_{l+1}=1}^{s} \pi_{b_l b_{l+1}} h^{b_{l+1}} \circ \phi_1\big(K^{(1)}, P_{\beta_l}\big)\pi(l) \\
&= \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{b_{l+1}=1}^{s} \pi_{b_l b_{l+1}} \\
&\quad \times A^{b_{l+1}}(A+K^{(1)}C)P_{\beta_l}(\,\cdot\,)^*(A^{b_{l+1}})'\pi(l) \\
&\quad + \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{b_{l+1}=1}^{s} \pi_{b_l b_{l+1}} \sum_{i=0}^{b_{l+1}-1} A^i Q (A^i)'\pi(l) \\
&\quad + \sum_{a_1=1}^{\infty}\sum_{b_1=1}^{s}\cdots\sum_{b_{l+1}=1}^{s} \pi_{b_l 0}\pi_{b_l b_{l+1}} \\
&\quad \times A^{b_{l+1}}\left([A\ \ K^{(1)}]J_1\,[A\ \ K^{(1)}]^*\right)(A^{b_{l+1}})'\pi(l) \\
&\triangleq \Gamma_3' + W_4 + W_5,
\end{aligned}
$$

where W_4 and W_5 can be readily shown to be bounded. In summary,

$$\mathbb{E}[P_{l+1}] \le \Gamma_2' + \Gamma_3' + V, \quad \text{for } j \ge 1, \tag{15}$$

where $V \triangleq W_1+W_2+W_3+W_4+W_5$. By a similar argument, we impose (15) to take equality and take vectorization. By (13) and (15), we calculate $\mathbb{E}[\mathrm{vec}(P_{\beta_{l+1}})]$ and convert it to the following form:

$$
\begin{aligned}
\mathbb{E}[\mathrm{vec}(P_{\beta_{l+1}})] =& T(H_K)^l \Psi \mathrm{vec}(\Sigma_0) + T(H_K)^l \mathrm{vec}(\Theta_l) \\
&+ T(H_K)^{l-1}\mathrm{vec}(\Theta_{l-1}) + \cdots + \mathrm{vec}(\Theta_0),
\end{aligned}
$$

where $\Theta_0, \ldots, \Theta_l$ are the functions of $Q, A, K^{(i)}$'s and are bounded. Similar to U and V, $\Theta_0, \ldots, \Theta_l$ can be readily shown to be bounded in a similar way. Therefore, $\sup_{k\le l}\mathbb{E}[\mathrm{vec}(P_{\beta_k})]$ is bounded as $l \to \infty$ if $\rho(H_K) < 1$. By some basic algebraic manipulations, one obtains that $\mathbb{E}\|P_{\beta_{l+1}}\|$ is uniformly bounded if $\rho(H_K) < 1$, which completes the proof. □

The stability condition in Theorem 1 is difficult to test. In the following proposition, we provide an equivalent condition. The proof has been omitted for the sake of limited space and can be found in the extended version of this work (see http://arxiv.org/abs/1501.05469). In view of this result, Theorem 1 can be recast as an LMI feasibility problem. As for the conversion to LMIs using Schur complements, we refer readers to Boyd, El Ghaoui, and Feron (1994) for details.

Proposition 1. *The following statements are equivalent:*

(i) *There exists $K \triangleq [K^{(1)}, \ldots, K^{(\mathrm{l_o}-1)}]$, where $K^{(i)}$'s are matrices with compatible dimensions, such that $\rho(H_K) < 1$, where H_K is defined in (10);*
(ii) *There exist $X_1 > 0, \ldots, X_s > 0$ and $K_1, \ldots, K_{\mathrm{l_o}-1}$ such that*

$$
\begin{aligned}
&\sum_{i=1}^{s}\pi_{ij}A^j\Delta_1 X_i \Delta_1^*(A^j)' + \sum_{i=1}^{s}\pi_{i0}\pi_{0j}\sum_{l=2}^{\mathrm{l_o}-1}(\pi_{00})^{l-2}A^j\Delta_l X_i\Delta_l^*(A^j)' \\
&\quad < X_j
\end{aligned}
$$

for all $j \in \mathcal{S}/\{0\}$, where $\Delta_l = A^l + K_l C^{(l)}$ with $C^{(l)}$ defined in (11).

It is worth noting the one-step observable (A, C) system, see Definition 1. In this case, we can find K_1 such that $A + K_1C = 0$. By Proposition 1, any $X > 0$ will automatically satisfy condition (ii), thereby implying that the Kalman filtering system with bounded Markovian packet losses is always peak-covariance stable. This observation is consistent with Xiao et al. (2009, Theorem 3.1).

4. Comparison with Xiao et al. (2009)

In this part, we compare our result with that of Xiao et al. (2009) and show the advantages of ours. Recall that the sufficient stability condition in Xiao et al. (2009) is $\rho(\Phi) < 1$ where

$$\Phi \triangleq \left[d_1^{(1)}\mathbf{P} + \sum_{l=2}^{\mathrm{l_o}-1}(\pi_{00})^{l-1}d_l^{(1)}\mathbf{Q}\right]\mathrm{diag}\left(\|A\|^2, \ldots, \|A^s\|^2\right),$$

with $\mathbf{P}$, $\mathbf{Q}$ being defined in (10) and $d_l^{(1)} \triangleq \min_{K^{(l)}}\|A^{(l)} + K^{(l)}C^{(l)}\|^2$.

4.1. Invariance with respect to similarity transformations

Theoretically, a state variable transformation (i.e., a similarity transformation from a linear system (A, B, C, D) to $(S^{-1}AS, S^{-1}B, CS, D)$ through the nonsingular matrix S) does not change the stability considered in this work. However, different state variable transformations may generate opposite conclusions from the stability condition given in Xiao et al. (2009). The invariance of stability behavior with respect to state variable transformations can be reflected well from the stability conditions presented by this work.

Proposition 2. *Let $S \in \mathbb{C}^{n\times n}$ be nonsingular. Suppose there exists $K \triangleq [K^{(1)}, \ldots, K^{(\mathrm{l_o}-1)}]$, where $K^{(i)}$'s are matrices with compatible dimensions, such that $\rho(H_K) < 1$, where H_K is defined in (10) for (A, C). Then, there always exists $\tilde{K} \triangleq S^{-1}[K^{(1)}, \ldots, K^{(\mathrm{l_o}-1)}]$ such that $\rho(\tilde{H}_{\tilde{K}}) < 1$, where $\tilde{H}_{\tilde{K}}$ is defined for $(\tilde{A}, \tilde{C}) \triangleq (S^{-1}AS, CS)$ in accordance with (10).*

The proof follows from Proposition 1 and direct calculation. We use the following example to illustrate this idea.

Example 1. Consider the system

$$A = \begin{bmatrix} 1.3 & 0.3 \\ 0 & 1.2 \end{bmatrix}, \qquad C = [\,1\ 1\,].$$

$Q = I_{2\times 2}$ and $R = 1$, and the bounded Markovian packet-loss process with transition probability matrix given by

$$\mathbf{\Pi} = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.8 & 0.1 & 0.1 \\ 0.8 & 0.1 & 0.1 \end{bmatrix}. \tag{16}$$

From Xiao et al. (2009), we have $d_1^{(1)} = 1.2200$ and $\rho(\Phi) = 0.7352 < 1$. Let

$$S = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}.$$

For the system $(\tilde{A}, \tilde{C}) \triangleq (S^{-1}AS, CS)$, we have $\tilde{d}_1^{(1)} = 1.3632$ and $\rho(\tilde{\Phi}) = 1.5202 > 1$.

4.2. Conservativity comparison

The stability condition given in this work is less conservative compared with that in Xiao et al. (2009), since the latter condition implies the former one. To show this, we need the following proposition.

Proposition 3. *Define*

$$\Phi_K \triangleq \left[d_1\mathbf{P} + \sum_{l=2}^{l_o-1}(\pi_{00})^{l-1} d_l\mathbf{Q} \right] \mathrm{diag}\left(\|A\|^2, \ldots, \|A^s\|^2 \right),$$

where $\mathbf{P}$ *and* $\mathbf{Q}$ *are defined in* (10) *and* $d_l \triangleq \|A^{(l)} + K^{(l)}C^{(l)}\|^2$, *and* $K \triangleq [K^{(1)}, \ldots, K^{(l_o-1)}]$ *with* $K^{(l)}$*'s of compatible dimensions. If there exists* K *such that* $\rho(\Phi_K) < 1$, *then* $\rho(H_K) < 1$.

Proof. If treating a scalar as the Kronecker product of two other scalars, similar to Proposition 1, the condition $\rho({\Phi_K}') < 1$ is equivalent to that there exists a vector

$$x \triangleq [x_1, \ldots, x_s],$$

where $x_j > 0$ for all $j \in \mathcal{S}/\{0\}$, such that

$$\sum_{i=1}^{s} \pi_{i0}\pi_{0j} \sum_{l=2}^{l_o-1} (\pi_{00})^{l-2} d_l \|A^j\|^2 x_i + \sum_{i=1}^{s} \pi_{ij} d_1 \|A^j\|^2 x_i < x_j.$$

The submultiplicativity and subadditivity of a matrix norm result in the following inequality:

$$\Bigg\| \sum_{i=1}^{s} \pi_{i0}\pi_{0j} \sum_{l=2}^{l_o-1} (\pi_{00})^{l-1} x_i A^j (A^l + K^{(l)}C^{(l)})(\cdot)^*(A^j)' + \sum_{i=1}^{s} \pi_{ij} x_i A^j (A + K^{(1)}C)(A + K^{(1)}C)^*(A^j)' \Bigg\| < x_j \tag{17}$$

for all $j \in \mathcal{S}/\{0\}$. Let $X_j = x_j I_{n\times n}$. Then we obtain from (17) that

$$\sum_{i=1}^{s} \pi_{i0}\pi_{0j} \sum_{l=2}^{l_o-1} (\pi_{00})^{l-2} A^j (A^l + K^{(l)}C^{(l)}) X_j (\cdot)^*(A^j)' + \sum_{i=1}^{s} \pi_{ij} A^j (A + K^{(1)}C) X_i (A + K^{(1)}C)^*(A^j)' < X_j.$$

Therefore $\rho(H_K) < 1$, which completes the proof. □

In virtue of Proposition 3, it is evident that $\rho(\Phi) < 1$ implies $\rho(H_{K^\star}) < 1$, where $K^\star \triangleq [K_1^\star, \ldots, K_{l_o-1}^\star]$ with $K_i^\star \triangleq \arg\min_{K^{(i)}} \|A^{(i)} + K^{(i)}C^{(i)}\|^2$.

Example 1 (*Cont'd*)**.** We continue to consider Example 1 with an alternative transition probability matrix

$$\mathbf{\Pi}_1 = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.6 & 0.2 & 0.2 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}.$$

From Xiao et al. (2009, Theorem 2), we obtain $\rho(\Phi) = 1.4704 > 1$. By solving an LMI feasibility problem using the **cvx** in Matlab, we see that the condition in Theorem 1 still holds with a group of feasible matrices

$$X_1 = X_2 = \begin{bmatrix} 0.1081 & 0.0243 \\ 0.0243 & 0.1042 \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} -0.8079 \\ -0.5914 \end{bmatrix}.$$

If we consider the transition probability matrix, which only allows the maximum length of consecutive packet losses to be 1, i.e.,

$$\mathbf{\Pi}_2 = \begin{bmatrix} 0.6 & 0.4 \\ 0.8 & 0.2 \end{bmatrix},$$

then $\rho(\Phi) = 0.49 < 1$ and the condition in our Theorem 1 holds. When we increase π_{11} in $\mathbf{\Pi}_2$ from 0.2 to 0.5, one can verify numerically that $\rho(\Phi) > 1$ while the condition in Theorem 1 still holds.

5. Conclusion

We have considered the bounded Markovian packet-loss process model and the notion of the peak-covariance stability for the Kalman filtering. A sufficient stability condition with bounded Markovian packet losses was established. Different from that of Xiao et al. (2009), this stability check can be recast as an LMI feasibility problem. Then we compared the proposed condition with that of Xiao et al. (2009), showing that our condition prevails from at least two aspects: (1) Our stability condition is invariant with respect to similarity state transformations, while the previous result is not; (2) Our condition is proved to be less conservative than the previous one. Numerical examples were provided to demonstrate the effectiveness of our result compared with the literature.

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References

Antsaklis, Panos J., & Michel, Anthony N. (2006). *Linear systems*. Springer.
Boyd, Stephen P., El Ghaoui, Laurent, Feron, Eric, & Balakrishnan, Venkataramanan (1994). *Linear matrix inequalities in system and control theory. Vol. 15*. SIAM.
Durrett, Rick (2010). *Probability: theory and examples. Vol. 3*. Cambridge University Press.
Elliott, E. O. (1963). Estimates of error rates for codes on burst-noise channels. *Bell System Technical Journal*, *42*(5), 1977–1997.
Gilbert, Edgar N. (1960). Capacity of a burst-noise channel. *Bell System Technical Journal*, *39*(5), 1253–1265.
Huang, M., & Dey, S. (2007). Stability of Kalman filtering with Markovian packet losses. *Automatica*, *43*, 598–607.
Kar, Soummya, Sinopoli, Bruno, & Moura, José M. F. (2012). Kalman filtering with intermittent observations: Weak convergence to a stationary distribution. *IEEE Transactions on Automatic Control*, *57*(2), 405–420.
Mo, Yilin, & Sinopoli, Bruno (2010). Towards finding the critical value for Kalman filtering with intermittent observations. ArXiv Preprint arXiv:1005.2442.
Mo, Yilin, & Sinopoli, B. (2012). Kalman filtering with intermittent observations: Tail distribution and critical value. *IEEE Transactions on Automatic Control*, *57*(3), 677–689.
Plarre, K., & Bullo, F. (2009). On Kalman filtering for detectable systems with intermittent observations. *IEEE Transactions on Automatic Control*, *54*(2), 386–390.
Shi, L., Epstein, M., & Murray, R. M. (2010). Kalman filtering over a packet-dropping network: A probabilistic perspective. *IEEE Transactions on Automatic Control*, *55*(9), 594–604.
Sinopoli, B., Schenato, L., Franceschetti, M., Poola, K., Jordan, M., & Sastry, S. (2004). Kalman filtering with intermittent observations. *IEEE Transactions on Automatic Control*, *49*(9), 1453–1464.
Wu, Jing, & Chen, Tongwen (2007). Design of networked control systems with packet dropouts. *IEEE Transactions on Automatic Control*, *52*(7), 1314–1319.
Wu, Junfeng, Shi, Guodong, Anderson, Brian D. O., & Johansson, Karl Henrik (0000). Kalman filtering over Gilbert–Elliott channels: Stability conditions and the critical curve. *IEEE Transactions on Automatic Control*, submitted for publication. ArXiv Preprint arXiv:1411.1217.
Xiao, Nan, Xie, Lihua, & Fu, Minyue (2009). Kalman filtering over unreliable communication networks with bounded Markovian packet dropouts. *International Journal of Robust and Nonlinear Control*, *19*(16), 1770–1786.
Xie, Li (2012). Stochastic comparison, boundedness, weak convergence, and ergodicity of a random Riccati equation with Markovian binary switching. *SIAM Journal on Control and Optimization*, *50*(1), 532–558.
Xie, Li, & Xie, Lihua (2007). Peak covariance stability of a random Riccati equation arising from Kalman filtering with observation losses. *Journal of Systems Science and Complexity*, *20*(2), 262–272.
Xie, Li, & Xie, Lihua (2008). Stability of a random Riccati equation with Markovian binary switching. *IEEE Transactions on Automatic Control*, *53*(7), 1759–1764.
Xiong, Junlin, & Lam, James (2007). Stabilization of linear systems over networks with bounded packet loss. *Automatica*, *43*(1), 80–87.
You, Keyou, Fu, Minyue, & Xie, Lihua (2011). Mean square stability for Kalman filtering with Markovian packet losses. *Automatica*, *47*(12), 2647–2657.

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