Multi-Sensor-Based Aperiodic Least-Squares Estimation for Networked Systems With Transmission Constraints

Haiyu Song, Wen-An Zhang, Li Yu, and Ling Shi

***Abstract*—This paper investigates the least-squares estimation problem for networked systems with transmission constraints. A group of sensors are deployed to measure the outputs of a plant and send the measurements to an estimator through a common communication channel. Due to the transmission constraints caused by the heterogenous or long-distance deployed sensors, only one sensor is allowed to transmit its measurement over one time slot. In this regard, a stochastic competitive transmission strategy is proposed to schedule the transmission permissions. By using the least-squares estimation approach, an aperiodic multi-step estimation algorithm is proposed for the estimator to aperiodically generate the estimates. Performance analysis is presented for the estimation system with bounded noises and random noises. An upper bound is derived for the expectation of the estimation error and a sufficient condition is presented to ensure the convergence of the obtained upper bound. An illustrative example is provided to demonstrate the effectiveness of the proposed results.**

***Index Terms*—Least-squares estimation, multi-sensor-based estimation, networked systems, stochastic competitive transmission, transmission constraints.**

I. INTRODUCTION

A multi-sensor-based estimation system is composed of several spatially distributed sensors with abilities of sensing and communication. Generally speaking, the use of a multi-sensor-based estimating structure is motivated by the following: 1) Outputs of a plant have to be observed by heterogenous sensors [1]–[3]; 2) The output terminals are long-distance deployed, or a plant consists of several separated parts and the outputs cannot be measured by one sensor [4]–[6]; 3) The robustness and estimation precision of a multi-sensor-based estimation system are much better than that of a single-sensor-based one [7]–[9]. Over the past decades, the multi-sensor-based estimation problem has attracted increasing research attention due to its wide applications in areas such as target tracking, battlefield surveillance, traffic monitoring, etc [10]–[12].

In a typical centralized estimation structure, a number of sensors perform sensing and data communication, while an estimator provides data processing. During two consecutive estimating instants, an estimator collects sampled information from the deployed sensors, and generates estimates at the end of the estimating interval using prior estimates and the received measurements. In a general state estimation problem, only one estimate is provided by the estimator at any estimating instant. Recently, several estimation algorithms have been presented for the multi-step estimation [13]–[17], i.e., the estimator is expected to generate more than one estimate at each estimating instant. There are two general classes of estimation schemes: periodic estimation and aperiodic estimation. In the former case, the estimator computes estimates with a fixed period, while the lengths of the estimating interval in the latter case are not necessarily the same. In those dynamic processes with low state updating rates, the states evolve very slowly during some time intervals. In this scenario, it is not necessary to frequently carry out the estimation and the estimating instants are not necessary periodic. To the best of the authors' knowledge, we are the first to address the aperiodic multi-step estimation problem, which gives us the first motivation in this paper.

Between two consecutive estimating instants, the sensors are able to convey the sampled information to the estimator through the communication channel. Note that it is wasteful and complicated to assign each sensor an individual communication channel. Letting all the sensors share a common communication channel is more economical and practical. When the sensors are heterogenous or long-distance deployed, it is impossible to encapsulate all the measurements from different sensors into one packet. In this scenario, the estimator is able to receive information from only one sensor over one time-slot. In other words, in a single-channel environment, any two sensors cannot send their measurements to the estimator simultaneously. This phenomenon is usually termed as transmission constraints or communication constraints. Recently, some results have been presented for estimation with the communication constraints, see for example, [17]–[20]. By minimizing the expected steady state estimation error covariance, a stochastic sensor selection strategy was proposed in [21] such that only one sensor takes a measurement at every time step. In [22], a multi-step sensor

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selection strategy was proposed to schedule sensors such that only a subset of them are able to send their observations to the fusion center at each time step. In [23], an optimal probabilistic measurement-independent strategy was derived to decide when to transmit estimates from each sensor. Two optimal sensor schedules are obtained in [24] by minimizing the terminal error covariance and the average error covariance of the estimator, respectively. In [25], an explicit construction of an optimal periodic sensor schedule is provided for two Gauss-Markov systems by minimizing the sum of the average estimation error covariance of each system. The sensor scheduling problems under limited communication resource can also be found in [26]–[28] and the references therein. According to the communication schemes considered in the literature, only one communication (or several sensors communicate with the estimator synchronously for one time) happens during each estimating interval, which implies that only the sampled information observed at current time step can be used by the estimator for data processing. A possible improvement is to supply the estimator with sampled information as much as possible before the estimating instant begins. Following this idea, the sensors can communicate with the estimator several times and thus more sampled information can be received by the estimator as long as the sensors' sampling frequency and transmitting frequency are not limited. In this multiple communications scenario, a critical issue is how to schedule the transmission permissions during each estimating interval, which is our second motivation to propose a competitive transmission strategy.

In this paper, the aperiodic multi-step estimation problem with transmission constraints is investigated. During each estimating interval, the sensors independently measure the outputs of a plant and send the measurements to an estimator through a common communication channel. At each estimating instant, the estimator runs a multi-sensor-based estimation algorithm to estimate the state of the plant during the current estimating interval. The main contributions of this paper are summarized as follows:

1) The transmission constraint is mathematically formulated, where the constraint conditions consist of two parts: firstly, only one transmission is allowed to take place at any specified time instant; secondly, each sensor has only one chance to obtain the transmission permission during each estimating interval for the sake of fairness.
2) To avoid the transmission constraint, a stochastic competitive transmission (SCT) strategy is proposed to schedule the transmission permissions during each estimating interval, where the transmission permissions are assigned randomly according to a conditional probability.
3) Note that the proposed SCT strategy is actually a strictly asynchronous communication mechanism by which the measurements from different sensors may not be received by the estimator simultaneously. In view of this, an aperiodic storage strategy is presented for each sensor to maintain its measurement if it has not obtained a transmission permission at that time instant.
4) An aperiodic least-squares estimation algorithm is proposed for the estimator to aperiodically generate estimates. Performance analysis is presented for the estimation system with both bounded and zero-mean random noises. Moreover, an upper bound of the estimation error is derived and a sufficient condition is provided to ensure the convergence of the proposed upper bound.

The remainder of the paper is organized as follows. Section II is devoted to the problem formulation. The analysis and synthesis for the multi-sensor-based aperiodic multi-step estimation algorithm are given in Section III. In Section IV, an illustrative example is provided to demonstrate the effectiveness of the proposed theoretical results. Finally, the conclusion is given in Section V.

Notation: $\mathbb{R}^r$ denotes the r-dimensional Euclidean space, the symbol T is the transpose, $\mathbf{E}\{\cdot\}$ is the mathematical expectation, $\mathrm{Prob}\{\cdot\}$ stands for the occurrence probability of the event "$\cdot$", $\mathrm{diag}\{\cdot\}$ denotes a block-diagonal matrix, $\mathrm{col}\{a_1,\ldots,a_r\} = [a_1^{\mathrm{T}},\ldots,a_r^{\mathrm{T}}]^{\mathrm{T}}$, where $a_1,\ldots,a_r$ can be scalars, vectors or matrices of appropriate dimensions. $\|A\|$ represents the Euclidian norm of the matrix or vector A, $\lambda_{\min}(B)$ stands for the minimum eigenvalue of the real symmetric matrix B. For any integers r_1 and r_2, assume that $\sum_{r=r_1}^{r_2}(\cdot)=0$ if $r_2<r_1$

II. Problem Formulation

Consider a linear discrete-time stochastic system described by the following state-space model:

$$\begin{cases} x(t+\Delta) = Ax(t) + Bw(t) \\ y_i(t) = C_i x(t) + D_i v_i(t), i = 1,2,\ldots,n \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^p$ is the state, Δ is the updating period of the state, $y_i(t) \in \mathbb{R}$ is the ith measured output, $w(t) \in \mathbb{R}$ and $v_i(t) \in \mathbb{R}$ are the process noise and the measurement noise, respectively. A, B, C_i and D_i are known real matrices of appropriate dimensions. A set of n sensors are deployed to observe the n measured outputs. In practice, the sensor failure phenomenon is unavoidable due to presence of obstacles, bad weather or other unreliable sensing environment [29]–[35]. During the sampling processes, the random failures are described by binary-valued random variables as follows:

$$\theta_i(t) = \begin{cases} 1, & \text{if failure does not occur in sensor } i \text{ at time } t; \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

where $\mathrm{Prob}\{\theta_i(t)=1\}=\lambda_i$, $\mathrm{Prob}\{\theta_i(t)=0\}=1-\lambda_i$, $\lambda_i\in[0,1]$ and $1-\lambda_i$ is called as the sensor failure rate. In this paper, the variables $\theta_i(t), i=1,2,\ldots,n$, are assumed to be independent of each other.

The n sensors communicate with a remote estimator via a common communication channel. Denote by $\{t_{k+1} : t_{k+1} - t_k = m_k\Delta, k = 0,1,2,\ldots\}$ the set of the estimator's estimating instants, where $t_0 = 0$, $m_k \in M$ is an integer and M is the set of the lengths of the estimating intervals[1]. The estimator collects the sampled information from the sensors in between two consecutive estimating instants, and carries out the estimation at the end of the estimating interval. Denote by $\mathcal{S}_k^{(i)}$ the estimator's received sampled information from sensor i during $(t_k, t_{k+1}]$. The overall structure of the estimation system is shown in Fig. 1, where $\theta_{k,j}^{(i)} = \theta_i(t_k + j\Delta)$ and $y_{k,j}^{(i)} = y_i(t_k +$

[1]In this paper, we assume that the estimating instants of the estimator are known and M is a subset of the integers. Moreover, we also assume that $m_k < \infty$, because $m_k = \infty$ implies that the $(k+1)$th estimating interval has a length of ∞, which is impossible in practice.

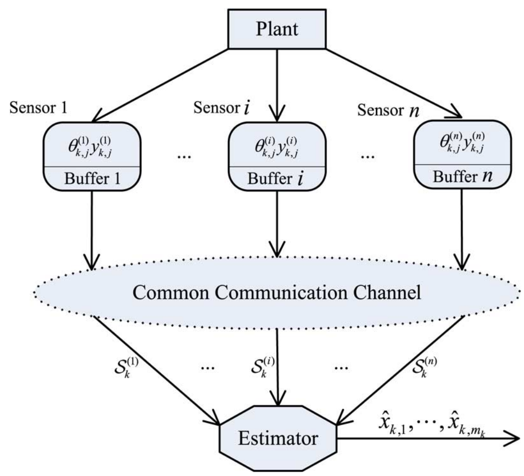

Fig. 1. The structure of the estimation system.

$j\Delta$). At the estimating instant t_{k+1}, the estimator is expected to generate m_k state estimates $\hat{x}_{k,1},\ldots,\hat{x}_{k,m_k}$ of the state vectors $x_{k,1},\ldots,x_{k,m_k}$, by using the received sampled information $\mathcal{S}_k^{(1)},\ldots,\mathcal{S}_k^{(n)}$, where $x_{k,j}=x(t_k+j\Delta)$, $j=1,2,\ldots,m_k$.

During each estimating interval, the n sensors are able to send their measurements to the estimator via the common communication channel. Due to the transmission constraint, however, only one transmission is allowed at any time instant. In view of this, the following random variables are introduced to describe the transmission situations in each sensor:

$$\sigma_{k,j}^{(i)}=\begin{cases}1, & \text{if sensor } i \text{ obtains the transmission permission at time } t_k+j\Delta;\\ 0, & \text{otherwise.}\end{cases} \tag{3}$$

In this paper, the transmission permissions are assigned in a competitive way. The architecture of the multi-sensor-based multi-step estimation process with competitive transmissions is shown in Fig. 2. Denote by $\mathcal{T}_{k,j}$ the set of those sensors that have obtained a transmission permission during $(t_k,t_k+j\Delta]$, $j=1,2,\ldots,m_k$, where $\mathcal{T}_{k,0}=\varnothing$. Then a stochastic competitive transmission strategy is introduced as follows.

Definition 1 (Stochastic Competitive Transmission, SCT): For a set of n sensors that contest the transmission permissions randomly with the following two constraints $\mathcal{C}_1$ and $\mathcal{C}_2$:

$$\mathcal{C}_1:\left(\sum_{j=\tilde{m}_k}^{m_k}\sigma_{k,j}^{(i)}\right)\in\{0,1\} \tag{4}$$

$$\mathcal{C}_2:\left(\sum_{i=1}^{n}\sigma_{k,j}^{(i)}\right)=\begin{cases}1, & \text{if } j\in\{\tilde{m}_k,\tilde{m}_k+1,\ldots,m_k\};\\ 0, & \text{otherwise.}\end{cases} \tag{5}$$

where $\tilde{m}_k=\max\{1,m_k-n+1\}$. The probability of obtaining a transmission permission is given by:

a) For $s\in\{1,2,\ldots,\tilde{m}_k-1\}$, $\text{Prob}\{\sigma_{k,s}^{(i)}=1|\mathcal{C}_1,\mathcal{C}_2\}=0$.

b) For $s\in\{\tilde{m}_k,\tilde{m}_k+1,\ldots,m_k\}$, $\text{Prob}\{\sigma_{k,s}^{(i)}=1|\mathcal{C}_1,\mathcal{C}_2,i\notin\mathcal{T}_{k,s-1}\}=\frac{1}{n-s+\tilde{m}_k}$.

c) For $s\in\{\tilde{m}_k+1,\tilde{m}_k+2,\ldots,m_k\}$, $\text{Prob}\{\sigma_{k,s}^{(i)}=1|\mathcal{C}_1,\mathcal{C}_2,i\in\mathcal{T}_{k,s-1}\}=0$.

Remark 1: The content of the SCT strategy consists of two parts: first, only one transmission is allowed at any time instant; second, each sensor has at most one chance to obtain the transmission permission during each estimating interval for the sake of fairness. When $n<m_k$, it follows from the setting in case a) that $\sigma_{k,s}^{(i)}\equiv 0$ for $s\in\{1,2,\ldots,\tilde{m}_k-1\}$, i.e., there is no transmission during $(t_k,t_{k,\tilde{m}_k-1})$. The purpose of such a setting is to ensure that only the most recent sampled information is received by the estimator from each sensor. Moreover, it will cut down $\tilde{m}_k-1$ communications during the $(k+1)$th estimating interval for energy saving. For those sensors that have not obtained transmission permissions during $(t_k,t_{k,s-1}]$, case b) specifies that each of them has the chance to obtain a transmission permission with a probability $\frac{1}{n-s+\tilde{m}_k}$ at time instant $t_{k,s}$. In case c), it specifies that a sensor will have no chance to compete for transmitting at $t_{k,s}$ if it has obtained a transmission permission before $t_{k,s-1}$.

Remark 2: According to the SCT strategy, the transmitting instants are related to the length of the estimating interval and the total number of sensors. When $n>m_k$, then there is a transmission at any time instant during the $(k+1)$th estimating interval, and $n-m_k$ sensors will have no chance to send their sampled information to the estimator. For the case $n\leq m_k$, all the sensors can forward their measurements to the estimator during $(t_k,t_{k+1}]$, and the transmissions happen at instants $t_k+(m_k-n+1)\Delta,\ldots,t_k+m_k\Delta$. An illustrative example is provided in Fig. 3 to show the transmission sequences in different estimating intervals with the cases $n>m_k$, $n=m_k$ and $n<m_k$, where $n=3$.

According to the definition of the SCT strategy, the following statement holds.

Proposition 1: During each estimating interval, the probability of obtaining a transmitting permission for each sensor is uniform, i.e., $\text{Prob}\{\sigma_{k,s}^{(i)}=1|\mathcal{C}_1,\mathcal{C}_2\}=\frac{1}{n}$ for any $s\in\{\tilde{m}_k,\tilde{m}_k+1,\ldots,m_k\}$.

Proof: See Appendix A. □

Remark 3: It follows from Proposition 1 that the probability of obtaining a transmitting permission is uniform. In this sense, there are two ways of realizing the SCT mechanism. The first way is a distributed one, where each sensor knows the estimator's estimating instants and the total number of sensors. At each transmitting instant, the sensors transmit their stored measurements to the estimator similar to that "several phones try to call to another phone at the same time", although they have the same probability to communicate with the estimator, only one of them can transmit successfully. The second way is a centralized one, where the transmission permissions are assigned by a host computer. At the beginning of the kth estimating interval, the host computer randomly generates a communication sequence satisfying the constraints $\mathcal{C}_1$ and $\mathcal{C}_2$, and sends orders to let $\tilde{m}_k$ sensors know when they should transmit their stored measurements. Though it is more appropriate to be called as a stochastic scheduling strategy in this case, we still use the term "SCT" to avoid confusion as it only causes the differences in the naming sense.

The SCT strategy specifies that each sensor has one chance at most to send its sampled information to the estimator during each estimating interval. In this scenario, although an output has been observed by the related sensor, it still cannot be transmitted to the estimator if the sensor has not obtained a transmitting permission at that instant. In view of this, a buffer is embedded in each sensor to store the sampled information. Denote by $\tilde{y}_{k,j}^{(i)}$

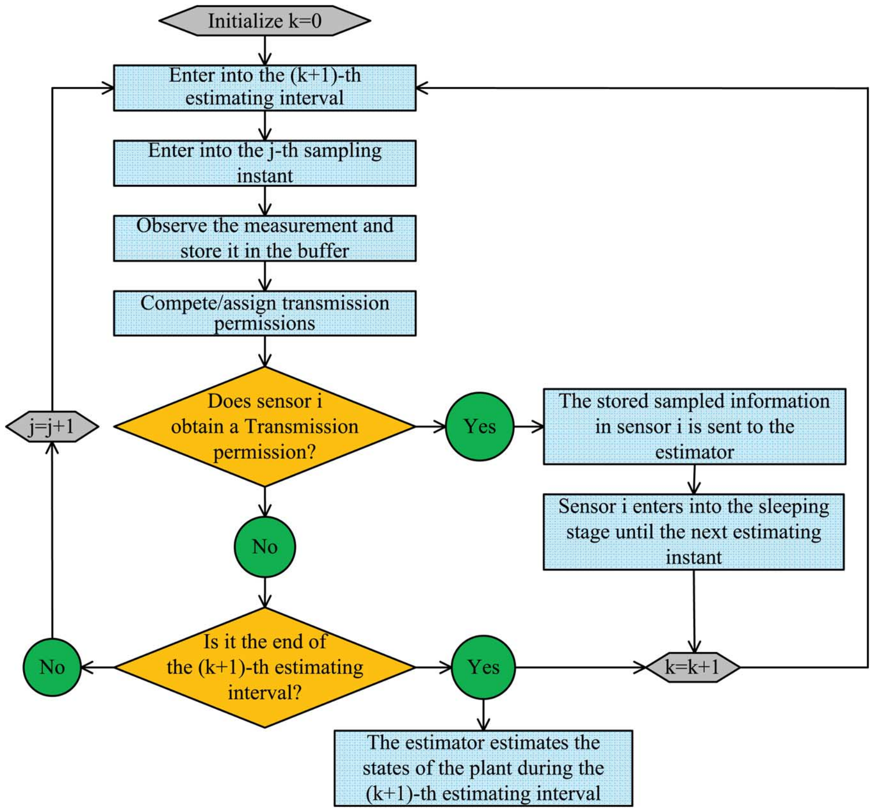

Fig. 2. The architecture of the multi-sensor-based estimation process with competitive transmission.

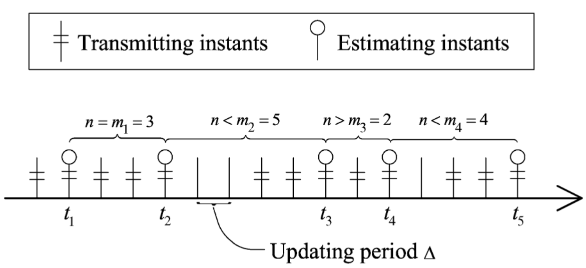

Fig. 3. The transmission sequences for $n=3$.

the stored sampled information in sensor i at time $t_k+j\Delta$, then an aperiodic mixed storage strategy is proposed as follows.

Definition 2 (Aperiodic Mixed Storage, AMS): For a set of n sensors with random failures, the storage process in each sensor's buffer during the estimating interval $(t_k, t_{k+1}]$ is set as follows:

$$\tilde{y}_{k,s}^{(i)}=\begin{cases}\theta_{k,1}^{(i)}y_{k,1}^{(i)}, & \text{if s=1;}\\ \theta_{k,s}^{(i)}y_{k,s}^{(i)}+\left(1-\theta_{k,s}^{(i)}\right)\tilde{y}_{k,s-1}^{(i)}, & \text{if } s=2,3,\ldots,m_k.\end{cases} \tag{6}$$

Remark 4: There are two general mechanisms for compensating the missing measurements: the zero-input mechanism and the hold-input mechanism [36]–[41]. In fact, the proposed AMS strategy is a combination of these two mechanisms (a zero-input mechanism at time $t_k+\Delta$ and a hold-input mechanism at time $t_k+2\Delta,\ldots,t_k+m_k\Delta$). According to the AMS strategy, the sampled information will only be used for one time at the current estimating interval, and the buffer in each sensor flushes out the oldest measurements at the beginning of the next estimating interval. It is quite different from the conventional compensation mechanism (for example, the measurements in a hold-input mechanism may be used repeatedly when consecutive sensor failures occur).

Remark 5: According to the SCT and the AMS strategies, the estimator receives $m_k-\tilde{m}_k+1$ packets during the $(k+1)$th estimating interval, and each packet contains the latest measurement that was collected by the corresponding sensor before it carries out the transmission. Once a sensor obtains the access to the communication channel, the stored measurement is immediately sent to the estimator. It should be pointed out if no successful sampling happens during the current estimating interval, then the sensor stops to carry out the transmission as no available measurement is obtained.

Considering the sensor failures, denote

$$f_{k,j,l}^{(i)}=\begin{cases}\theta_{k,l}^{(i)}, & \text{if } j=l;\\ \theta_{k,l}^{(i)}\prod_{s=l+1}^{j}\left(1-\theta_{k,s}^{(i)}\right), & \text{if } j>l.\end{cases} \tag{7}$$

where $j\geq l$ and $j,l\in\{1,\ldots,m_k\}$. Then it follows from the AMS strategy that

$$\tilde{y}_{k,j}^{(i)}=\sum_{l=1}^{j}f_{k,j,l}^{(i)}y_{k,l}^{(i)} \tag{8}$$

Thus, the estimator's received sampled information from sensor i during $(t_k,t_{k+1}]$ is $\mathcal{S}_k^{(i)}=\{\sigma_{k,1}^{(i)}\tilde{y}_{k,1}^{(i)},\ldots,\sigma_{k,m_k}^{(i)}\tilde{y}_{k,m_k}^{(i)}\}$, which can be recombined as $\mathcal{S}_k^{(i)}=\{\Psi_{k,1}^{(i)}y_{k,1}^{(i)},\ldots,\Psi_{k,m_k}^{(i)}y_{k,m_k}^{(i)}\}$, where $\Psi_{k,j}^{(i)}=\sum_{l=j}^{m_k}\sigma_{k,l}^{(i)}f_{k,l,j}^{(i)}$. Denote by $y_{k,j}=\mathrm{col}\{y_{k,j}^{(1)},\ldots,y_{k,j}^{(n)}\}$, then the estimator's received sampled information from the n sensors during $(t_k, t_{k+1}]$ can be expressed in the form of $\mathcal{S}_k = \{\Psi_{k,1}y_{k,1}, \ldots, \Psi_{k,m_k}y_{k,m_k}\}$, where $\Psi_{k,j} = \sum_{l=j}^{m_k}\sigma_{k,l}f_{k,l,j}$, $\sigma_{k,l} = \mathrm{diag}\{\sigma_{k,l}^{(1)}, \ldots, \sigma_{k,l}^{(n)}\}$ and $f_{k,l,j} = \mathrm{diag}\{f_{k,l,j}^{(1)}, \ldots, f_{k,l,j}^{(n)}\}$.

At each estimating instant t_{k+1}, the objective of the estimator is to generate m_k estimates of the state vectors $x_{k,1}, \ldots, x_{k,m_k}$ by using the sampled information $\mathcal{S}_k$ and the prediction $\bar{x}_{k,1}$ of the state $x_{k,1}$. In this paper, the least-squares estimation approach will be used to generate the estimates. To this end, the following cost function is introduced:

$$J_k = \mu\|\hat{x}_{k,1} - \bar{x}_{k,1}\|^2 + \eta\sum_{j=1}^{m_k}\|\Psi_{k,j}y_{k,j} - Q\Psi_{k,j}C\hat{x}_{k,j}\|^2 \quad (9)$$

where the weighted parameters μ and η are positive scalars that represent the weights of the prior estimated information and the sampled information, respectively. Then, at any estimating instant, a multi-sensor-based aperiodic multi-step estimation problem is stated as follows.

Problem 1: Based on the prediction $\bar{x}_{k,1}$ and the sampled information $\mathcal{S}_k$, find the optimal estimates $\hat{x}_{k,j}$, $j = 1, 2, \ldots, m_k$, that minimize the cost (9) and satisfy the constraints

$$\hat{x}_{k,j+1} = A\hat{x}_{k,j},\ j = 1, 2, \ldots, m_k - 1 \quad (10)$$

where the prediction $\bar{x}_{k,1}$ is obtained as $\bar{x}_{k,1} = A\hat{x}_{k-1,m_{k-1}}$, $\bar{x}_{0,1} = A\hat{x}_0$ and $\hat{x}_0$ is the initial value of the estimate.

Remark 6: At the estimating instant t_{k+1}, only the sampled information $\mathcal{S}_k$ and the prediction $\bar{x}_{k,1}$ of the state $x_{k,1}$ can be obtained by the estimator. In other words, the estimates $\hat{x}_{k,1}, \ldots, \hat{x}_{k,m_k}$ are generated based on $\mathcal{S}_k$ and $\bar{x}_{k,1}$. In view of this, only the estimation error in time $t_{k,1}$ is considered in the cost function (9). Note that the cost function (9) consists of the prediction estimation error and the output estimation error. Thus, the estimates are obtained by minimizing the weighted value of these two classes of errors.

Remark 7: As the measurements are time-stamped and the transmitting processes are real-time, the value of $\Psi_{k,j}$ is known to the estimator. Here the "multi-step estimation" means that more than one estimates will be generated at any estimating instant. Note that there are three rates in the estimating system: sampling rate, estimating rate and transmitting rate, where the former two have the period of Δ and $m_k\Delta$, respectively, while the latter one is stochastic. Lifting technique is a typical approach to deal with the multi-rate problems, see [42]–[44]. Instead of augmenting the system model as in these literatures, the dimensions of the state vectors in this paper are the same as the original system, which reduces the computational complexity significantly.

III. Main Results

In this section, a multi-step least-squares estimation algorithm is proposed for the estimator to aperiodically generate the state estimates. By regarding the noises as bounded ones and random ones, respectively, the estimation performance is analyzed and an upper bound of the expectation of the estimation error is derived. Moreover, a sufficient condition is given to ensure the convergence of the obtained upper bound.

Denote $C = \mathrm{col}\{C_1, \ldots, C_n\}$, $D = \mathrm{diag}\{D_1, \ldots, D_n\}$ and $v_{k,j} = \mathrm{col}\{v_{k,j}^{(1)}, \ldots, v_{k,j}^{(n)}\}$, where $v_{k,j}^{(i)} = v_i(t_k + j\Delta)$. Then a multi-sensor-based aperiodic multi-step least-squares estimation algorithm is obtained as follows.

Theorem 1: For the given weighted parameters μ and η, Problem 1 has a unique solution

$$\hat{x}_{k,j} = A^{j-1}(\mu I + \eta Q_k)^{-1}\left(\mu\bar{x}_{k,1} + \eta\sum_{s=1}^{m_k}H_{k,s}y_{k,s}\right),\quad j = 1, 2, \ldots, m_k \quad (11)$$

where $Q_k = \sum_{s=1}^{m_k}(A^{s-1})^{\mathrm{T}}C^{\mathrm{T}}\Psi_{k,s}\Psi_{k,s}CA^{s-1}$ and $H_{k,s} = (A^{s-1})^{\mathrm{T}}C^{\mathrm{T}}\Psi_{k,s}\Psi_{k,s}$. Moreover, denote the estimation error at $t_k + j\Delta$ by $e_{k,j} = x_{k,j} - \hat{x}_{k,j}$, then one has

$$e_{0,j} = A^{j-1}(\mu I + \eta Q_0)^{-1}(\mu A(x_0 - \hat{x}_0) + \mu Bw_{0,0} + \Omega_{0,3} + \Omega_{0,4}) + \sum_{r=1}^{j-1}A^{j-1-r}Bw_{0,r} \quad (12)$$

$$e_{k,j} = A^{j-1}(\mu I + \eta Q_k)^{-1}\left(\mu A^{m_{k-1}}e_{k-1,1} + \sum_{i=1}^{4}\Omega_{k,i}\right) + \sum_{r=1}^{j-1}A^{j-1-r}Bw_{k,r},\ k = 1, 2, \ldots \quad (13)$$

where

$$\Omega_{k,1} = \mu Bw_{k-1,m_{k-1}}$$
$$\Omega_{k,2} = \mu\sum_{r=1}^{m_{k-1}-1}A^{m_{k-1}-r}Bw_{k-1,r}$$
$$\Omega_{k,3} = -\eta\sum_{s=1}^{m_k}\sum_{r=1}^{s-1}H_{k,s}CA^{s-1-r}Bw_{k,r}$$
$$\Omega_{k,4} = -\eta\sum_{s=1}^{m_k}H_{k,s}Dv_{k,s}$$

Proof: See Appendix B. □

Theorem 1 provides an aperiodic multi-step estimation algorithm for the estimator to aperiodically generate the estimates. At estimating instant t_{k+1}, the estimator generates m_k state estimates $\hat{x}_{k,1}, \hat{x}_{k,2}, \ldots, \hat{x}_{k,m_k}$ by using the sampled information $\mathcal{S}_k$ and the prior predicted information $\bar{x}_{k,1}$. The detailed estimating process is summarized in Algorithm 1.

Remark 8: The matrix $\mu I + Q_k$ in (11) is positive-definite for any sensor failure situations and transmission sequences, which means that the inverse of the matrix $\mu I + Q_k$ always exists. Thus, (11) always holds. Note that the derived estimation algorithm can be used to estimate a dynamic process with any unknown process noises and measurement noises, which is different from the well-known Kalman filtering method [45], where the statistics of the noises are assumed to be known. Moreover, in Algorithm 1, it does not need to compute the expected estimation error covariance matrix at each estimating instant, which will not increase the computation cost of the estimator.

Remark 9: In Algorithm 1, Steps 2–6 are related to the sensors that have not obtained transmission permissions during the current estimating interval. The time instants in Steps 3–4 are

Algorithm 1: The multi-sensor-based aperiodic multi-step estimation algorithm with SCT and AMS

Step 1. Initialize: $\hat{x}_0$, $k=0$.

Step 2. Set $j=1$ and $\mathcal{T}_{k,0}=\varnothing$.

Step 3. For the sensors $i \in \{1,\ldots,n\}/\mathcal{T}_{k,j-1}$, update the stored sampled information in their buffers as follows:

$\tilde{y}^{(i)}_{k,1} \leftarrow \theta^{(i)}_{k,1} y^{(i)}_{k,1}, (j=1)$

$\tilde{y}^{(i)}_{k,j} \leftarrow \theta^{(i)}_{k,j} y^{(i)}_{k,j} + (1-\theta^{(i)}_{k,j})\tilde{y}^{(i)}_{k,j-1}, (j \in \{2,\ldots,m_k\})$

Step 4. If $j \in \{1,2,\ldots,\tilde{m}_k-1\}$, then $j \leftarrow j+1$, and return to *Step 3*; and if $j \notin \{1,2,\ldots,\tilde{m}_k-1\}$, go to *Step 5*.

Step 5. For the sensors $i \in \{1,\ldots,n\}/\mathcal{T}_{k,j-1}$, assign the transmission permissions according to the SCT strategy and set:

$r \leftarrow \arg_i\{\sigma^{(i)}_{k,j}=1, i \notin \mathcal{T}_{k,j-1}\}$

$\mathcal{S}^{(r)}_k \leftarrow \tilde{y}^{(r)}_{k,j}$

$\mathcal{T}_{k,j} \leftarrow \mathcal{T}_{k,j-1} \cup \{r\}$

Step 6. If $j \neq m_k$, then $j \leftarrow j+1$ and return to *Step 3*; and if $j=m_k$, then go to *Step 7*.

Step 7. Run the equation (11) in the estimator to generate $\hat{x}_{k,1}, \hat{x}_{k,2}, \ldots, \hat{x}_{k,m_k}$.

Step 8. $k \leftarrow k+1$, return to *Step 2*.

$t_{k,j}$, where $j \in \{1,2,\ldots,m_k\}$, and the ones in Steps 5–6 are $t_{k,j}$, where $j \in \{\tilde{m}_k, \tilde{m}_k+1,\ldots,m_k\}$. Step 7 is executed in the estimator.

Denote

$$\begin{aligned}
\bar{\sigma}^{(i,r)}_{k,l,q} &= \mathrm{Prob}\left\{\sigma^{(i)}_{k,l}\sigma^{(r)}_{k,q}=1 \middle| \mathcal{C}_1,\mathcal{C}_2\right\}\\
\tilde{\sigma}^{(i,r)}_{k,l,q} &= \mathrm{Prob}\left\{\sigma^{(i)}_{k,l}=1 \middle| \mathcal{C}_1,\mathcal{C}_2,\sigma^{(r)}_{k,q}=1\right\}\\
\Upsilon^{(i)}_{k,l,j} &= \mathrm{Prob}\left\{f^{(i)}_{k,l,j}=1\right\}\\
\tilde{\Psi}^{(i)}_{k,j} &= \sum_{l=j}^{m_k}\left(\sigma^{(i)}_{k,l}f^{(i)}_{k,l,j}\right)^2\\
\tilde{\Upsilon}^{(i)}_{k,j} &= \mathbf{E}\left\{\tilde{\Psi}^{(i)}_{k,j}\middle|\mathcal{C}_1,\mathcal{C}_2\right\}\\
\Theta^{(i,r)}_{k,l,j} &= \mathbf{E}\left\{\tilde{\Psi}^{(i)}_{k,l}\left(\tilde{\Psi}^{(r)}_{k,j}\right)^{-1}\middle|\mathcal{C}_1,\mathcal{C}_2\right\}
\end{aligned}$$

Before giving the analysis of the estimation performance, three useful propositions related to the SCT and AMS strategies are first presented as follows.

Proposition 2: Under the constraints $\mathcal{C}_1$ and $\mathcal{C}_2$, the values of $\bar{\sigma}^{(i,r)}_{k,l,q}$ and $\tilde{\sigma}^{(i,r)}_{k,l,q}$, $i,r \in \{1,2,\ldots,n\}$, $l,q \in \{1,2,\ldots,m_k\}$, can be computed as follows:

$$\bar{\sigma}^{(i,r)}_{k,l,q} = \begin{cases} \frac{1}{n}, & \text{if } i=r \text{ and} \\ & \quad l=q \in \{\tilde{m}_k,\tilde{m}_k+1,\ldots,m_k\};\\ \frac{1}{n(n-1)}, & \text{if } i\neq r \text{ and } l \neq q \text{ and}\\ & \quad l,q \in \{\tilde{m}_k,\tilde{m}_k+1,\ldots,m_k\};\\ 0, & \text{otherwise.}\end{cases} \tag{14}$$

$$\tilde{\sigma}^{(i,r)}_{k,l,q} = \begin{cases} \frac{1}{n-1}, & \text{if } i\neq r \text{ and } l\neq q \text{ and}\\ & \quad l,q \in \{\tilde{m}_k,\tilde{m}_k+1,\ldots,m_k\};\\ 0, & \text{otherwise.}\end{cases} \tag{15}$$

Proof: See Appendix C. □

Proposition 3: According to the AMS strategy, the values of $\Upsilon^{(i)}_{k,l,j}$, $l \geq j$, $l,j \in \{1,2,\ldots,m_k\}$, can be computed as follows:

$$\Upsilon^{(i)}_{k,l,j} = \lambda_i(1-\lambda_i)^{l-j} \triangleq \Upsilon^{(i)}_{l,j} \tag{16}$$

Proof: The results follow from the definitions of sensor failure rate and the AMS strategy, and the proof is thus omitted here.

Proposition 4: It follows from the definition of the SCT strategy that

$$\Psi_{k,j}\Psi_{k,j} = \sum_{i=1}^{n}\tilde{\Psi}^{(i)}_{k,j}\Lambda_i \tag{17}$$

where $\Lambda_i = \mathrm{diag}\{\underbrace{0,\ldots,0}_{i-1},1,\underbrace{0,\ldots,0}_{n-i}\}$. Moreover, the expectations of $\tilde{\Upsilon}^{(i)}_{k,j}$ and $\Theta^{(i,r)}_{k,l,j}$, $i,r \in \{1,2,\ldots,n\}$, $l,j \in \{1,2,\ldots,m_k\}$, can be computed as follows:

$$\tilde{\Upsilon}^{(i)}_{k,j} = \begin{cases} \frac{1}{n}\sum_{l=\tilde{m}_k}^{m_k}\Upsilon^{(i)}_{l,j}, & j \in \{1,2,\ldots,\tilde{m}_k-1\};\\ \frac{1}{n}\sum_{l=j}^{m_k}\Upsilon^{(i)}_{l,j}, & j \in \{\tilde{m}_k,\tilde{m}_k+1,\ldots,m_k\}\end{cases} \tag{18}$$

$$\Theta^{(i,r)}_{k,l,j} = \begin{cases} \tilde{\Upsilon}^{(i)}_{k,l}, & \text{if } i=r \text{ and } l\geq j;\\ \tilde{\Upsilon}^{(i)}_{k,j}, & \text{if } i=r \text{ and } l<j;\\ \tilde{\Theta}^{(i,r)}_{k,l,j}, & \text{if } i\neq r \text{ and } j\neq m_k;\\ \bar{\Theta}^{(i,r)}_{k,l,m_k}, & \text{if } i\neq r \text{ and } j=m_k \text{ and } l\neq m_k;\\ 0, & \text{if } i\neq r \text{ and } j=m_k \text{ and } l=m_k.\end{cases} \tag{19}$$

where

$$\begin{aligned}
\tilde{\Theta}^{(i,r)}_{k,l,j} &= \sum_{q=l}^{m_k}\bar{\sigma}^{(i,i)}_{k,q,q}\Upsilon^{(i)}_{q,l}\left(\sum_{p=j,p\neq q}^{m_k}\tilde{\sigma}^{(i,r)}_{k,p,q}\Upsilon^{(r)}_{p,j}\right)^{-1}\\
\bar{\Theta}^{(i,r)}_{k,l,m_k} &= \left(\sum_{q=l}^{m_k-1}\tilde{\sigma}^{(i,r)}_{k,q,m_k}\Upsilon^{(i)}_{q,l}\right)\left(\bar{\sigma}^{(r,r)}_{k,m_k,m_k}\Upsilon^{(r)}_{m_k,m_k}\right)^{-1}
\end{aligned}$$

Proof: See Appendix D. □

Remark 10: According to the definition of the SCT strategy, the computations of $\bar{\sigma}^{(i,r)}_{k,l,q}$, $\tilde{\sigma}^{(i,r)}_{k,l,q}$, $\tilde{\Upsilon}^{(i)}_{k,j}$ and $\Theta^{(i,r)}_{k,l,j}$ are subjected to the constraints $\mathcal{C}_1$ and $\mathcal{C}_2$, which are different from that of a conventional transmission strategy without these two constraints. For example, $\mathrm{Prob}\{\sigma^{(i)}_{k,l}\sigma^{(r)}_{k,q}=1\} = \frac{1}{n^2}$ if $i\neq r$ or $l\neq q$ and is equivalent to $\frac{1}{n}$ if $i=r$ and $l=q$, whereas the one subjected to $\mathcal{C}_1$ and $\mathcal{C}_2$ is given by (14). Compare with the value of $\tilde{\sigma}^{(i,r)}_{k,l,q}$ in (15), $\mathrm{Prob}\{\sigma^{(i)}_{k,l}=1|\sigma^{(r)}_{k,q}=1\} = \frac{1}{n}$ if no constraint is introduced.

Remark 11: In Proposition 3, the meaning of the probability in $\Upsilon^{(i)}_{k,l,j}$ consists of two parts: first, the time-stamp of the transmission permission obtained by sensor i during $(t_k, t_{k+1}]$ is $t_{k,j}$; second, the measurement received by the estimator at time $t_{k,j}$ is $y^{(i)}_{k,l}$.

Firstly, we focus on the analysis of the estimation performance when the process noises and measurement noises are norm-bounded. An upper bound of the norm of the estimation error and a sufficient condition for the convergence of the proposed upper bound are presented in the following theorem.

Theorem 2: Suppose that $w_{k,j} \in W$ and $v_{k,j}^{(i)} \in V$, where W and V are bounded sets. Then the norm of the estimation error is bounded as

$$\|\mathbf{E}\{e_{k,j}\}|\,\mathcal{C}_1, \mathcal{C}_2\| \leq \tilde{e}_{k,j} \tag{20}$$

where the sequence $\tilde{e}_{k,j}$ is given by

$$\tilde{e}_{k,j} = a_{k,j}\tilde{e}_{k-1,1} + b_{k,j}, k = 1, 2, \ldots \tag{21}$$

$$\tilde{e}_{0,j} = a_{0,j}\|x_0 - \hat{x}_0\| + b_{0,j} \tag{22}$$

with

$$\begin{aligned}
r_w &= \max_{w\in W}\|w\|,\ r_v = \max_{v\in V}\|v\|,\ f_s = \|A^s\| \\
a_{k,j} &= \frac{\mu f_{m_{k-1}+j-1}}{\mu + \eta\lambda_{\min}(\tilde{Q}_k)} \\
b_{k,j} &= \frac{\mu r_w}{\mu + \eta\lambda_{\min}(\tilde{Q}_k)}\left(\Xi_j^{(1)} + \Xi_{k,j}^{(2)}\right) \\
&\quad + \left(\Xi_j^{(3)} + \eta\Xi_{k,j}^{(4)}\right) r_w + \eta r_v \Xi_{k,j}^{(5)} \\
a_{0,j} &= \frac{\mu f_j}{\mu + \eta\lambda_{\min}(\tilde{Q}_0)} \\
b_{0,j} &= \frac{\mu r_w}{\mu+\eta\lambda_{\min}(\tilde{Q}_0)}\Xi_j^{(1)} + \left(\Xi_j^{(3)} + \eta\Xi_{0,j}^{(4)}\right) r_w + \eta r_v \Xi_{0,j}^{(5)} \\
\tilde{Q}_k &= \sum_{s=1}^{m_k}\sum_{i=1}^{n}\left(A^{s-1}\right)^{\mathrm{T}} C^{\mathrm{T}} \tilde{\Upsilon}_{k,s}^{(i)} \varLambda_i C A^{s-1} \\
\Xi_j^{(1)} &= \|A^{j-1}B\| \\
\Xi_{k,j}^{(2)} &= \left\|\sum_{r=1}^{m_{k-1}-1} A^{m_{k-1}+j-r-1} B\right\| \\
\Xi_j^{(3)} &= \left\|\sum_{r=1}^{j-1} A^{j-1-r} B\right\| \\
\Xi_{k,j}^{(4)} &= \left\|\sum_{s=1}^{m_k}\sum_{r=0}^{s-1} A^{j-1}\varGamma_{k,s} C A^{s-r-1} B\right\| \\
\Xi_{k,j}^{(5)} &= \left\|A^{j-1}\sum_{s=1}^{m_k}\varGamma_{k,s} D\right\| \\
\varGamma_{k,s} &= \sum_{r=1}^{n}\left(\mu\left(\tilde{\Upsilon}_{k,s}^{(r)}\right)^{-1} I + \eta\varPi_{k,r,s}\right)^{-1}\left(A^{s-1}\right)^{\mathrm{T}} C^{\mathrm{T}} \varLambda_r \\
\varPi_{k,r,s} &= \sum_{l=1}^{m_k}\left(A^{l-1}\right)^{\mathrm{T}} C^{\mathrm{T}}\left(\sum_{i=1}^{n}\Theta_{k,l,s}^{(i,r)}\varLambda_i\right) C A^{l-1}
\end{aligned}$$

and x_0 is the initial value of the state.

Moreover, if the weighted parameters μ and η satisfy

$$\mu(\tilde{f} - 1) < \eta\min\left\{\lambda_{\min}(\tilde{Q}_k)\right\} \tag{23}$$

where $\tilde{f} = \max_{s\in\{1,2,\ldots,2\tilde{m}-1\}} f_s$, then $\tilde{e}_{k,j}$ converges to a ball $\mathcal{B}$ of radius

$$R = \max_{j\in\{1,\ldots,\tilde{m}\}}\left\{\frac{\tilde{a}_j\tilde{b}_1}{1 - \tilde{a}_1} + \tilde{b}_j\right\} \tag{24}$$

centered at the origin, where $\tilde{m} = \max\{m_k : m_k \in M\}$, $\tilde{a}_j = \max\{a_{k,j} : m_k \in M\}$ and $\tilde{b}_j = \max\{b_{k,j} : m_k \in M\}$. Especially, when $m_k \equiv m$, then

$$\lim_{k\to\infty}\tilde{e}_{k,j} = \frac{\tilde{a}_j\tilde{b}_1}{1 - \tilde{a}_1} + \tilde{b}_j \triangleq \tilde{e}_{\infty,j} \tag{25}$$

Proof: See Appendix E. □

Remark 12: In Theorem 2, an upper bound is derived for the expectation of the estimation error at any time instant. It is shown that the proposed upper bound will converge to a ball $\mathcal{B}$ of radius R if the weighted parameters μ and η satisfy (23). In fact, the condition (23) is easy to be satisfied for any value of $\tilde{f}$. For the case $\tilde{f} \leq 1$, then (23) holds for any $\mu \geq 0$ and $\eta \geq 0$. For the case $\tilde{f} > 1$, then (23) holds if the parameters μ and η satisfy $\mu \geq 0$, $\eta \geq 0$ and $\mu/\eta < \min\{\lambda_{\min}(\tilde{Q}_k)\}/(\tilde{f} - 1)$.

Remark 13: It can be seen from (21) and (22) that the upper bound at each time instant monotonically increases with the increasing of the initial estimation error $\|x_0 - \hat{x}_0\|$. In view of this, the initial estimate $\hat{x}_0$ should be chosen as close to x_0 as possible. Moreover, it follows from (24) that the initial estimation error does not influence on the convergence radius R.

Remark 14: Denote by

$$\begin{aligned}
\varphi_{k,j} &= \frac{\mu(\Xi_j^{(1)} + \Xi_{k,j}^{(2)})}{\mu + \eta\lambda_{\min}(\tilde{Q}_k)} + \Xi_j^{(3)} + \eta\Xi_{k,j}^{(4)}, \\
\varphi_{0,j} &= \frac{\mu\Xi_j^{(1)}}{\mu + \eta\lambda_{\min}(\tilde{Q}_0)} + \Xi_j^{(3)} + \eta\Xi_{0,j}^{(4)}, \\
\varphi_j &= \arg_{\varphi_{k,j}}\{\tilde{b}_j\}
\end{aligned}$$

and

$$\phi_j = \arg_{\eta\Xi_{k,j}^{(5)}}\{\tilde{b}_j\}.$$

Then one has $b_{k,j} = r_w\varphi_{k,j} + r_v\eta\Xi_{k,j}^{(5)}$ and $b_{0,j} = r_w\varphi_{0,j} + r_v\eta\Xi_{0,j}^{(5)}$. Thus, the convergence radius can be rewritten as $R = r_w\max_{j\in\{1,\ldots,\tilde{m}\}}\{\frac{\tilde{a}_j\varphi_1}{1-\tilde{a}_1+\varphi_j}\} + r_v\max_{j\in\{1,\ldots,\tilde{m}\}}\{\tilde{a}_j\eta\frac{\phi_1}{1-\tilde{a}_1} + \eta\phi_j\}$, from which we can see that the convergence radius is proportional to the bounds, r_w and r_v, of the noises.

Remark 15: Generally speaking, there are two classes of estimation performance indexes: the first one is absolute estimation error or expected estimation error, and the second one is mean square estimation error. When the mean-square error (MSE) is used to analyze the convergence of the proposed estimation algorithm, there is a term $(\mu I + \eta Q_k)^{-1}\mathcal{X}(\mu I + \eta Q_k^{\mathrm{T}})^{-1}$ in the expression of $\mathbf{E}\{e_{k,j}e_{k,j}^{\mathrm{T}}|\mathcal{C}_1, \mathcal{C}_2\}$, where $\mathcal{X} = A^{m_{k-1}}e_{k-1,1}e_{k-1,1}^{\mathrm{T}}(A^{m_{k-1}})^{\mathrm{T}}$. Note that the elements of Q_k are stochastic and correlated with each other, then the strongly coupled stochastic elements of $(\mu I + \eta Q_k)^{-1}\mathcal{X}(\mu I + \eta Q_k^{\mathrm{T}})^{-1}$ lead to the difficulty in computing $\mathbf{E}\{e_{k,j}e_{k,j}^{\mathrm{T}}|\mathcal{C}_1, \mathcal{C}_2\}$, which is one of the reasons that we prefer to the expected estimation error analysis. In the simulation section, the MSE is used to measure and compare the estimation performance.

Remark 16: There are some related results to deal with the estimation problem where the statistics of the process noises and measurement noises are not necessarily to be known, e.g.,

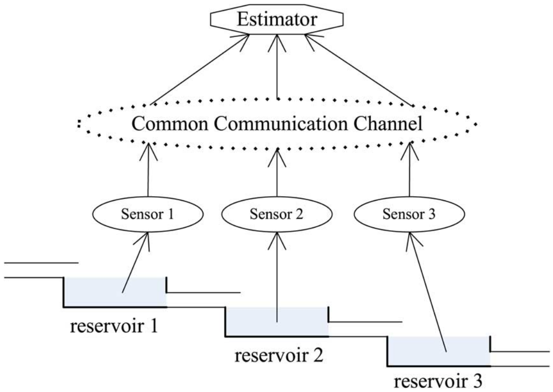

Fig. 4. Structure of the weir system.

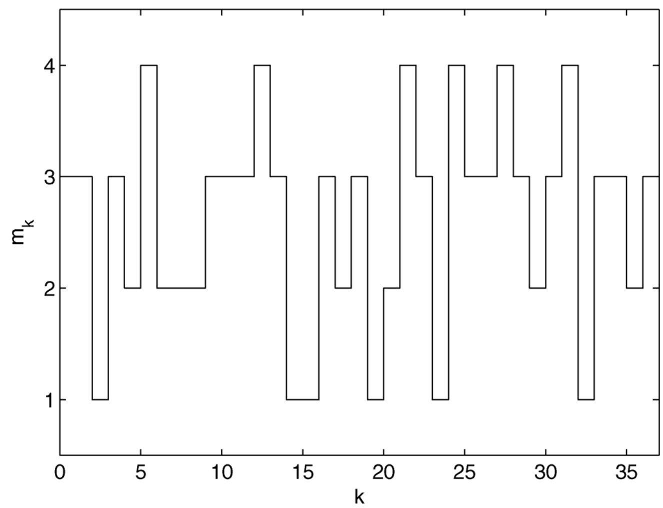

Fig. 5. Lengths of estimating intervals.

[46]–[49]. The main differences of the proposed estimation algorithm and those approaches in [46]–[49] lie in the following three aspects:

- The proposed estimation algorithm is based on multiple sensor data, while the ones in [46]–[49] are related to single sensor.
- In [46]–[49], the estimator has to compute the expected estimation error covariance matrix at each estimating instant, which will greatly increase the computation cost of the estimator. Thus, the computation complexity of the proposed estimation algorithm is lower than the ones in [46]–[49].
- As is pointed out in Remark 15, it is difficult to compute the expected estimation error covariance matrix if the proposed stochastic competitive transmission and aperiodic mixed storage strategies are used. In this sense, the approaches in [46]–[49] are difficult to be implemented to solve those estimation problems with transmission constraints.

In what follows, we will consider the case where the process noises and measurement noises are zero-mean. The properties of the expectation of the estimation error and its norm are presented in the following corollary.

Corollary 1: Suppose that $w_{k,j}$ and $v_{k,j}$ are zero-mean noises, then

a) If the initial values satisfy $\hat{x}_0 = \mathbf{E}\{x_0\}$, then $\mathbf{E}\{e_{k,j}|\mathcal{C}_1,\mathcal{C}_2\} = 0$.
b) If the condition (23) holds, then $\lim_{k\to\infty} \|\mathbf{E}\{e_{k,j}|\mathcal{C}_1,\mathcal{C}_2\}\| = 0$.

Proof: The results follow directly from Theorem 2, and the proof is thus omitted here. □

IV. Illustrative Examples

Consider an artificial weir system [50] consisting of three water reservoirs as shown in Fig. 4. The dynamic of the weir system is described by a three-order state-space model as follows:

$$x(t+\Delta) = \begin{bmatrix} 0.90 & 0 & 0 \\ 0.43 & 0.80 & 0 \\ 0.15 & 0.35 & 0.75 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w(t) \quad (26)$$

where $x(t) = \mathrm{col}\{x_1(t), x_2(t), x_3(t)\}$, $x_1(t) \in \mathbb{R}$, $x_2(t) \in \mathbb{R}$ and $x_3(t) \in \mathbb{R}$ represent the fill levels of the three water reservoirs. Three sensors are deployed to observe the three outputs with the observation matrices $C_1 = [1,0,0]$, $C_2 = [0,1,0]$, $C_3 = [0,0,1]$, and the measurement noise matrices $D_1 = D_2 = D_3 = 0.1$. The sensor failure rates are 0.2, i.e., $\lambda_1 = \lambda_2 = \lambda_3 = 0.8$.

The estimating instants of the estimator is $t_{k+1} = m_k\Delta$, $k = 0,1,2,\ldots$, where the estimating intervals are depicted in Fig. 5. It can be seen that there are six estimating intervals that have the length of 4Δ, which implies that six communications in all are reduced during the whole estimation process. In the simulation, the centralized method stated in Remark 3 is used to implement the SCT mechanism. Note that $m_k \in \{1,2,3,4\}$, then one can calculate that $\tilde{f} = 2.0352$ and $\min\{\lambda_{\min}(\tilde{Q}_k) : m_k \in \{1,2,3,4\}\} = 0.229$. According to condition (23), the weighted parameters μ and η of the cost function (9) should satisfy $0 < 4.521\mu < \eta$. Choose $\mu = 1$ and $\eta = 5$. At each estimating instant t_{k+1}, the estimator is expected to generate m_k state estimates $\hat{x}_{k,1},\ldots,\hat{x}_{k,m_k}$, given the initial condition $x_0 = \mathrm{col}\{15,10,20\}$ and $\hat{x}_0 = \mathrm{col}\{0,0,0\}$. The noises $w_{k,j}$ and $v_{k,j}^{(i)}$ are independently uniformly distributed with $w_{k,j} \in [-r_w, r_w]$ and $v_{k,j}^{(i)} \in [-r_v, r_v]$, where $r_w = r_v = 0.02$. To compare the actual estimation error with the derived upper bound, we define the MSEs of the state estimates as follows:

$$MSE_{k,j} = \sqrt{\frac{\sum_{r=1}^{N}\sum_{i=1}^{3}(x_{i,k,j} - \hat{x}_{i,r,k,j})^2}{N}},\ j = 1,2,\ldots,m_k \quad (27)$$

where N is the number of the Monte Carlo runs and $\hat{x}_{i,r,k,j}$ represents the estimate of $x_{i,k,j} = x_i((j + \sum_{s=0}^{k-1 m_s})\Delta)$ from the j-th Monte Carlo simulation. Through 10000 simulation runs, the MSEs and the corresponding upper bounds are depicted in Fig. 6. The actual states and their estimates are shown in Fig. 7. It can be seen from the simulations that the proposed estimation algorithm performs well and the estimation errors are bounded in a mean sense.

Next, we turn to illustrate the relationship between the weighted parameters and the upper bounds of the estimation errors. Consider the case $m_k = 4$, then the set of the optional transmitting instants is $\{t_k + j\Delta : j = 2,3,4, k = 0,1,2,\ldots\}$. By fixing $\mu = 1$ and $\eta = 1$, respectively, the trajectories of $e_{\infty,1}$, $e_{\infty,2}$, $e_{\infty,3}$ and $e_{\infty,4}$ are depicted in Figs. 8 and 9.

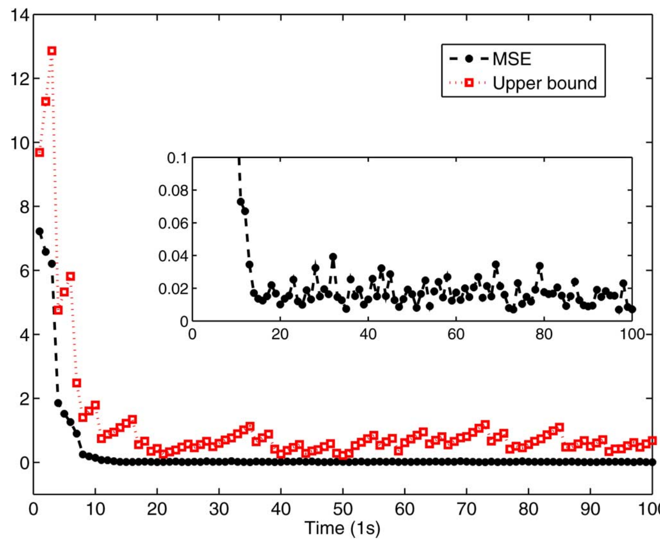

Fig. 6. The MSEs and the corresponding upper bounds.

It can be seen from Figs. 8 and 9 that the upper bounds of the estimation errors and the ratio of μ to η are not directly related. In this example, the values of $e_{\infty,j}$, $j = 1, 2, 3, 4$, are minimized when the ratio of μ to η is about 1:10. The results imply that the estimation performance is not only related to the prior estimates, but also related to the sampled information that was obtained during the current estimating interval.

By setting the initial estimation error $x_{\text{error}} = x_0 - \hat{x}_0$ as $\text{col}\{0,0,0\}$, $\text{col}\{1.5,1,2\}$, $\text{col}\{15,10,20\}$ and $\text{col}\{150,100,200\}$, respectively, the corresponding upper bounds of the norm of the estimation errors are depicted in Fig. 10. It is shown that a larger initial estimation error will lead to a more delayed convergence of the derived upper bounds. However, it can be seen from Fig. 10 that the limits of the upper bounds are not influenced by the initial estimation error, which is consistent with the analysis in Remark 13.

Note that the proposed SCT strategy in this paper allows that multiple communications occur during each estimating interval, while the single communication scheme in [17], [18] and [19] specifies that the sensors' sampling instants and transmission instants are the same as the estimator's estimating instants. To compare the estimation precisions of the algorithm (11) with the single communication scheme and the multiple communications scheme, the MSEs through 10000 simulation runs are illustrated in Fig. 11. It is shown in Fig. 11 that the estimation precision with multiple communications are higher than that of the single communication. A reasonable explanation of the benefit from the multiple communications scheme is that more sampled information is used by the estimator during each estimating interval.

Consider the case that the estimator has a fixed estimation rate, i.e., the lengths of estimating intervals are uniform. It should be pointed out that when $\tilde{m}_k = 1$, the estimator's received sampled information is real-time. When $\tilde{m}_k \geq 2$, then $\tilde{m}_k - 1$ measurements are transmitted with delays. For different $\tilde{m}_k$, the MSEs and the corresponding upper bounds with the initial estimation error $x_{\text{error}} = \text{col}\{0,0,0\}$ are depicted in Fig. 12, and the values of $\bar{e}_{\infty,j}$ are given in Table I. It can be concluded from Fig. 12 and Table I that a larger length of the estimating interval leads to a more conservative upper bound of the estimation error. In other words, the proposed upper bounds are more conservative if the estimator has a larger estimating period.

Lastly, we illustrate the effect of the bounds of noises on the convergence radius. Choose $\mu = 1$ and $\eta = 10$. The convergence radiuses with different bounds of noises are shown in Fig. 13, where r_w and r_v are the same. As is consistent with Remark 14, the convergence radius is proportional to the bounds of noises.

V. Conclusion

In this paper, the multi-sensor-based aperiodic multi-step estimation problem was investigated. A stochastic competitive transmission strategy was proposed to deal with the transmission constraints, where only one transmission happened at each transmitting instant and each sensor had one chance at most to send its measurement to the estimator for the sake of fairness. An aperiodic mixed storage strategy was proposed for the sensors to store the measurements. Based on the received sampled information from the sensors, an aperiodic multi-step estimation algorithm was derived for the estimator to aperiodically generate several estimates. Estimation performance analysis was presented for both cases where the noises are bounded and random. Moreover, an upper bound was provided for the expectation of the estimation error and a sufficient condition was given to ensure the convergence of the proposed estimation algorithm.

Appendix A
Proof of Proposition 1

Note that

$$\begin{aligned}&\text{Prob}\left\{\sigma_{k,s}^{(i)} = 1 \middle| \mathcal{C}_1, \mathcal{C}_2\right\}\\&\quad= \text{Prob}\left\{\sigma_{k,s}^{(i)} = 1 \middle| \mathcal{C}_1, \mathcal{C}_2, i \notin \mathcal{T}_{k,s-1}\right\}\\&\qquad\times \text{Prob}\{i \notin \mathcal{T}_{k,s-1} | \mathcal{C}_1, \mathcal{C}_2\}\end{aligned} \tag{28}$$

When $s = \tilde{m}_k$, it follows from the case a) of Definition 1 that

$$\text{Prob}\{i \notin \mathcal{T}_{k,\tilde{m}_k-1} | \mathcal{C}_1, \mathcal{C}_2\} = 1 \tag{29}$$

When $s \in \{\tilde{m}_k + 1, \tilde{m}_k + 2, \cdots, m_k\}$, then

$$\begin{aligned}&\text{Prob}\{i \notin \mathcal{T}_{k,s-1} | \mathcal{C}_1, \mathcal{C}_2\}\\&\quad= \prod_{l=\tilde{m}_k}^{s-1} \text{Prob}\left\{\sigma_{k,l}^{(i)} = 0 \middle| \mathcal{C}_1, \mathcal{C}_2, i \notin \mathcal{T}_{k,l-1}\right\}\\&\quad= \prod_{l=\tilde{m}_k}^{s-1} \left(1 - \text{Prob}\left\{\sigma_{k,l}^{(i)} = 1 \middle| \mathcal{C}_1, \mathcal{C}_2, i \notin \mathcal{T}_{k,l-1}\right\}\right)\\&\quad= \prod_{l=\tilde{m}_k}^{s-1} \left(1 - \frac{1}{n - l + \tilde{m}_k}\right)\\&\quad= \frac{n - s + \tilde{m}_k}{n}\end{aligned} \tag{30}$$

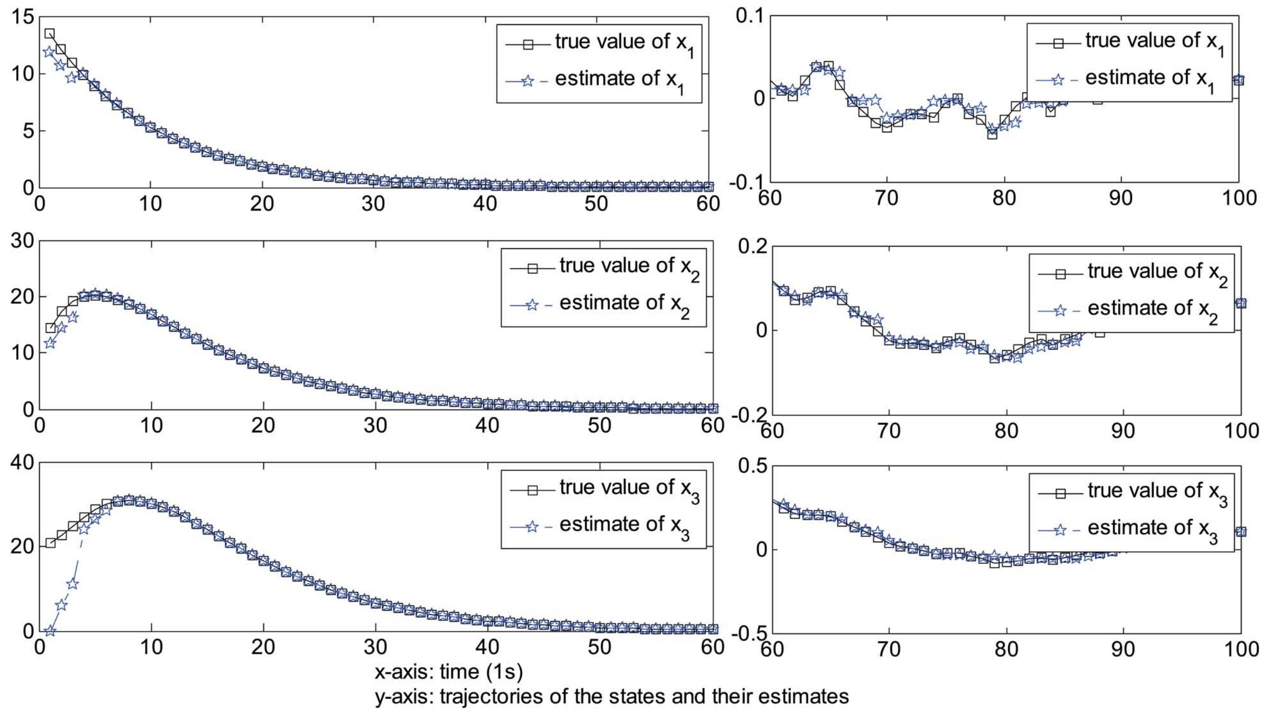

Fig. 7. Trajectories of the states and their estimates.

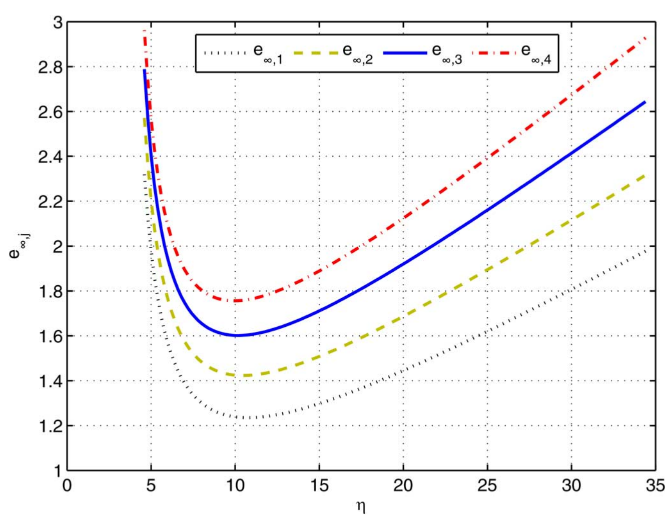

Fig. 8. Relationship between $e_{\infty,j}$ and $\eta(\mu = 1)$.

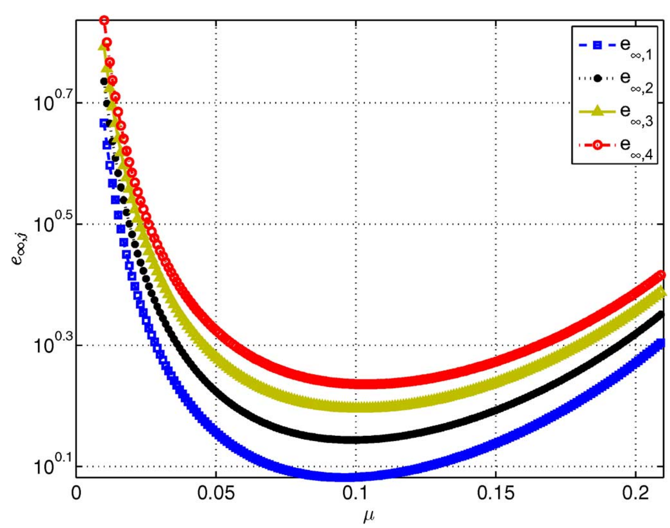

Fig. 9. Relationship between $e_{\infty,j}$ and $\mu(\eta = 1)$.

For any $s \in \{\tilde{m}_k, \tilde{m}_k + 1, \cdots, m_k\}$, it follows from the case b) of Definition 1 that

$$\text{Prob}\left\{ \sigma_{k,s}^{(i)} = 1 \middle| \mathcal{C}_1, \mathcal{C}_2, i \notin \mathcal{T}_{k,s-1} \right\} = \frac{1}{n - s + \tilde{m}_k} \quad (31)$$

Substituting (29), (30) and (31) into (28) leads that

$$\text{Prob}\left\{ \sigma_{k,s}^{(i)} = 1 \middle| \mathcal{C}_1, \mathcal{C}_2 \right\} = \frac{1}{n} \quad (32)$$

Thus, the proof is completed.

Appendix B
Proof of Theorem 1

The first order partial derivative of the cost function (9) with respect to $\hat{x}_{k,1}$ is

$$\nabla_{\hat{x}_{k,1}} J_k = 2\mu(\hat{x}_{k,1} - \bar{x}_{k,1}) \\ -2\eta \sum_{s=1}^{m_k} \left(A^{s-1}\right)^{\mathrm{T}} C^{\mathrm{T}} \Psi_{k,s} (\Psi_{k,s} y_{k,s} - \Psi_{k,s} C A^{s-1} \hat{x}_{k,1}) \quad (33)$$

Note that the necessary condition on the minimum of (9) is $\nabla_{\hat{x}_{k,1}} J_k = 0$, from which one has

$$(\mu I + \eta Q_k)\hat{x}_{k,1} - \mu \bar{x}_{k,1} - \eta \sum_{s=1}^{m_k} H_{k,s} y_{k,s} = 0 \quad (34)$$

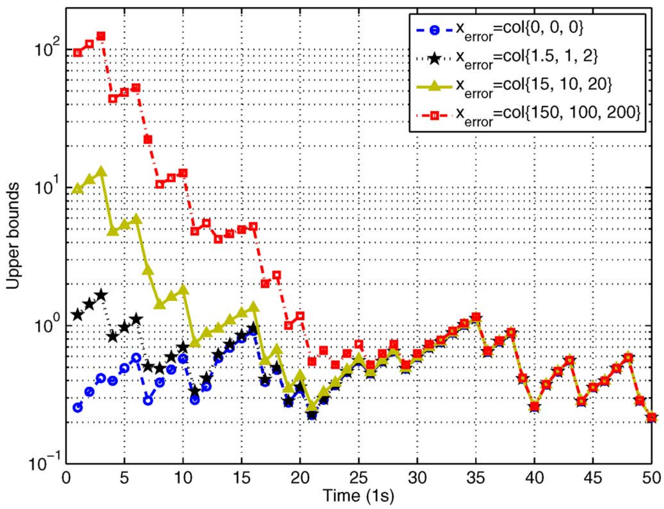

Fig. 10. Upper bounds under different initial estimation errors.

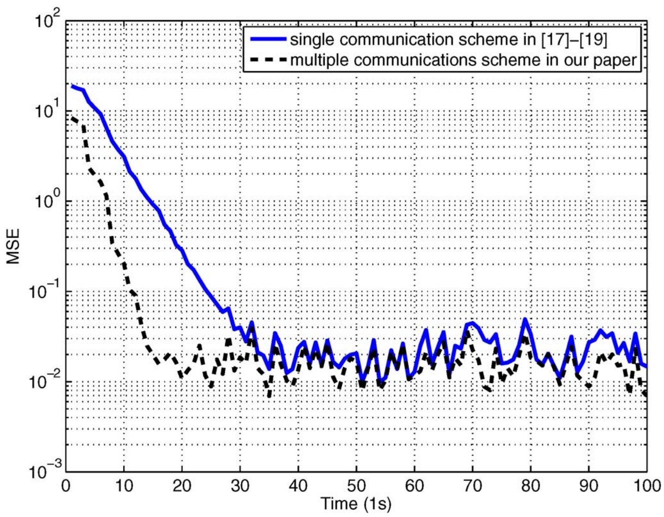

Fig. 11. Comparison of the MSEs with the single communication scheme and the multiple communication scheme.

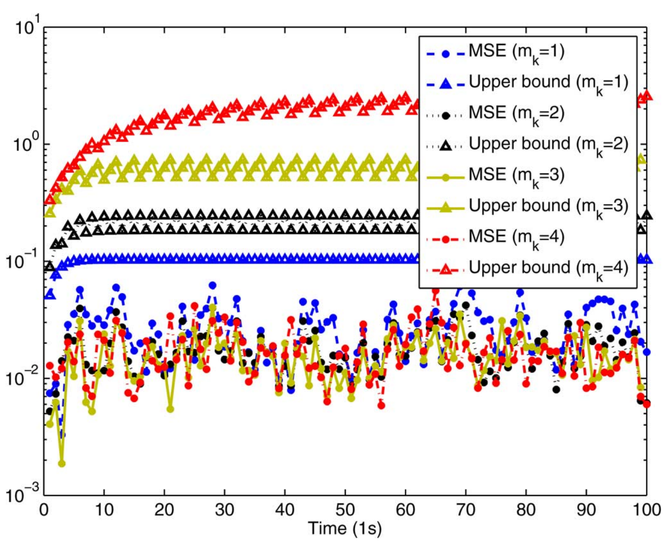

Fig. 12. The MESs and the corresponding upper bounds with different lengths of estimating intervals ($x_{\text{error}} = \text{col}\{0, 0, 0\}$).

TABLE I
VALUES OF $\bar{e}_{\infty,j}$ WITH DIFFERENT $\tilde{m}_k$

$\tilde{m}_k$	$e_{\infty,1}$	$e_{\infty,2}$	$e_{\infty,3}$	$e_{\infty,4}$
1	0.1026	/	/	/
2	0.1835	0.2446	/	/
3	0.5203	0.6270	0.7331	/
4	1.9787	2.2042	2.4031	2.5646

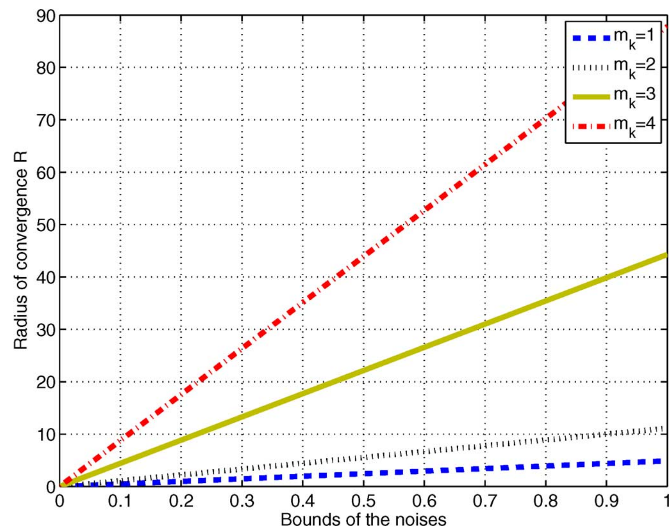

Fig. 13. Convergence radiuses with different bounds of noises.

As μ and η are positive and Q_k is positive semi-definite, then the inverse of $\mu I + \eta Q_k$ exists, which is sufficient to ensure the unique optimal value $\hat{x}_{k,1}$. Thus, one obtains (11) by applying (10).

Note that

$$x_{k,j} = A^{j-1}x_{k,1} + \sum_{r=1}^{j-1} A^{j-1-r}Bw_{k,r} \tag{35}$$

Then

$$y_{k,j} = CA^{j-1}x_{k,1} + C\sum_{r=1}^{j-1} A^{j-1-r}Bw_{k,r} + Dv_{k,j} \tag{36}$$

Substituting (36) into (34) leads to

$$(\mu I + \eta Q_k)(\hat{x}_{k,1} - x_{k,1}) + \mu(x_{k,1} - \bar{x}_{k,1}) - \eta\sum_{s=1}^{m_k}\sum_{r=1}^{s-1} H_{k,s}CA^{s-1-r}Bw_{k,r} - \eta\sum_{s=1}^{m_k} H_{k,s}Dv_{k,s} = 0 \tag{37}$$

Due to the fact that

$$x_{k,1} = A^{m_{k-1}}x_{k-1,1} + Bw_{k-1,m_{k-1}} + \sum_{r=1}^{m_{k-1}-1} A^{m_{k-1}-r}Bw_{k-1,r} \tag{38}$$

and

$$\bar{x}_{k,1} = A\hat{x}_{k-1,m_{k-1}} = A^{m_{k-1}}\hat{x}_{k-1,1} \tag{39}$$

Then it follows from (37) that

$$\begin{aligned}&(\mu I+\eta Q_k)(\hat{x}_{k,1}-x_{k,1})+\mu A^{m_{k-1}}(x_{k-1,1}-\hat{x}_{k-1,1})\\&+\mu Bw_{k-1,m_{k-1}}+\mu\sum_{r=1}^{m_{k-1}-1}A^{m_{k-1}-r}Bw_{k-1,r}\\&-\eta\sum_{s=1}^{m_k}\sum_{r=1}^{s-1}H_{k,s}CA^{s-1-r}Bw_{k,r}-\eta\sum_{s=1}^{m_k}H_{k,s}Dv_{k,s}=0\end{aligned}\tag{40}$$

According to

$$e_{k,j}=A^{j-1}e_{k,1}+\sum_{r=1}^{j-1}A^{j-1-r}Bw_{k,r}\tag{41}$$

(13) holds. Similarly, (12) can be obtained. Thus, the proof is completed.

Appendix C
Proof of Proposition 2

According to Proposition 1, one has

$$\begin{aligned}\text{Prob}\left\{\sigma_{k,l}^{(i)}\sigma_{k,l}^{(i)}=1\middle|\mathcal{C}_1,\mathcal{C}_2\right\}&=\text{Prob}\left\{\sigma_{k,l}^{(i)}=1\middle|\mathcal{C}_1,\mathcal{C}_2\right\}\\&=\frac{1}{n}\end{aligned}\tag{42}$$

It follows from the SCT strategy that $\sigma_{k,l}^{(i)}\sigma_{k,q}^{(i)}\equiv 0$ for all $l\neq q$ and $\sigma_{k,l}^{(i)}\sigma_{k,l}^{(r)}\equiv 0$ for all $i\neq r$. Additionally, the case a) in Definition 1 implies that $\sigma_{k,l}^{(i)}\sigma_{k,q}^{(r)}\neq 0$ only if $l,q\in\{\tilde{m}_k,\tilde{m}_k+1,\cdots,m_k\}$. Note that for any $l,q\in\{\tilde{m}_k,\tilde{m}_k+1,\cdots,m_k\}$, one has

$$\begin{aligned}&\text{Prob}\left\{\sigma_{k,l}^{(i)}\sigma_{k,q}^{(r)}=1\middle|\mathcal{C}_1,\mathcal{C}_2,i\neq r,l\neq q\right\}\\&=\text{Prob}\left\{\sigma_{k,q}^{(r)}=1\middle|\mathcal{C}_1,\mathcal{C}_2\right\}\\&\quad\times\text{Prob}\left\{\sigma_{k,l}^{(i)}=1\middle|\mathcal{C}_1,\mathcal{C}_2,\sigma_{k,q}^{(r)}=1,\ i\neq r,l\neq q\right\}\end{aligned}\tag{43}$$

Similar to the proof of Proposition 1, one can show that

$$\text{Prob}\left\{\sigma_{k,l}^{(i)}=1\middle|\mathcal{C}_1,\mathcal{C}_2,\sigma_{k,q}^{(r)}=1,i\neq r,l\neq q\right\}=\frac{1}{n-1}\tag{44}$$

from which it can be concluded that (14) and (15) hold.

Appendix D
Proof of Proposition 4

Note that

$$\begin{aligned}\Psi_{k,j}\Psi_{k,j}&=\text{diag}\left\{\Psi_{k,j}^{(1)},\ldots,\Psi_{k,j}^{(n)}\right\}\text{diag}\left\{\Psi_{k,j}^{(1)},\ldots,\Psi_{k,j}^{(n)}\right\}\\&=\left(\sum_{i=1}^{n}\Psi_{k,j}^{(i)}\Lambda_i\right)\left(\sum_{i=1}^{n}\Psi_{k,j}^{(i)}\Lambda_i\right)\\&=\left(\sum_{l=j}^{m_k}\sum_{i=1}^{n}\sigma_{k,l}^{(i)}f_{k,l,j}^{(i)}\Lambda_i\right)^2\end{aligned}\tag{45}$$

According to Definition 1, $\sigma_{k,l}^{(i)}\sigma_{k,j}^{(i)}\equiv 0$ for all $l\neq j$, then (17) holds.

It follows from the case a) in Definition 1 that

$$\tilde{\Psi}_{k,j}^{(i)}=\begin{cases}\sum\limits_{l=\tilde{m}_k}^{m_k}\left(\sigma_{k,l}^{(i)}f_{k,l,j}^{(i)}\right)^2, & \text{if } j\in\{1,2,\ldots,\tilde{m}_k-1\}\\ \sum\limits_{l=j}^{m_k}\left(\sigma_{k,l}^{(i)}f_{k,l,j}^{(i)}\right)^2, & \text{if } j\in\{\tilde{m}_k,\tilde{m}_k+1,\ldots,m_k\}\end{cases}\tag{46}$$

By using

$$\begin{aligned}&\text{Prob}\left\{\left(\sigma_{k,l}^{(i)}f_{k,l,j}^{(i)}\right)^2=1\middle|\mathcal{C}_1,\mathcal{C}_2\right\}\\&=\text{Prob}\left\{\left(\sigma_{k,l}^{(i)}\right)^2=1\middle|\mathcal{C}_1,\mathcal{C}_2\right\}\text{Prob}\left\{\left(f_{k,l,j}^{(i)}\right)^2=1\right\}\end{aligned}\tag{47}$$

Then it follows from Propositions 2 and 3 that (47) holds.

Note that

$$\mathbf{E}\left\{\tilde{\Psi}_{k,l}^{(i)}\left(\tilde{\Psi}_{k,j}^{(r)}\right)^{-1}\middle|\mathcal{C}_1,\mathcal{C}_2\right\}=\mathbf{E}\left\{\frac{\sum_{q=l}^{m_k}\left(\sigma_{k,q}^{(i)}f_{k,q,l}^{(i)}\right)^2}{\sum_{p=j}^{m_k}\left(\sigma_{k,p}^{(r)}f_{k,p,j}^{(r)}\right)^2}\middle|\mathcal{C}_1,\mathcal{C}_2\right\}\tag{48}$$

For the case $i=r$ and $l\geq j$, one has

$$\begin{aligned}&\mathbf{E}\left\{\tilde{\Psi}_{k,l}^{(i)}\left(\tilde{\Psi}_{k,j}^{(i)}\right)^{-1}\middle|\mathcal{C}_1,\mathcal{C}_2,l\geq j\right\}\\&=\text{Prob}\left\{\frac{\sum_{q=l}^{m_k}\left(\sigma_{k,q}^{(i)}f_{k,q,l}^{(i)}\right)^2}{\sum_{p=j}^{m_k}\left(\sigma_{k,p}^{(i)}f_{k,p,j}^{(i)}\right)^2}=1\middle|\mathcal{C}_1,\mathcal{C}_2,\ l\geq j\right\}\\&=\sum_{q=l}^{m_k}\text{Prob}\left\{\frac{\left(\sigma_{k,q}^{(i)}f_{k,q,l}^{(i)}\right)^2}{\sum_{p=j}^{m_k}\left(\sigma_{k,p}^{(i)}f_{k,p,j}^{(i)}\right)^2}=1\middle|\mathcal{C}_1,\mathcal{C}_2,\ l\geq j\right\}\end{aligned}\tag{49}$$

When $q\geq j$, it follows from $\mathcal{C}_1$ that $\frac{(\sigma_{k,q}^{(i)}f_{k,q,l}^{(i)})^2}{\sum_{p=j}^{m_k}(\sigma_{k,p}^{(i)}f_{k,p,j}^{(i)})^2}=1$ implies that $\sigma_{k,q}^{(i)}=1$ and $\sigma_{k,j}^{(i)}=\sigma_{k,j+1}^{(i)}=\cdots=\sigma_{k,q-1}^{(i)}=\sigma_{k,q+1}^{(i)}=\sigma_{k,q+2}^{(i)}=\cdots=\sigma_{k,m_k}^{(i)}=0$. Thus, one has

$$\begin{aligned}&\mathbf{E}\left\{\tilde{\Psi}_{k,l}^{(i)}\left(\tilde{\Psi}_{k,j}^{(i)}\right)^{-1}\middle|\mathcal{C}_1,\mathcal{C}_2,\ l\geq j\right\}\\&=\sum_{q=l}^{m_k}\text{Prob}\left\{\left(\sigma_{k,q}^{(i)}f_{k,q,l}^{(i)}\right)^2=1\middle|\mathcal{C}_1,\mathcal{C}_2\right\}\end{aligned}\tag{50}$$

which follows from (18) that

$$\mathbf{E}\left\{\tilde{\Psi}_{k,l}^{(i)}\left(\tilde{\Psi}_{k,j}^{(i)}\right)^{-1}\middle|\mathcal{C}_1,\mathcal{C}_2,\ l\geq j\right\}=\tilde{\Upsilon}_{k,l}^{(i)}\tag{51}$$

Similarly, for the case $i=r$ and $l<j$, one can obtain that

$$\begin{aligned}&\mathbf{E}\left\{\tilde{\Psi}_{k,l}^{(i)}\left(\tilde{\Psi}_{k,j}^{(i)}\right)^{-1}\middle|\mathcal{C}_1,\mathcal{C}_2,l<j\right\}\\&=\sum_{p=j}^{m_k}\text{Prob}\left\{\left(\sigma_{k,p}^{(i)}f_{k,p,j}^{(i)}\right)^2=1\middle|\mathcal{C}_1,\mathcal{C}_2\right\}=\tilde{\Upsilon}_{k,j}^{(i)}\end{aligned}\tag{52}$$

For the case $i \neq r$, one has (53) at the bottom of the page, and

$$\mathbf{E}\left\{ \tilde{\Psi}_{k,l}^{(i)} \left(\tilde{\Psi}_{k,m_k}^{(r)}\right)^{-1} \middle| \mathcal{C}_1,\mathcal{C}_2,\ i\neq r, l\neq m_k \right\} = \sum_{q=l}^{m_k-1} \mathbf{E}\left\{ \frac{\left(\sigma_{k,q}^{(i)} f_{k,q,l}^{(i)}\right)^2}{\left(\sigma_{k,m_k}^{(r)} f_{k,m_k,m_k}^{(r)}\right)^2} \middle| \mathcal{C}_1,\mathcal{C}_2, i\neq r \right\} = \sum_{q=l}^{m_k-1} \frac{\mathrm{Prob}\left\{ \left(\sigma_{k,q}^{(i)} f_{k,q,l}^{(i)}\right)^2 = 1 \middle| \mathcal{C}_1,\mathcal{C}_2, i\neq r, \sigma_{k,m_k}^{(r)}=1 \right\}}{\mathrm{Prob}\left\{ \left(\sigma_{k,m_k}^{(r)} f_{k,m_k}, m_k^{(r)}\right)^2 = 1 \middle| \mathcal{C}_1,\mathcal{C}_2 \right\}} \tag{54}$$

By using Propositions 2 and 3, it can be concluded that

$$\mathbf{E}\left\{ \tilde{\Psi}_{k,l}^{(i)} \left(\tilde{\Psi}_{k,j}^{(r)}\right)^{-1} \middle| \mathcal{C}_1,\mathcal{C}_2, i\neq r, j\neq m_k \right\} = \tilde{\Theta}_{k,l,j}^{(i,r)} \tag{55}$$

and

$$\mathbf{E}\left\{ \tilde{\Psi}_{k,l}^{(i)} \left(\tilde{\Psi}_{k,m_k}^{(r)}\right)^{-1} \middle| \mathcal{C}_1,\mathcal{C}_2, i\neq r, l\neq m_k \right\} = \bar{\Theta}_{k,l,m_k}^{(i,r)} \tag{56}$$

When $j = l = m_k$, it follows from the SCT strategy that $\mathrm{Prob}\{\sigma_{k,m_k}^{(i)} = 1, \sigma_{k,m_k}^{(r)} = 1 | \mathcal{C}_1,\mathcal{C}_2\} = 0$ for all $i \neq r$. Thus, (19) holds and the proof is completed.

Appendix E
Proof of Theorem 2

It follows from (13) that

$$\begin{aligned} \mathbf{E}\{e_{k,j}|\mathcal{C}_1,\mathcal{C}_2\} = {} & \mu A^{j-1}\mathbf{E}\left\{ (\mu I+\eta Q_k)^{-1} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} \\ & \times A^{m_{k-1}}\mathbf{E}\{e_{k-1,1}|\mathcal{C}_1,\mathcal{C}_2\} \\ & + \sum_{i=1}^{4} \mathbf{E}\left\{ A^{j-1} (\mu I+\eta Q_k)^{-1}\Omega_{k,i} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} \\ & + \sum_{r=1}^{j-1} A^{j-1-r}Bw_{k,r} \end{aligned} \tag{57}$$

Note that

$$\begin{aligned} & \mathbf{E}\left\{ A^{j-1} (\mu I+\eta Q_k)^{-1}\Omega_{k,1} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} \\ & \quad = A^{j-1}\mathbf{E}\left\{ (\mu I+\eta Q_k)^{-1} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\}\Omega_{k,1} \end{aligned} \tag{58}$$

$$\begin{aligned} & \mathbf{E}\left\{ A^{j-1} (\mu I+\eta Q_k)^{-1}\Omega_{k,2} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} \\ & \quad = A^{j-1}\mathbf{E}\left\{ (\mu I+\eta Q_k)^{-1} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\}\Omega_{k,2} \end{aligned} \tag{59}$$

$$\begin{aligned} & \mathbf{E}\left\{ A^{j-1} (\mu I+\eta Q_k)^{-1}\Omega_{k,3} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} \\ & \quad = -\eta A^{j-1}\sum_{s=1}^{m_k}\sum_{r=1}^{s-1}\left(\mathbf{E}\left\{ (\mu I+\eta Q_k)^{-1}H_{k,s} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} \right. \\ & \qquad \left. \times CA^{s-1-r}Bw_{k,r}\right) \end{aligned} \tag{60}$$

$$\begin{aligned} & \mathbf{E}\left\{ A^{j-1} (\mu I+\eta Q_k)^{-1}\Omega_{k,4} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} \\ & \quad = -\eta A^{j-1}\sum_{s=1}^{m_k}\mathbf{E}\left\{ (\mu I+\eta Q_k)^{-1}H_{k,s} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} Dv_{k,s} \end{aligned} \tag{61}$$

By using (18), one has $\mathbf{E}\{Q_k|\mathcal{C}_1,\mathcal{C}_2\} = \tilde{Q}_k$. Thus, (58) and (59) are equivalent to

$$\begin{aligned} & \mathbf{E}\left\{ A^{j-1} (\mu I+\eta Q_k)^{-1}\Omega_{k,1} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} \\ & \quad = A^{j-1}(\mu I+\eta \tilde{Q}_k)^{-1}\Omega_{k,1} \end{aligned} \tag{62}$$

$$\begin{aligned} & \mathbf{E}\left\{ A^{j-1} (\mu I+\eta Q_k)^{-1}\Omega_{k,2} \middle| \mathcal{C}_1,\mathcal{C}_2 \right\} \\ & \quad = A^{j-1}(\mu I+\eta \tilde{Q}_k)^{-1}\Omega_{k,2} \end{aligned} \tag{63}$$

According to (17), one has

$$\begin{aligned} & (\mu I+\eta Q_k)^{-1}H_{k,s} \\ & = \left(\mu I + \eta\sum_{l=1}^{m_k}(A^{l-1})^{\mathrm{T}}C^{\mathrm{T}}\left(\sum_{i=1}^{n}\tilde{\Psi}_{k,l}^{(i)}\Lambda_i\right)CA^{l-1}\right)^{-1} \\ & \quad \times (A^{s-1})^{\mathrm{T}}C^{\mathrm{T}}\sum_{r=1}^{n}\tilde{\Psi}_{k,s}^{(r)}\Lambda_r \\ & = \sum_{r=1}^{n}\left(\left(\mu I+\eta\sum_{l=1}^{m_k}(A^{l-1})^{\mathrm{T}}C^{\mathrm{T}}\left(\sum_{i=1}^{n}\tilde{\Psi}_{k,l}^{(i)}\Lambda_i\right)CA^{l-1}\right)^{-1} \right. \\ & \qquad \left. \times (A^{s-1})^{\mathrm{T}}C^{\mathrm{T}}\tilde{\Psi}_{k,s}^{(r)}\Lambda_r\right) \\ & = \sum_{r=1}^{n}\left(\mu\left(\tilde{\Psi}_{k,s}^{(r)}\right)^{-1}I + \eta\sum_{l=1}^{m_k}(A^{l-1})^{\mathrm{T}}C^{\mathrm{T}} \right. \\ & \qquad \left. \times\left(\sum_{i=1}^{n}\tilde{\Psi}_{k,l}^{(i)}\left(\tilde{\Psi}_{k,s}^{(r)}\right)^{-1}\Lambda_i\right)CA^{l-1}\right)^{-1} \\ & \quad \times (A^{s-1})^{\mathrm{T}}C^{\mathrm{T}}\Lambda_r \end{aligned} \tag{64}$$

$$\begin{aligned} \mathbf{E}\left\{ \tilde{\Psi}_{k,l}^{(i)} \left(\tilde{\Psi}_{k,j}^{(r)}\right)^{-1} \middle| \mathcal{C}_1,\mathcal{C}_2,\ i\neq r, j\neq m_k \right\} & = \sum_{q=l}^{m_k} \mathbf{E}\left\{ \frac{\left(\sigma_{k,q}^{(i)} f_{k,q,l}^{(i)}\right)^2}{\sum_{p=j}^{m_k}\left(\sigma_{k,p}^{(r)} f_{k,p,j}^{(r)}\right)^2} \middle| \mathcal{C}_1,\mathcal{C}_2,\ i\neq r, j\neq m_k \right\} \\ & = \sum_{q=l}^{m_k} \frac{\mathrm{Prob}\left\{ \left(\sigma_{k,q}^{(i)} f_{k,q,l}^{(i)}\right)^2 = 1 \middle| \mathcal{C}_1,\mathcal{C}_2 \right\}}{\sum_{p=j,p\neq q}^{m_k}\mathrm{Prob}\left\{ \left(\sigma_{k,p}^{(r)} f_{k,p,j}^{(r)}\right)^2 = 1 \middle| \mathcal{C}_1,\mathcal{C}_2, \sigma_{k,q}^{(i)}=1, i\neq r, j\neq m_k \right\}} \end{aligned} \tag{53}$$

Thus, it follows from (18) and (19) that (60) and (61) are equivalent to

$$\mathbf{E}\left\{A^{j-1}\left(\mu I+\eta Q_k\right)^{-1}\Omega_{k,3}\middle|\,\mathcal{C}_1,\mathcal{C}_2\right\} = -\eta A^{j-1}\sum_{s=1}^{m_k}\sum_{r=0}^{s-1}\Gamma_{k,s}CA^{s-1-r}Bw_{k,r} \tag{65}$$

$$\mathbf{E}\left\{A^{j-1}\left(\mu I+\eta Q_k\right)^{-1}\Omega_{k,4}\middle|\,\mathcal{C}_1,\mathcal{C}_2\right\} = -\eta A^{j-1}\sum_{s=1}^{m_k}\Gamma_{k,s}Dv_{k,s} \tag{66}$$

Substituting (62), (63), (65) and (66) into (57) leads to

$$\begin{aligned}\mathbf{E}\{e_{k,j}|\mathcal{C}_1,\mathcal{C}_2\} &= \mu A^{j-1}(\mu I+\eta\tilde{Q}_k)^{-1}A^{m_{k-1}}\mathbf{E}\{e_{k-1,1}|\mathcal{C}_1,\mathcal{C}_2\} \\ &\quad + A^{j-1}(\mu I+\eta\tilde{Q}_k)^{-1}\Omega_{k,1} + A^{j-1}(\mu I+\eta\tilde{Q}_k)^{-1}\Omega_{k,2} \\ &\quad + \sum_{r=1}^{j-1}A^{j-1-r}Bw_{k,r} - \eta A^{j-1}\sum_{s=1}^{m_k}\sum_{r=0}^{s-1}\Gamma_{k,s}CA^{s-1-r}Bw_{k,r} \\ &\quad - \eta A^{j-1}\sum_{s=1}^{m_k}\Gamma_{k,s}Dv_{k,s}\end{aligned} \tag{67}$$

Thus,

$$\begin{aligned}\|\mathbf{E}\{e_{k,j}|\mathcal{C}_1,\mathcal{C}_2\}\| &\le \mu\|A^{m_{k-1}+j-1}\|\left\|(\mu I+\eta\tilde{Q}_k)^{-1}\right\|\|\mathbf{E}\{e_{k-1,1}|\mathcal{C}_1,\mathcal{C}_2\}\| \\ &\quad + \mu\left\|(\mu I+\eta\tilde{Q}_k)^{-1}\right\| \\ &\quad \times\left(\|A^{j-1}B\| + \left\|\sum_{r=1}^{m_{k-1}-1}A^{m_{k-1}+j-r-1}B\right\|\right)r_w \\ &\quad + \left(\left\|\sum_{r=1}^{j-1}A^{j-1-r}B\right\|\right. \\ &\qquad \left. +\eta\left\|\sum_{s=1}^{m_k}\sum_{r=0}^{s-1}A^{j-1}\Gamma_{k,s}CA^{s-r-1}B\right\|\right)r_w \\ &\quad + \eta r_v\left\|A^{j-1}\sum_{s=1}^{m_k}\Gamma_{k,s}D\right\|\end{aligned} \tag{68}$$

Note that

$$\left\|(\mu I+\eta\tilde{Q}_k)^{-1}\right\| = \frac{1}{\mu+\eta\lambda_{\min}(\tilde{Q}_k)} \tag{69}$$

Then one has

$$\|\mathbf{E}\{e_{k,j}|\mathcal{C}_1,\mathcal{C}_2\}\| \le a_{k,j}\,\|\mathbf{E}\{e_{k-1,1}|\mathcal{C}_1,\mathcal{C}_2\}\| + b_{k,j} \tag{70}$$

Similarly, one has

$$\mathbf{E}\{e_{0,j}|\mathcal{C}_1,\mathcal{C}_2\} \le a_{0,j}\|x_0-\bar{x}_0\| + b_{0,j} \tag{71}$$

Thus, (20)–(22) hold.

If condition (23) holds, then $0 \le a_{k,j} < 1$, from which one obtains

$$\begin{aligned}\tilde{e}_{k,j} &= a_{k,j}\left(\prod_{s=1}^{k-1}a_{s,1}\right)\tilde{e}_{0,1} \\ &\quad + a_{k,j}\sum_{l=1}^{k-1}\left(\prod_{s=l+1}^{k-1}a_{s,1}\right)b_{l,1} + b_{k,j} \\ &\le \tilde{a}_j\tilde{a}_1^{k-1}\tilde{e}_{0,1} + \tilde{a}_j\sum_{l=1}^{k-1}\tilde{a}_1^{k-l-1}\tilde{b}_1 + \tilde{b}_j \\ &\to \frac{\tilde{a}_j\tilde{b}_1}{1-\tilde{a}_1} + \tilde{b}_j\end{aligned} \tag{72}$$

Thus, the proof is completed.

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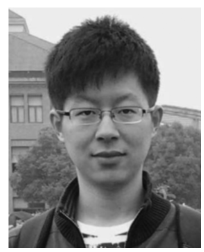

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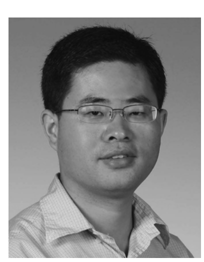

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