Deterministic Sensor Selection for Centralized State Estimation Under Limited Communication Resource

Chao Yang, Junfeng Wu, Xiaoqiang Ren, Wen Yang, Hongbo Shi, and Ling Shi

***Abstract*—This paper studies a sensor selection problem. A group of sensors measure the state of a process and send their measurements to a remote estimator. Due to communication constraints, only limited sensors are allowed to communicate with the estimator. The paper intends to answer which sensors should be chosen such that the estimation performance of the estimator is optimized. Both reliable and packet-dropping channels are considered. It is required to minimize the steady-state estimation error covariance for reliable channels and to minimize the upper bound of the expected estimation error covariance for packet-dropping channels. For both scenarios, the original optimization problems are transformed to problems which can be solved by convex optimization techniques.**

***Index Terms*—Networked state estimation, sensor scheduling, sensor selection, convex optimization, modified algebraic Riccati equation (MARE).**

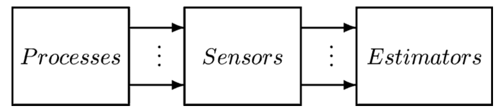

Fig. 1. The general model of sensor scheduling.

I. Introduction

In the last 60s and 70s, the theory of optimal filtering in signal processing had been well studied. In 1960, as the connection of two streams of development, digital filtering and statistical filtering, the theory of Kalman filtering was proposed [1]. In the subsequent two decades, the theory of optimal filtering further got substantially investigated and had become a matured topic. Looking into the future at that time, the incorporation of the practical constraints associated with filter realization into the mathematical statement of the statistical filtering problem was of desire as mentioned in [2].

In the late 90s, networked control systems (NCSs) became an attractive research topic. In its framework, the components of a control system, namely actuator, controller, sensor, estimator, etc, communicate through shared channels. Such a structure brings in great flexibilities and it facilities the installation and maintenance of the system, and benefits a number of practical applications, such as unmanned vehicle, surveillance, environment monitoring, and smart grid [3]. Simultaneously, it introduces the issue in communication into the design of the system and the analysis of the system performance, which has never been of concern before. Accordingly, the filtering problem in NCSs turns out to be in need of taking the modeling of communication into the classical setup, which has been expected in the past as the future direction of traditional filtering as mentioned before. The framework of NCSs brings new challenges to the issue of filtering, which will see its new development.

Consequently, a number of novel problems related to filtering in NCSs have been inspired. One class of them is the sensor scheduling problem. Briefly speaking, in a sensor scheduling problem, a group of sensors, usually constrained by such as limited sensor battery power, limited bandwidth, or heavy work load, need a communication or service plan to optimize some objective of the estimation performance of the system. A model involved in a sensor scheduling problem generally contains the following components: state processes, sensors, and estimators, each of which can be single or multiple. Two stages of scheduling of sensors may be in need: which sensors take measurements of the processes and which sensors transmit data to the estimators (Fig. 1).

In general, the sensor scheduling problems are challenging. First, this is due to the large size of the domain of feasible schedules. If one sensor needs to be chosen out of N ones at each time within a horizon T, then N^T schedules are feasible. It is almost impossible to directly compare the candidate schedules. Second is the high nonlinearity in the form of estimation performance criterion which usually appears in the objective function. For example, for a Gauss-Markov system (one can refer to (1)–(2)), the estimation error covariance is given by the recursion of algebraic Riccati functions, which have a high nonlinearity. Consequently, explicit solutions are difficult to obtain, unless the system has special properties. Numerical algorithms are usually applied to solve more general models, while relaxations may be involved.

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The study on the model of a single state process, sensor, and estimator provides basic results of the sensor scheduling problem. In this model, one constraint of limited communication times is often taken into consideration where within a finite time horizon N only $d < N$ measurements can be transmitted, which is motivated by the limited battery power of the sensor. In [4], Savage and La Scala considered a particular scalar Gauss-Markov system under this constraint and proposed the optimal schedule to minimize the terminal estimation error variance. Yang and Shi [5] also studied a general scalar Gauss-Markov system under the constraint of limited communication times and used the average estimation error variance as the performance metric, and proposed a necessary condition for the optimal schedules. In part of [6], a general higher-order system with a smart sensor which has local computation capability was considered. The authors gave the optimal schedule of the smart sensor to minimize the average estimation error covariance under the constraint of limited communication times. Besides limited communication times, other forms of constraints are also investigated. Ren *et al.* [7] considered that the sensor has two transmission energy levels and the high level corresponds to a high packet reception ratio. They gave the optimal dynamic schedule to minimize the average estimation error for a fixed overall communication energy budget. Online schedules are also applied to improve the estimation performance, such as [8], [9], which made use of real-time measurements based on the optimal offline schedules.

Another model is that multiple sensors measure one process and communicate with a single estimator, which is often related to the problem of centralized state estimation. For this model, usually limited sensors are allowed to communicate at each time instance, mainly resulted from constrained bandwidth. Gupta *et al.* [10] investigated how to *stochastically* select one sensor for estimating the dynamic state of a linear system and provided upper and lower bounds of the expected error covariance. Joshi and Boyd [11] studied a problem to select a subset out of a group of sensors to estimate a *static vector*. They relaxed the original problem and proposed solution by means of convex optimization techniques. Mo *et al.* [12] considered how to select a subset of sensors in a tree topology to communicate with an estimator at the root at each time step, in order to minimize the asymptotic expected estimation error covariance. They relaxed the objective to a lower bound of the original one and proposed a *stochastic* sensor scheduling algorithm to randomly select a subset of sensors according to a probability distribution to be designed, which is solved by convex optimization algorithms. Huber [13] investigated a scheduling problem where only one sensor can transmit the measurement at each time and the average estimation error covariance over a finite time horizon is to be minimized. The author viewed this scheduling problem as tree searching and tackled the problem using standard tree pruning algorithms. Zhao *et al.* [14] proved that when considering the cost function of the limit supremum of the average error covariances over the infinite horizon, the optimal cost and the corresponding optimal schedules are independent of the covariance of the initial state, and the optimal cost can be approached arbitrarily close by a periodic schedule. Smart sensors are also involved in the model, such as in [6]. In [15], the sensors are allowed to send a packet of past measurements to the estimator and the optimal periodic schedule are proposed explicitly.

More complex models have also been investigated. For one with a single process and multiple sensors and estimators, Yang and Shi [16] considered an agent network where each agent owns a sensor and an estimator, and limited communication channels between them are available. They proposed the optimal schedules explicitly to minimize the global estimation performance for scalar systems with general sensors and for higher-order systems with sensors having identical sensing capability. For the model with multiple processes, Xu and Hespanha [17] studied a system consisting of two coupled processes each of which is able to compute the estimates of both processes and sends its local state to the other. They used dynamic programming to solve the stochastic optimal schedule of data transmission which minimizes a cost counting both the estimation performance and the transmission rates. Savage and La Scala [4] also considered a group of state processes to be measured by corresponding sensors and only one sensor is able to send the data to an estimator. For several special systems, the authors gave the optimal schedules to minimize a terminal cost at a particular time. Shi and Zhang [18] studied a case of two processes measured by associated sensors which send the measurements to a remote estimator, and they provided an explicit optimal periodic schedule of the sensor transmission to minimize the average estimation error over a given time horizon.

In this paper, we focus on the sensor selection problem (alternatively static sensor scheduling problem) for centralized state estimation. A group of sensors take measurement of a state process and send the data to a remote estimator for state estimation while a limited number of sensors are allowed to transmit the data. This paper intends to answer a basic question: if fixed sensors are used during the working process, which ones should be chosen? This question has not been satisfactorily answered yet in the literature and needs to be deeply investigated. Besides, seeing the nature that optimal time varying schedules are usually difficult and complicated to obtain, the solution to this problem provides a satisfactory suboptimal option for sensor scheduling due to the simplicity in its realization.

The methods in the existing literature are not yet sufficient to solve this problem. The bounds given in [10] are obtained by selecting only one sensor at each time. In [11], though the problem is solved efficiently by convex optimization algorithms, it only applies to the estimation of a static vector and cannot be extended easily to the estimation of a dynamic process. For a general setting without special properties of the sensors, the tree pruning method in [13] cannot be applied and inefficient searching algorithms have to be involved. In [12], the relaxed objective to find a schedule to minimize a lower bound of the objective is also not satisfactory. The novelty and contributions of this paper are summarized as follows:

1) For reliable communication channels, we give a closed-form optimal selection for a class of systems where the sensors satisfy some special conditions; and we transform the problem into a convex optimization one for general systems without special restrictions on the sensors.
2) This paper also investigates a model with packet-dropping channels, while reliable channels were mostly considered in sensor scheduling/selection problems in the existing literature. In this case, as the asymptotic estimation error does not exist due to the uncertainty of the channel, one upper bound of the expected estimation error covariance is found and taken as the objective to minimize, and the problem is also transformed to a convex one and solved efficiently.

The remainder of the paper is organized as follows. Section II presents the basic mathematical setup and description of the problem to be studied. Sections III and IV consider the sensor selection problem over reliable communications and packet-dropping channels, respectively. Examples and conclusions are given in the end.

Notations: $\mathbb{Z}_+$ is the set of non-negative integers. $k \in \mathbb{Z}_+$ is the time index. $\mathbb{R}$ is the set of real numbers. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{S}_+^n$ (and $\mathbb{S}_{++}^n$) is the set of n by n positive semi-definite matrices (and positive definite matrices). When $X \in \mathbb{S}_+^n$ (and $\mathbb{S}_{++}^n$), it is written as $X \geq 0$ (and $X > 0$). $X \geq Y$ if $X - Y \in \mathbb{S}_+^n$. $\boldsymbol{E}[X]$ or $\boldsymbol{E}X$ is the expectation of a random variable X and $\boldsymbol{E}[\cdot|\cdot]$ denotes the conditional expectation. $\mathrm{Tr}(\cdot)$ is the trace of a matrix. $\lfloor x \rfloor$ denotes the largest integer which is smaller or equal to x. For functions f, f_1, f_2 with appropriate domains, $f_1 f_2(x)$ stands for the function composition $f_1(f_2(x))$, and $f^n(x) \triangleq f(f^{n-1}(x))$ with $f^0(x) \triangleq x$.

II. Problem Setup

A. System Model

We consider a discrete linear time-invariant system:

$$x_{k+1} = Ax_k + w_k, \tag{1}$$

$$y_k^i = C_i x_k + v_k^i, \quad i = 1, 2, \ldots, N, \tag{2}$$

which is a single dynamic process measured by N sensors. In (1) and (2), $x_k \in \mathbb{R}^n$ is the system state at time k and $y_k^i \in \mathbb{R}^{p_i}$ is the measurement taken by sensor i. Both $\{w_k\}$ and $\{v_k^i\}$ are noise processes which are white, zero-mean, Gaussian and $\boldsymbol{E}[w_k w_j'] = \delta_{kj} Q (Q \geq 0)$ and $\boldsymbol{E}[v_k^i (v_j^i)'] = \delta_{kj} R_i (R_i > 0)$. They are also independent processes, i.e., $\boldsymbol{E}[w_k (v_j^i)'] = 0, \forall j, k$. The initial state x_0 is a zero-mean Gaussian random vector that is uncorrelated with w_k and v_k^i for any k and i and has covariance $\Pi \geq 0$. Assume that C_i has full row rank.

By defining

$$\begin{aligned} y_k &\triangleq \left((y_k^1)', (y_k^2)', \ldots, (y_k^N)' \right)', \\ C &\triangleq (C_1', C_2', \ldots, C_N')', \\ v_k &\triangleq \left((v_k^1)', (v_k^2)', \ldots, (v_k^N)' \right)', \\ R &\triangleq \mathrm{diag}\{R_1, R_2, \ldots, R_N\}, \end{aligned}$$

the overall measurement equation can be written as

$$y_k = Cx_k + v_k. \tag{3}$$

We assume that the pair (C_i, A) is detectable.

B. Sensor Scheduling

The sensors, after taking measurements, transmit their data to a remote estimator via wireless communication channels. In practical applications, the transmission is likely to be constrained by limited resources such as finite communication bandwidth or limit communication energy. As such, one sensor may not be assigned to transmit its measurement at some time instances. To specify the measurement transmission, for sensor i at time k, define the *sensor scheduling variable* γ_k^i as:

$$\gamma_k^i = \begin{cases} 1, & y_k^i \text{ is sent,} \\ 0, & y_k^i \text{ is not sent.} \end{cases}$$

Moreover, we denote the set of all the sensor scheduling variables at time k by $\boldsymbol{\gamma}_k$:

$$\boldsymbol{\gamma}_k \triangleq \left\{ \gamma_k^1, \gamma_k^2, \ldots, \gamma_k^N \right\},$$

and let a *sensor schedule* $\boldsymbol{\theta}$ be the set of $\boldsymbol{\gamma}_k$ over the entire time horizon:

$$\boldsymbol{\theta} \triangleq \{\boldsymbol{\gamma}_k\}_{k=1}^{\infty}.$$

Two types of communication constraints are often considered:

1) *Limited number of available channels*: Only $d (d < N)$ channels are available, or equivalently only d sensors can transmit their data, at each time instant. This model applies when the communication bandwidth is limited, e.g., in a large sensor network using FDMA or TDMA protocol, the available channels are less than the sensors.
2) *Limited power budget with fixed individual power cost*: For sensor i, each transmission incurs a fixed power cost c_i. A power budget $\mathcal{E}$ is imposed at each time instant, i.e.,

$$\sum_{i=1}^{N} \gamma_k^i c_i \leq \mathcal{E}, \quad \forall k. \tag{4}$$

This model can be viewed as an extension of the previous one, with c_i's being identical. The model also has practical applications. The sensors may be distributed in a large terrain and the cost for each one to communicate with the remote estimator is likely to vary. It is also reasonable to specify a power budget for the sensor network at each time to extend its operating time.

C. Channel Model

In practical applications, channels may be reliable or have packet delays or droppings. In this paper, the cases of reliable channels and packet-dropping channels are considered, and we leave the one of delays in our future work.

For packet-dropping channels, when sensor i is selected, we use the *packet-arrival variable* τ_k^i to indicate the arrival of the measurement y_k^i at time k:

$$\tau_k^i = \begin{cases} 1, & y_k^i \text{ is received,} \\ 0, & y_k^i \text{ is dropped,} \end{cases}$$

We assume that $\{\tau_k^i\}$ is a Bernoulli process with mean $\boldsymbol{E}\tau_k^i = \lambda_i$. For reliable channels, data packet will not be dropped which is mathematically equivalent to $\tau_k^i \equiv 1, \forall k \in \mathbb{Z}_+$.

D. Estimation Process

At each time k, the remote estimator runs a Kalman filter to calculate the minimum mean-squared error (MMSE) estimate of the state x_k based on the measurements received from the sensors. We consider packet-dropping channel first. Define

$$\tilde{y}_k \triangleq \left(\gamma_k^1\tau_k^1\left(y_k^1\right)', \gamma_k^2\tau_k^2\left(y_k^2\right)', \ldots, \gamma_k^N\tau_k^N\left(y_k^N\right)'\right)', \quad (5)$$

and let $\tilde{\boldsymbol{Y}}_k \triangleq \{\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_k\}$ be the set of measurements the estimator has received by time k.

Define $\hat{x}_{k|k-1}$ as the *a priori* estimate of x_k, which is the predicted state estimate when the estimator only receives $\tilde{\boldsymbol{Y}}_{k-1}$, and $\hat{x}_{k|k}$ as the *a posteriori* estimate of x_k after updating the measurement $\tilde{y}_k$ further:

$$\hat{x}_{k|k-1} \triangleq \boldsymbol{E}[x_k|\tilde{\boldsymbol{Y}}_{k-1}],$$
$$\hat{x}_{k|k} \triangleq \boldsymbol{E}[x_k|\tilde{\boldsymbol{Y}}_k].$$

Let $P_{k|k-1}$ and $P_{k|k}$ be the estimation error covariance matrices associated with $\hat{x}_{k|k-1}$ and $\hat{x}_{k|k}$, respectively:

$$P_{k|k-1} \triangleq \boldsymbol{E}[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})'|\tilde{\boldsymbol{Y}}_{k-1}],$$
$$P_{k|k} \triangleq \boldsymbol{E}[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})'|\tilde{\boldsymbol{Y}}_k].$$

In the following content, the procedure the estimator calculates these quantities is provided. The estimator first calculates $\hat{x}_{k|k-1}$ and $P_{k|k-1}$ according to the following equations:

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1}, \quad (6)$$
$$P_{k|k-1} = AP_{k-1|k-1}A' + Q, \quad (7)$$

where the recursion starts from $\hat{x}_{0|0} = 0$ and $P_{0|0} = \Pi$.

After receiving the measurements from the sensors, the estimator first fuses the measurements and obtains $\tilde{y}_k$ and meanwhile calculates the following quantities:

$$\tilde{C}_k \triangleq \left(\gamma_k^1\tau_k^1 C_1', \gamma_k^2\tau_k^2 C_2', \ldots, \gamma_k^N\tau_k^N C_N'\right)', \quad (8)$$
$$\tilde{R}_k \triangleq \operatorname{diag}\left\{\gamma_k^1\tau_k^1 R_1, \gamma_k^2\tau_k^2 R_2, \ldots, \gamma_k^N\tau_k^N R_N\right\}. \quad (9)$$

Then it computes $\hat{x}_{k|k}$ and $P_{k|k}$ as follows:

$$P_{k|k} = \left(P_{k|k-1}^{-1} + \sum_{i=1}^{N}\gamma_k^i\tau_k^i C_i' R_i^{-1} C_i\right)^{-1}, \quad (10)$$
$$K_k = P_{k|k}\tilde{C}_k'\tilde{R}_k^{\dagger}, \quad (11)$$
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(\tilde{y}_k - \tilde{C}_k\hat{x}_{k|k-1}), \quad (12)$$

where † represents the Moore-Penrose pseudo-inverse.

For reliable channels, the estimation process are identical to (6)–(12) except that $\tau_k^i = 1$ for all i and k.

E. Problem Description

We consider the following cost function:

$$J(\boldsymbol{\theta}) = \limsup_{T\to\infty}\frac{1}{T}\sum_{k=1}^{T}\operatorname{Tr}(\boldsymbol{E}P_{k|k}), \quad (13)$$

where the expectation is included for covering the packet-dropping scenario with respect to τ_k^i. The main problem studied in this paper is cast as follows.

Problem 1:

$$\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$$
$$\text{s.t.} \sum_{i=1}^{N}\gamma_k^i c_i \le \mathcal{E}, \quad \forall k,$$
$$\gamma_k^i \in \{0,1\}, \quad i = 1, 2, \ldots, N.$$

Generally, in an optimal schedule, γ_k^i should be time varying. However, it is difficult to analyze or obtain a deterministic time varying optimal schedule ([12], [13], etc). In this paper, we seek for time-invariant schedules, i.e., once a sensor is chosen, it keeps working within the whole time horizon, Consequently, γ_k^i is identical for all k for each i and hence it can be denoted as γ^i. Accordingly, the presentation of a sensor schedule is reduced to a *sensor selection* $\boldsymbol{\theta} = \{\gamma^1, \gamma^2, \ldots, \gamma^N\}$.

III. Sensor Selection Over Reliable Channels

This section considers the sensor selection problem over reliable communications, i.e., $\tau_k^i = 1$ for all i and k. In this section we assume that the pair (C_i, A) is detectable and $(A, \sqrt{Q})$ is controllable. As a result, $P_{k|k}$ converges to a steady-state value exponentially fast according to standard Kalman filtering analysis [2]. Define

$$\overline{P} \triangleq \lim_{k\to\infty} P_{k|k}. \quad (14)$$

The recursion (6)–(12) shows that $\overline{P}$ depends on the underlying sensor selection $\boldsymbol{\theta}$. Therefore, $\overline{P}$ can be also written as $\overline{P}(\boldsymbol{\theta})$. Since the estimation error covariance enters steady state exponentially fast, the objective function (13) is transformed to:

$$J(\boldsymbol{\theta}) = \operatorname{Tr}(\overline{P}(\boldsymbol{\theta})). \quad (15)$$

We propose the first problem as follows. We seek for optimal selections minimizing the trace of the steady-state estimation error covariance under the constraint of limited available channels:

Problem 2:

$$\min_{\boldsymbol{\theta}} \operatorname{Tr}(\overline{P}(\boldsymbol{\theta}))$$
$$\text{s.t.} \sum_{i=1}^{N}\gamma^i \le d,$$
$$\gamma^i \in \{0,1\},$$

where the given $d(d \in \mathbb{N}, d < N)$ is the number of available channels.

Remark 1: Our previous paper [16] investigated a sensor selection problem with multiple estimators: each sensor is attached with an estimator and the global estimation performance is considered. Comparatively, in this paper, we consider how to select sensors to optimize the performance of a single estimator, which is a different problem. Moreover, the previous paper focused on the models with some restrictions, such as identical sensors or sensors with comparable sensing capability, and proposed closed-form optimal selections based on the analysis of the estimation quantities. In this paper, the setting of the sensors removes those restrictions and hence is more general. Accordingly, the optimal selections are given by the solutions of convex optimization problems instead of explicit specification.

Let the solution of Problem 2 be $\boldsymbol{\theta}^\star = \{\gamma_\star^1, \gamma_\star^2, \ldots, \gamma_\star^N\}$ and denote the optimal value by $J^\star$.

A. Explicit Solution for a Class of Systems

We begin to study Problem 2 by analyzing the properties of the steady error covariance $\overline{P}$. For sensor i, define the *sensing precision matrix* [16] S_i as

$$S_i \triangleq C_i' R_i^{-1} C_i$$

and the *assimilated sensing precision matrix* $\tilde{S}$ as

$$\tilde{S} \triangleq \sum_{i=1}^N \gamma^i C_i' R_i^{-1} C_i.$$

Remark 2: From (10), one can see the meaning of the sensing precision matrix S_i: it indicates the contribution from sensor i to the estimation performance of the remote estimator.

Further define two operators $h : \mathbb{S}_+^n \to \mathbb{S}_+^n$ and $\tilde{g} : \mathbb{S}_+^n \times \mathbb{S}_+^n \to \mathbb{S}_+^n$ as follows:

$$\begin{aligned} h(X) &\triangleq AXA' + Q, \\ \tilde{g}(X;S) &\triangleq ([h(X)]^{-1} + S)^{-1}, \end{aligned}$$

and define

$$W(S) \triangleq \lim_{k\to\infty} \tilde{g}^k(X;S),$$

where $\tilde{g}^k(X;S) = \tilde{g}(\tilde{g}^{k-1}(X;S);S)$. Then recursive update (10) for $P_{k|k}$ can be written as

$$P_{k|k} = \tilde{g}(P_{k-1|k-1}; \tilde{S}).$$

Accordingly,

$$\overline{P} = W(\tilde{S}),$$

from which one can see that $\overline{P}$ is determined by $\tilde{S}$. One important result about $W(S)$ is given by [16] presented in the following lemma.

Lemma 1: $W(S)$ is matrix monotonically decreasing with respect to S in $\mathbb{S}_+^n$.

This property implies that, for sensor i and j, if partial orders exists between their sensing precision matrices, the contribution of the two to the quality of remote estimation can be compared. We have the following result.

Lemma 2: Assume that $S_i \geq S_j$. For two selections $\boldsymbol{\theta}_{(1)} = \left\{\gamma_{(1)}^1, \gamma_{(1)}^2, \ldots, \gamma_{(1)}^N\right\}$ and $\boldsymbol{\theta}_{(2)} = \left\{\gamma_{(2)}^1, \gamma_{(2)}^2, \ldots, \gamma_{(2)}^N\right\}$, let $\gamma_{(1)}^n = \gamma_{(2)}^n$ for all n except $n = i, j$, where $\gamma_{(1)}^i = 1, \gamma_{(1)}^j = 0$, and $\gamma_{(2)}^i = 0, \gamma_{(2)}^j = 1$. Let $\overline{P}_{(1)}$ and $\overline{P}_{(2)}$ be the steady-state estimation error covariances associated with $\boldsymbol{\theta}_{(1)}$ and $\boldsymbol{\theta}_{(2)}$, respectively. Then

$$\overline{P}_{(1)} \leq \overline{P}_{(2)}. \tag{16}$$

Proof: The argument directly follows from Lemma 1. ■

Remark 3: Notice that (16) holds for matrices, which directly leads to the trace inequality.

Intuitively, sensor i provides more precise measurements than sensor j and one should exclude sensor j. As a direct extension, if all the sensors have partial orders, we have following result.

Theorem 1: If the sensing precision matrices of all sensors have partial orders, without loss of generality, let $S_1 \geq S_2 \geq \cdots \geq S_N$, then the solution $\boldsymbol{\theta}_\star$ to Problem 2 is given as follows:

$$\gamma_\star^i = \begin{cases} 1, & i = 1, 2, \ldots, d, \\ 0, & i = d+1, d+2, \ldots, N, \end{cases} \tag{17}$$

i.e., sensor 1 to sensor d are chosen.

Remark 4: For a first-order system, the sensing precision matrices (in this case are all scalars) can be totally ordered, hence Theorem 1 applies.

B. Numerical Solution for General Systems

In this subsection, we study the more general scenario where a partial order among the sensing precision matrices does not necessarily exist. We first present some relevant analysis. Another important result on $W(S)$ is given in [16].

Theorem 2: $W(S)$ is matrix convex with respect to S in $\mathbb{S}_+^n$.

Theorem 2 implies the following result.

Corollary 1: Define $\mathcal{W}(t_1, \ldots, t_N) \triangleq W\left(\sum_{i=1}^N t_i S_i\right), t_i \in \mathbb{R}_+$. It is matrix monotonically decreasing and convex in each t_i.

Notice that $\overline{P} = W(\tilde{S}) = \mathcal{W}(\gamma^1, \ldots, \gamma^N)$. Then Corollary 1 shows that the objective of Problem 2 is convex in the optimization variables. Hence, we are likely to apply convex optimization to solve it. However, the implicit form of $\overline{P}$ with respect to the optimization variables prevents the application of numerically tractable algorithms. Therefore, we turn to alternative methods to obtain $\overline{P}$. Using Cholesky factorization, S_i can be factorized as

$$S_i = H_i' H_i.$$

Let

$$H \triangleq (H_1', H_2', \ldots, H_N')'.$$

Define

$$\Gamma \triangleq \operatorname{diag}\left\{\gamma^1 I_{p_1}, \gamma^2 I_{p_2}, \ldots, \gamma^N I_{p_N}\right\},$$

where I_{p_i} is the identity matrix with order p_i, i.e., the order of y_k^i. We define the operator $\hat{g}(X;S)$ as

$$\hat{g}(X;\Gamma) \triangleq ([h(X)]^{-1} + H'\Gamma H)^{-1}. \tag{18}$$

Notice that $P_{k|k}$ satisfies $P_{k|k} = \hat{g}(P_{k-1|k-1};\Gamma)$. Hence $\overline{P}$ satisfies $\overline{P} = \hat{g}(\overline{P};\Gamma)$.

Problem 2 is equivalent to the following problem:

Problem 3:

$$\begin{aligned} &\min_{\boldsymbol{\theta}, X} \operatorname{Tr}(X) \\ &\text{s.t. } X \geq \hat{g}(X;\Gamma), \\ &\qquad \sum_{i=1}^N \gamma^i \leq d, \\ &\qquad \gamma^i \in \{0,1\}. \end{aligned}$$

Problem 3 is still not solvable using any efficient numerical algorithm since the feasible domains given by $X \geq \hat{g}(X;\Gamma)$

and $\gamma^i \in \{0,1\}$ are not convex. For the former inequality, one has the following result.

Lemma 3: If (A, C) is detectable and $(A, \sqrt{Q})$ is controllable, the following statements are equivalent:

1) $\exists X$ such that $X \geq \hat{g}(X;\Gamma)$.
2) $\exists Y > 0$ and Z, such that

$$\begin{bmatrix} Y & YA - ZHA & Y - ZH & Z \\ A'Y - A'H'Z' & Y & 0 & 0 \\ Y - H'Z' & 0 & Q^{-1} & 0 \\ Z' & 0 & 0 & \Gamma \end{bmatrix} \geq 0.$$

Moreover, for Y satisfying the inequality in (2), $X = Y^{-1}$ is a solution to the inequality in (1). It is also true conversely.

Proof: See the Appendix. ■

Based on Lemma 3, Problem 2 is equivalent to the following one:

Problem 4:

$$\min_{\boldsymbol{\theta}, X, Y, Z} \operatorname{Tr}(X)$$
$$\text{s.t.} \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} Y & * & * & * \\ A'Y - A'H'Z' & Y & 0 & 0 \\ Y - H'Z' & 0 & Q^{-1} & 0 \\ Z' & 0 & 0 & \Gamma \end{bmatrix} \geq 0,$$
$$\sum_{i=1}^{N} \gamma^i \leq d, \gamma^i \in \{0,1\},$$

where the entries filled with stars can be recovered according to the symmetry of the matrix.

Since the feasible domains of γ^i's are discrete, Problem 4 is a Boolean-convex problem which is a common issue for the selection problem when applying numerical methods. A relaxation on the feasible domains is often used to obtain a convex problem [11]. By relaxation we have the following problem:

Problem 5: The problem statement is the same as Problem 4 except that the constraint $\gamma^i \in \{0,1\}$ is replaced by $0 \leq \gamma^i \leq 1$.

Denote the solution by $\boldsymbol{\theta}_\dagger = \{\gamma_\dagger^1, \gamma_\dagger^2, \ldots, \gamma_\dagger^N\}$ and the optimal value by $J^\dagger$. Problem 5 can be efficiently solved by proper numerical algorithms. Although it is not equivalent to the original problem, the optimal objective value of this relaxed problem is clearly seen to be a lower bound of Problem 2: $J^\dagger \leq J^\star$. The elements of $\boldsymbol{\theta}_\dagger$ may be fractional. We use $\boldsymbol{\theta}_\dagger$ to obtain a feasible solution to Problem 2, denoted as $\boldsymbol{\theta}_f$, which chooses the first d largest element of $\boldsymbol{\theta}_\dagger$. Although Problem 5 is a relaxed one, the discretized solution $\boldsymbol{\theta}_f$ should be close to or may even coincide with the optimal selection $\boldsymbol{\theta}^\star$.

Remark 5: The algorithm can be improved by using Lemma 2. If $S_i \geq S_j$, sensor i is always prior to sensor j to be chosen.

Remark 6: We extend the communication constraints to the limited power budget with fixed individual power budget and have the following problem:

$$\min_{\boldsymbol{\theta}} \operatorname{Tr}(\overline{P}(\boldsymbol{\theta}))$$
$$\text{s.t.} \sum_{i=1}^{N} \gamma^i c_i \leq \mathcal{E},$$
$$\gamma^i \in \{0,1\}.$$

The problem can be solved by the similar method in the previous subsections. If the cost of each sensor is identical, this problem is equivalent to Problem 2. Moreover, similarly to Lemma 2, for sensor i and j, if $S_i \geq S_j$ and $c_i \leq c_j$, then sensor i is better than sensor j.

IV. Sensor Selection Over Packet-Dropping Channels

In the previous section, the sensor selection problem is considered when the channels are reliable. In many practical applications, channels are likely to be unreliable, such as fading, packet delays, and droppings. In this section, we take the unreliable channels into consideration and investigate the sensor selection problem over packet-dropping channels.

In this part, we study the *a priori* error covariance $P_{k+1|k}$ instead of the *a posteriori* one to simplify the analysis. Define

$$T_k \triangleq \operatorname{diag}\left\{\tau_k^1 I_{p_1}, \tau_k^2 I_{p_2}, \ldots, \tau_k^N I_{p_N}\right\},$$

where I_{p_i} are identity matrix with order p_i, the order of y_k^i. The corresponding recursive update equation of $P_{k+1|k}$ can be written as:

$$P_{k+1|k} = A\left(P_{k|k-1}^{-1} + C\Gamma T_k R^{-1} T_k \Gamma C'\right)^{-1} A' + Q. \tag{19}$$

The unreliable property of the channels brings in the following new complexities compared to the case of reliable channels. First, $P_{k+1|k}$ depends on T_k and is hence time-varying. Consequently, the steady-state error covariance $\overline{P}$ does not exist, which enhances the complexity in the indication of estimation quality. Second, the stability of the filter needs to be considered, as there is a probability that the estimation error diverges if the packet-dropping rate is too high.

A. Stability Analysis

For the case of one single channel, [19] studied the filter stability regarding the intermittent measurements. Similarly, for the case of multiple unreliable channels considered here, we analyze the filter stability for some given sensor selection $\boldsymbol{\theta}$ in this subsection.

For notation simplicity, denote $P_{k|k-1}$ as P_k. Since P_k is stochastic, only statistical properties can be deduced. We analyze the property of the mean error covariance $\boldsymbol{E}P_k$. In the following part, we provide an upper bound of $\boldsymbol{E}P_k$. Furthermore, we give a necessary and sufficient condition on the convergence of the upper bound and hence that of $\limsup_{k\to\infty} \boldsymbol{E}P_k$.

We first assume that $\gamma^i = 1$ for all i, i.e., $\Gamma = I$. The case with general Γ will be analyzed later. Then (19) can be reduced to

$$P_{k+1} = A\left(P_k^{-1} + C' T_k R^{-1} T_k C\right)^{-1} A' + Q. \tag{20}$$

Before we state the main result, we present some preliminaries. Denote the set of λ_i by

$$\boldsymbol{\lambda} \triangleq \{\lambda_1, \lambda_2, \ldots, \lambda_N\},$$

where λ_i is the mean of the Bernoulli process $\{\tau_k^i\}$. For the channel of sensor i, denote the variance of the random process $\{\tau_k^i\}$ as σ_i^2, which is given by $\sigma_i^2 = \lambda_i(1-\lambda_i)$. Further define

$$q_i \triangleq \frac{\lambda_i^2}{\sigma_i^2} = \frac{\lambda_i}{1-\lambda_i}.$$

Moreover, let

$$\begin{aligned} D_{SNR} &\triangleq \operatorname{diag}\{q_1 I_{p_1}, q_2 I_{p_2}, \ldots, q_N I_{p_N}\}, \\ \mathcal{I} &\triangleq \operatorname{diag}\left\{\mathbf{1}_{p_1}\mathbf{1}_{p_1}', \mathbf{1}_{p_2}\mathbf{1}_{p_2}', \ldots, \mathbf{1}_{p_N}\mathbf{1}_{p_N}'\right\}, \end{aligned}$$

$\mathbf{1}_{p_i}$ is the column vector of dimension p_i with all components equal to 1.

Define the modified algebraic Riccati equation (MARE) for the Kalman filter with intermittent measurements through multiple channels as follows:

$$\begin{aligned} g_{\boldsymbol{\lambda}}(X) \triangleq & AXA' + Q \\ & - AXH'\left(W \odot (HXH' + I)\right)^{-1} HXA', \end{aligned} \tag{21}$$

where $W = \mathbf{1}\mathbf{1}' + D_{SNR}^{-1}\mathcal{I}$, and $\odot$ is the Hadamard product representing the elementwise matrix multiplication. The subscript $\boldsymbol{\lambda}$ of $g_{\boldsymbol{\lambda}}(X)$ means that it is also determined by the mean set $\boldsymbol{\lambda}$. We provide an upper bound of $\boldsymbol{E}P_k$ in the next theorem.

Theorem 3: For a sequence $\{V_k\}$ satisfying $V_1 = \boldsymbol{E}P_1$ and $V_{k+1} = g_{\boldsymbol{\lambda}}(V_k)$, we have

$$\boldsymbol{E}P_k \le V_k, \quad \forall k.$$

Proof: See the Appendix. ■

Define

$$\begin{aligned} \Lambda &\triangleq \operatorname{diag}\{\lambda_1 I_{p_1}, \lambda_2 I_{p_2}, \ldots, \lambda_N I_{p_N}\}, \\ \Sigma &\triangleq \operatorname{diag}\{\sigma_1 I_{p_1}, \sigma_2 I_{p_2}, \ldots, \sigma_N I_{p_N}\}. \end{aligned}$$

The following theorem provides a necessary and sufficient condition for the convergence of $\{V_k\}$.

Theorem 4: A necessary and sufficient condition for $\{V_k\}$ to converge, namely $X = g_{\boldsymbol{\lambda}}(X)$ has one unique positive definite solution, is given as follows.

1) $\exists P > 0$ and L, such that

$$\begin{aligned} P > & (A - L\Lambda H)P(A - L\Lambda H)' \\ & + L((\Sigma^2\mathcal{I}) \odot (HPH'))L'. \end{aligned} \tag{22}$$

2) The system

$$\begin{aligned} x_{k+1} &= A'x_k, \\ y_k &= B'x_k, \end{aligned} \tag{23}$$

has no unobservable eigenvalue on the unit circle, where $BB' = Q$.

Proof: According to [20], the MARE (21) has one unique positive definite solution if and only if

1) for the stochastic system

$$e_{k+1} = (A - LT_kH)'e_k \tag{24}$$

there exists a static L, such that $\lim_{k\to\infty} \boldsymbol{E}[e_k e_k'] = 0$,

2) the system (23) has no unobservable eigenvalue on the unit circle.

For the system (24), let $\Pi_k \triangleq \boldsymbol{E}[e_k e_k']$. We have

$$\begin{aligned} \Pi_k = & (A - L\Lambda H)\Pi_{k-1}(A - L\Lambda H)' \\ & + L((\Sigma^2\mathcal{I}) \odot (H\Pi_{k-1}H'))L'. \end{aligned}$$

According to [21], that $\lim_{k\to\infty} \Pi_k = 0$ holds is equivalent to $\exists P > 0$ and L, such that (22) is satisfied. ■

Theorem 5: $\limsup_{k\to\infty} \boldsymbol{E}P_k$ converges if the conditions in Theorem 4 are satisfied. Let $\overline{V}$ satisfies $\overline{V} = g_{\boldsymbol{\lambda}}(\overline{V})$. When $\overline{V}$ exists,

$$\limsup_{k\to\infty} \boldsymbol{E}P_k \le \overline{V}. \tag{25}$$

Proof: Since $\{V_k\}$ is an upper bound of $\{\boldsymbol{E}P_k\}$, the convergence of $\{V_k\}$ implies the boundedness of $\{\boldsymbol{E}P_k\}$. When the limit of $\{V_k\}$ exists, it is also an upper bound of $\limsup_{k\to\infty} \boldsymbol{E}P_k$. ■

For a general Γ where only a subset of the sensors is selected, the discussion will be the same. Without loss of generality, we assume that only sensor $1, 2, \ldots, q$ are chosen. Let

$$\begin{aligned} \hat{C} &\triangleq (C_1', C_2', \ldots, C_q')', \\ \hat{R} &\triangleq \operatorname{diag}\{R_1, R_2, \ldots, R_q\}, \\ \hat{T}_k &\triangleq \operatorname{diag}\left\{\tau_k^1 I_{p_1}, \tau_k^2 I_{p_2}, \ldots, \tau_k^q I_{p_q}\right\}. \end{aligned}$$

Then (19) is reduced to

$$P_{k+1} = A\left(P_k^{-1} + \hat{C}\hat{T}_k\hat{R}^{-1}\hat{T}_k\hat{C}'\right)^{-1} A' + Q,$$

which has the same form as (20), and hence the remaining discussion is the same. P_k can be also analyzed in an alternative way. We define

$$\begin{aligned} \tilde{g}_{\boldsymbol{\lambda}}(X;\Gamma) \triangleq & AXA' + Q \\ & - AXH'\Gamma(W \odot (\Gamma HXH'\Gamma + I))^{-1}\Gamma HXA', \end{aligned}$$

and construct a sequence $\{V_k(\Gamma)\}$ with $V_1(\Gamma) = \boldsymbol{E}P_1$ and $V_{k+1}(\Gamma) = \tilde{g}_{\boldsymbol{\lambda}}(V_k(\Gamma);\Gamma)$. For P_k given by (19), we have

$$\boldsymbol{E}P_k \le V_k(\Gamma), \quad \forall k.$$

If $\{V_k(\Gamma)\}$ converges, let

$$\overline{V}(\Gamma) = \lim_{k\to\infty} V_k(\Gamma),$$

and we have

$$\limsup_{k\to\infty} \boldsymbol{E}P_k \le \overline{V}(\Gamma).$$

B. Optimization Problem

Generally, it is difficult to analyze $\limsup_{k\to\infty} \boldsymbol{E}P_k$ due to the complex iterations. Fortunately, we obtain an asymptotic upper bound $\overline{V}(\Gamma)$ which bounds the performance of $\limsup_{k\to\infty} \boldsymbol{E}P_k$. In this subsection, we use the asymptotic upper bound $\overline{V}(\Gamma)$ of $\limsup_{k\to\infty} \boldsymbol{E}P_k$ as the objective function:

$$J(\boldsymbol{\theta}) = \operatorname{Tr}(\overline{V}(\Gamma)). \tag{26}$$

We propose the sensor selection problem over packet-dropping channels as follows:

Problem 6:

$$\begin{aligned}&\min_{\boldsymbol{\theta}} \operatorname{Tr}(\overline{V}(\Gamma))\\ &\text{s.t. } \sum_{i=1}^{N} \gamma^i c_i \le \mathcal{E},\\ &\gamma^i \in \{0,1\}, i = 1,2,\ldots,N.\end{aligned}$$

This optimization problem is not solvable, since the explicit form of $\overline{V}$ is unknown. We present the following result to make it numerically solvable.

Lemma 4: The following statements are equivalent:

1) $\exists X > 0$, such that $X \ge \tilde{g}_{\boldsymbol{\lambda}}(X;\Gamma)$.

2) $\exists Y > 0, Z$ such that

$$\Theta = \begin{bmatrix} \Theta_1 & \Theta_2 \\ \Theta_2' & \Theta_3 \end{bmatrix} \ge 0, \tag{27}$$

where

$$\Theta_1 = \begin{bmatrix} Y & YA - ZH & Y & ZF \\ AY - H'Z' & Y & 0 & 0 \\ Y & 0 & Q^{-1} & 0 \\ FZ' & 0 & 0 & \Gamma \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} ZG\overline{H}_1 & \cdots & ZG\overline{H}_N \\ & \mathbf{0} & \end{bmatrix},$$

and

$$\Theta_3 = \begin{bmatrix} Y & & \\ & \ddots & \\ & & Y \end{bmatrix},$$

in which $\overline{H}_i \triangleq [\mathbf{0}, H_i', \mathbf{0}]'$, obtained by removing all the other blocks in H except $H_i, G \triangleq \sqrt{D_{SNR}^{-1}}$ and $F \triangleq \Lambda^{-\frac{1}{2}}$ which are diagonal matrices.

Moreover, for Y satisfying the inequality in (2), $X = Y^{-1}$ is a solution to the inequality in (1). It is also true conversely.

Proof: See the Appendix. ■

According to Lemma 4, Problem 6 is equivalent to the following problem:

Problem 7:

$$\begin{aligned}&\min_{X,Y,Z,\boldsymbol{\theta}} \operatorname{Tr}(X)\\ &\text{s.t. } \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \ge 0,\\ &\begin{bmatrix} \Theta_1 & \Theta_2 \\ \Theta_2' & \Theta_3 \end{bmatrix} \ge 0,\\ &\Theta_1 = \begin{bmatrix} Y & YA - ZH & Y & ZF \\ AY - H'Z' & Y & 0 & 0 \\ Y & 0 & Q^{-1} & 0 \\ FZ' & 0 & 0 & \Gamma \end{bmatrix},\\ &\Theta_2 = \begin{bmatrix} ZG\overline{H}_1 & \cdots & ZG\overline{H}_N \\ & \mathbf{0} & \end{bmatrix},\\ &\Theta_3 = \begin{bmatrix} Y & & \\ & \ddots & \\ & & Y \end{bmatrix},\\ &\sum_{i=1}^{N} \gamma^i c_i \le \mathcal{E},\\ &\gamma^i \in \{0,1\}, \quad i = 1,2,\ldots,N.\end{aligned}$$

Problem 7 is not convex since the feasible domains for γ^i's are discrete. Similar to Problem 4, we relax this constraint and obtain a convex problem.

Problem 8: The problem statement is the same as Problem 7 except that the constraint $\gamma^i \in \{0,1\}$ is replaced by $0 \le \gamma^i \le 1$.

Denote the solution by $\boldsymbol{\theta}_\ddagger = \{\gamma_\ddagger^1, \gamma_\ddagger^2, \ldots, \gamma_\ddagger^N\}$. Problem 8 can be solved by proper numerical algorithms and the discretization of its solution to achieve a suboptimal selection $\boldsymbol{\theta}_b$ is similar to that in the previous section. Moreover, the stability of the filter needs to be checked when applying $\boldsymbol{\theta}_b$ according to Theorem 4.

V. Examples

In this section, the effectiveness of the numerical algorithms we proposed is illustrated by two examples. To this end, in each example, the optimal sensor selection scheme obtained by brute-force search and a random policy are compared with ours. Note that under a certain constraint, there are many admissible static sensor selection strategies. The random policy refers to a policy that randomly picks the admissible strategies with equal probabilities. In our simulations, the performance of the random policy is computed as the average performance of all the admissible strategies. Through the two examples, we assume there are 10 sensors and the number of available channels N_a varies from 1 to 10. The power cost of each transmission for each sensor is identically one unit, i.e., $c_i = 1, \forall i$. The order of the system is 2, while the dimension of each sensor is either 1 or 2, which is generated randomly.

In the first example, the numerical algorithm proposed in Section III for the scenario where the communication channels are reliable is simulated. The parameters used are generated randomly, the details of which are summarized as follows:

$$\begin{aligned}&A = \begin{bmatrix} 1.24 & 1.17 \\ 0.98 & 0.93 \end{bmatrix}, \quad Q = \begin{bmatrix} 2.23 & 1.02 \\ 1.02 & 3.45 \end{bmatrix},\\ &R_1 = \begin{bmatrix} 0.57 & -0.17 \\ -0.17 & 0.35 \end{bmatrix}, \quad R_4 = \begin{bmatrix} 0.43 & -0.18 \\ -0.18 & 0.41 \end{bmatrix},\\ &R_5 = \begin{bmatrix} 0.53 & -0.15 \\ -0.15 & 0.33 \end{bmatrix}, \quad R_6 = \begin{bmatrix} 0.46 & -0.07 \\ -0.07 & 0.31 \end{bmatrix},\\ &R_9 = \begin{bmatrix} 0.29 & -0.08 \\ -0.08 & 0.32 \end{bmatrix}, R_2 = 0.42,\ R_3 = 0.4,\\ &R_7 = 0.70, \quad R_8 = 0.54, \quad R_{10} = 0.6,\\ &C_1 = \begin{bmatrix} -0.01 & -1.38 \\ 0.61 & 0.7 \end{bmatrix}, \quad C_4 = \begin{bmatrix} 0.32 & -0.14 \\ -0.25 & 0.72 \end{bmatrix},\\ &C_5 = \begin{bmatrix} 0.63 & -0.67 \\ 1.61 & 0.54 \end{bmatrix}, \quad C_6 = \begin{bmatrix} -1.67 & 0.1 \\ 0.15 & 0.5 \end{bmatrix},\\ &C_9 = \begin{bmatrix} -0.75 & -0.62 \\ 1.65 & 0.08 \end{bmatrix}, \quad C_2 = [0.69, 1.34],\\ &C_3 = [-0.28, -0.13], \quad C_7 = [1.16, 0.59],\\ &C_8 = [1.21, 1.39], \quad C_{10} = [0.19, 0.13].\end{aligned}$$

The simulation result is shown in Fig. 2, from which we can see that when the number of available channels is 2 or more, the estimation error covariances of ours and the optimal one are the same. When the number of available sensors is small, our algorithm outperforms significantly the random policy.

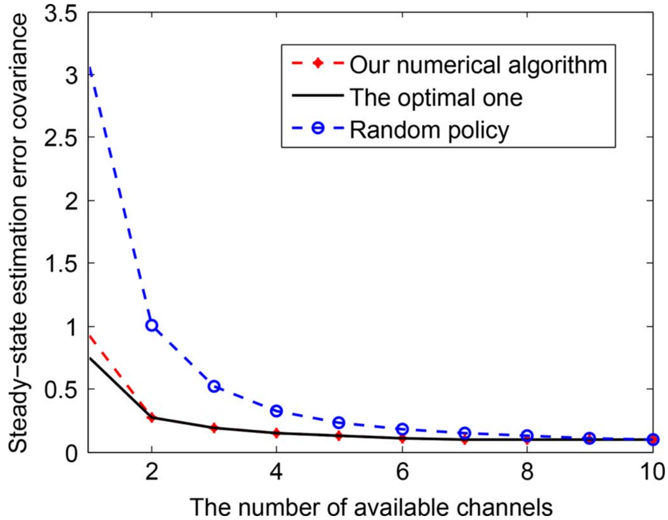

Fig. 2. Steady-state estimation error covariance of different sensor selection schemes as a function of the number of available channels.

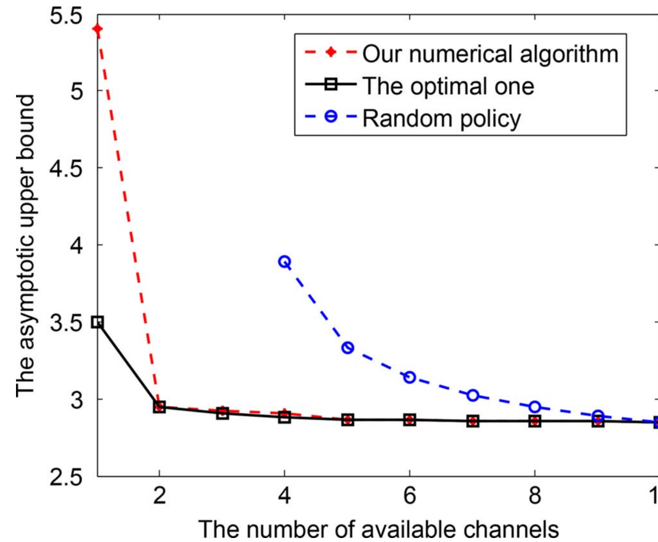

Fig. 3. The asymptotic upper bound of expectation of estimation error covariance, $\overline{V}(\Gamma)$, for different sensor selection schemes as a function of the number of available channels, N_a.

In the second example, the numerical algorithm for the packet-dropping channel proposed in Section IV is simulated. The detailed parameters are listed as follows:

$$
\begin{aligned}
&A=\begin{bmatrix}0.71 & 0.33\\ 0.33 & 1.11\end{bmatrix}, \quad Q=\begin{bmatrix}1.14 & -0.2\\ -0.2 & 1.5\end{bmatrix},\\
&R_1=\begin{bmatrix}0.81 & -0.53\\ -0.53 & 0.66\end{bmatrix}, \quad R_5=\begin{bmatrix}1.23 & -0.95\\ -0.95 & 1.01\end{bmatrix},\\
&R_{10}=\begin{bmatrix}0.45 & -0.17\\ -0.17 & 0.3\end{bmatrix}, \quad R_2=0.32, \quad R_3=0.3,\\
&R_4=1.11, \quad R_6=0.26, \quad R_7=1.28, \quad R_8=1.22, \quad R_9=0.44,\\
&C_1=\begin{bmatrix}2.74 & 4.45\\ 4.01 & 3.32\end{bmatrix}, \quad C_5=\begin{bmatrix}3.82 & 2.35\\ 3 & 0.3\end{bmatrix},\\
&C_{10}=\begin{bmatrix}0.77 & 0.98\\ -0.45 & -0.5\end{bmatrix}, \quad C_2=[3.92, 3.2],\\
&C_3=[-0.33, 2.05], \quad C_4=[0.61, 3.82],\\
&C_6=[2.76, 2.85], \quad C_7=[2.44, 3.13],\\
&C_8=[1.25, -0.63], \quad C_9=[3.04, 3.83],\\
&\boldsymbol{\lambda}=\{0.22, 0.25, 0.9, 0.7, 0.56, 0.18, 0.21, 0.07, 0.91, 0.71\}.
\end{aligned}
$$

As depicted in Fig. 3, the estimation performance of our algorithm is very close to that of the optimal one, both of which are smaller than that of the random policy. Due to packet dropouts of the communication channels, the mean of $\overline{V}(\Gamma)$ for the random policy is unbounded when $N_a \leq 3$. Take $N_a = 1$ for example. If a sensor is chosen whose channel has too low a packet-arrival rate to guarantee the stability of the filter, the corresponding $\overline{V}(\Gamma)$ will be unbounded. Consequently, the average value of $\overline{V}(\Gamma)$ for the random policy is also unbounded.

VI. Conclusion

This paper considered the sensor selection problem for centralized state estimation. A group of sensors take measurement of a state process and send the data to a remote estimator and only limited sensors are allowed to do the transmission. This paper intended to find which sensors should be chosen such that the estimation performance of the estimator is optimized. Both reliable and packet-dropping channels were considered. For both cases, the selection problems were transformed to convex optimization ones which can be efficiently solved and satisfactory suboptimal selections were obtained after discretization of the solution to the convex optimization problems.

In our current work, we focused on the deterministic sensor selection, i.e., the selection is determined offline. As a future work, we will study stochastic selection, i.e., the sensors are chosen online according to some probability, which is to be assigned. Other general communication channel models including packet-delay and fading ones will also be considered.

Appendix

Before we give the complete proofs of lemmas and theorems omitted in the main content, we present some preliminary lemmas.

Lemma 5 (Matrix Inversion Lemma): Let $X > 0$. If $X = A + BD^{-1}C'$, where $B, D > 0$, then

$$X^{-1} = A^{-1} - A^{-1}B(D + C'A^{-1}B)^{-1}C'A^{-1}.$$

Lemma 6: Define the operator

$$\begin{aligned}\psi_{\boldsymbol{\lambda}}(L, X, \Gamma) = AXA' + Q + L\Lambda(W \odot (HXH' + \tilde{\Gamma}))\Lambda L'\\ -AXH'\Lambda L' - L\Lambda HXA', \quad (28)\end{aligned}$$

where $\tilde{\Gamma} = \Gamma + \sigma^2(I - \Gamma)$, and σ is a scalar. Let

$$L^\star(\sigma) = AXH'(W \odot (HXH' + \tilde{\Gamma}))^{-1}\Lambda^{-1}. \quad (29)$$

Then

$$\tilde{g}_{\boldsymbol{\lambda}}(X; \Gamma) = \lim_{\sigma\to\infty} \psi_{\boldsymbol{\lambda}}(L^\star(\sigma), X, \Gamma). \quad (30)$$

Moreover, $L^\star \triangleq \lim_{\sigma\to\infty} L^\star(\sigma)$ exists,

$$L^\star = AXH'\Gamma\left(W \odot (\Gamma HXH'\Gamma + I)\right)^{-1}\Gamma\Lambda^{-1}. \quad (31)$$

Proof: We have

$$\psi_{\boldsymbol{\lambda}}(L^\star(\sigma), X, \Gamma) = AXA' + Q - AXH'(W \odot (HXH' + \tilde{\Gamma}))^{-1}HXA'.$$

Let

$$M(\sigma) \triangleq (W \odot (HXH' + \tilde{\Gamma}))^{-1},$$
$$\bar{M} \triangleq \Gamma(W \odot (\Gamma HXH'\Gamma + I))^{-1}\Gamma.$$

We just need to verify $\lim_{\sigma\to\infty} M(\sigma) = \bar{M}$. We have

$$M(\sigma) = (W \odot (HXH') + W \odot \tilde{\Gamma})^{-1}.$$

It can be easily verified that $W \odot (HXH') > 0$ and is hence invertible. Since $\tilde{\Gamma}$ is diagonal,

$$W \odot \tilde{\Gamma} = (\mathbf{11}' + D_{SNR}^{-1}\mathcal{I}) \odot \tilde{\Gamma} = (I + D_{SNR}^{-1})\tilde{\Gamma} = \Lambda^{-1}\tilde{\Gamma} > 0$$

and is invertible. Moreover,

$$\begin{aligned}\lim_{\sigma\to\infty}(W \odot \tilde{\Gamma})^{-1} &= \lim_{\sigma\to\infty}\tilde{\Gamma}^{-1}\Lambda \\ &= \lim_{\sigma\to\infty}\left(\Gamma + \frac{1}{\sigma^2}(I-\Gamma)\right)(W \odot I)^{-1} \\ &= \Gamma(W \odot I)^{-1} \\ &= \Gamma(W \odot I)^{-1}\Gamma.\end{aligned}$$

By Matrix Inversion Lemma, we have

$$\begin{aligned}&\lim_{\sigma\to\infty} M(\sigma) \\ &= \lim_{\sigma\to\infty}\left[(W \odot \tilde{\Gamma})^{-1} - (W \odot \tilde{\Gamma})^{-1}\left((W \odot \tilde{\Gamma})^{-1} + (W \odot (HXH'))^{-1}\right)^{-1}(W \odot \tilde{\Gamma})^{-1}\right] \\ &= \Gamma(W \odot I)^{-1}\Gamma - \Gamma(W \odot I)^{-1}\Gamma\Big(\Gamma(W \odot I)^{-1}\Gamma + (W \odot (HXH'))^{-1}\Big)^{-1}\Gamma(W \odot I)^{-1}\Gamma \\ &= \Gamma\left[(W \odot I)^{-1} - (W \odot I)^{-1}\Gamma\Big(\Gamma(W \odot I)^{-1}\Gamma + (W \odot (HXH'))^{-1}\Big)^{-1}\Gamma(W \odot I)^{-1}\right]\Gamma \\ &= \Gamma\left(W \odot I + \Gamma\left(W \odot (HXH')\right)\Gamma\right)^{-1}\Gamma \\ &= \Gamma\left(W \odot I + W \odot (\Gamma HXH'\Gamma)\right)^{-1}\Gamma \\ &= \Gamma\left(W \odot (\Gamma HXH'\Gamma + I)\right)^{-1}\Gamma \\ &= \bar{M},\end{aligned}$$

where the fourth equality follows from Matrix Inversion Lemma. Notice that

$$L^\star(\sigma) = AXH'M(\sigma)\Lambda^{-1}.$$

Hence,

$$\begin{aligned}\lim_{\sigma\to\infty} L^\star(\sigma) &= AXH'\bar{M}\Lambda^{-1} \\ &= AXH'\Gamma\left(W \odot (\Gamma HXH'\Gamma + I)\right)^{-1}\Gamma\Lambda^{-1}.\end{aligned}$$

■

We give the proofs in the following subsections.

A. Proof of Lemma 3

Define

$$\phi(K, X, \Gamma) \triangleq (I - KH)(AXA' + Q)(I - KH)' + K\tilde{\Gamma}K',$$

where $\tilde{\Gamma} = \Gamma + \sigma^2(I - \Gamma)$, and σ is a scalar. We make the following statement: ($\star$) $\exists X > 0$ and $K(\sigma)$, such that $\lim_{\sigma\to\infty} K(\sigma) = K_\infty$ exists and $X \geq \lim_{\sigma\to\infty}\phi\left(K(\sigma), X, \Gamma\right)$ holds.

We show that (1), (2), and ($\star$) are equivalent.

$(\mathbf{1}) \Rightarrow (\star)$*:* When $\exists X > 0$ such that $X \geq \hat{g}(X;\Gamma)$, let

$$K^\star(\sigma) = AXH'(HXH' + \tilde{\Gamma})^{-1}. \tag{32}$$

Notice that

$$\hat{g}(X;\Gamma) = \lim_{\sigma\to\infty}\phi\left(K^\star(\sigma), X, \Gamma\right). \tag{33}$$

This can be shown by either direct calculation (one can refer to the proof of Lemma 6) or by an alternative formulation of $P_{k|k}$ as follows. The measurement received by the estimator from sensor i can be written as

$$\tilde{y}_k^i = C_i x_k + \tilde{v}_k^i,$$

where the probability distribution of $\tilde{v}_k^i$ follows

$$p\left(\tilde{y}_k^i|\gamma^i\right) = \begin{cases}\mathcal{N}(0, R_i), & \gamma^i = 1,\\ \mathcal{N}(0, \sigma^2 R_i), & \gamma^i = 0,\end{cases}$$

for some σ. The scenario that sensor i is not chosen corresponds to the case of $\sigma \to \infty$. Then we have an alternative formulation for $P_{k|k}$:

$$P_{k|k} = \lim_{\sigma\to\infty}[P_{k|k-1} - P_{k|k-1}H'(HP_{k|k-1}H' + \tilde{\Gamma})^{-1}HP_{k|k-1}],$$

which is indeed

$$P_{k|k} = \lim_{\sigma\to\infty}\phi\left(K^\star(\sigma), P_{k-1|k-1}, \Gamma\right).$$

Notice that

$$P_{k|k} = \hat{g}(P_{k-1|k-1};\Gamma).$$

Hence,

$$\hat{g}(X;\Gamma) = \lim_{\sigma\to\infty}\phi\left(K^\star(\sigma), X, \Gamma\right).$$

As a result, X and $K^\star(\sigma)$ satisfy $X \geq \lim_{\sigma\to\infty}\phi\left(K^\star(\sigma), X, \Gamma\right)$.

$(\star) \Rightarrow (\mathbf{1})$*:* Assume that $\exists X > 0$ and $K_0(\sigma)$, such that $X \geq \lim_{\sigma\to\infty}\phi(K_0(\sigma), X, \Gamma)$ holds. Notice that

$$\phi\left(K^\star(\sigma), X, \Gamma\right) = \min_{K(\sigma)}\phi\left(K(\sigma), X, \Gamma\right), \tag{34}$$

where $K^\star(\sigma)$ is given by (32). Then $\phi\left(K_0(\sigma), X, \Gamma\right) \geq \phi\left(K^\star(\sigma), X, \Gamma\right)$. Since $\lim_{\sigma\to\infty}\phi\left(K_0(\sigma), X, \Gamma\right)$ exists, we have

$$\lim_{\sigma\to\infty}\phi\left(K_0(\sigma), X, \Gamma\right) \geq \lim_{\sigma\to\infty}\phi\left(K^\star(\sigma), X, \Gamma\right) = \hat{g}(X;\Gamma).$$

Hence, X also satisfies $X \geq \hat{g}(X;\Gamma)$.

$(\star) \Rightarrow (\mathbf{2})$*:* Assume that $\exists X > 0$ and $K(\sigma)$, such that we have $X \geq \lim_{\sigma\to\infty}\phi\left(K(\sigma), X, \Gamma\right)$. According to the property of limits, $\exists\Delta > 0$, when $\sigma > \Delta, X \geq \phi(K(\sigma), X, \Gamma)$ holds. By the similar approach in Theorem 5 of [19], the inequality $X \geq \phi(K(\sigma), X, \Gamma)$ can be shown to be equivalent to

$$\begin{bmatrix} Y & YA - Z(\sigma)HA & Y - Z(\sigma)H & Z(\sigma) \\ * & Y & 0 & 0 \\ * & 0 & Q^{-1} & 0 \\ * & 0 & 0 & \tilde{\Gamma}^{-1} \end{bmatrix} \geq 0,$$

where $Y = X^{-1}, Z(\sigma) = X^{-1}K(\sigma)$, and the entries replaced by $\star$ can be recovered by the symmetry of the matrix. We denote it as $\Psi(\sigma) \geq 0$ for short.

Since $\Psi(\sigma) \geq 0$ when $\sigma > \Delta$, then $\lim_{\sigma\to\infty} \Psi(\sigma) \geq 0$ holds, according to the property of limits. Notice that

$$\lim_{\sigma\to\infty} \tilde{\Gamma}^{-1} = \lim_{\sigma\to\infty} \left[\Gamma + \frac{1}{\sigma^2}(I - \Gamma)\right] = \Gamma.$$

Consequently, for $Y = X^{-1}$ and $Z = X^{-1} \lim_{\sigma\to\infty} K(\sigma) = X^{-1}K_\infty$,

$$\begin{bmatrix} Y & YA - ZHA & Y - ZH & Z \\ A'Y - A'H'Z' & Y & 0 & 0 \\ Y - H'Z' & 0 & Q^{-1} & 0 \\ Z' & 0 & 0 & \Gamma \end{bmatrix} \geq 0$$

holds. Thus (2) is verified.

$(\mathbf{2}) \Rightarrow (\star)$*:* The proof is similar to that of $(\star) \Rightarrow (2)$.

Consequently, (1) is equivalent to (2), and for Y satisfying the inequality in (2), $X = Y^{-1}$ is a solution to the inequality in (1).

B. Proof of Theorem 3

Define an operator

$$\Phi(L, X, T) \triangleq (A - LTH)X(A - LTH)' + Q + LTITL'.$$

Equation (20) can be further written as

$$\begin{aligned} P_{k+1} &= A(P_k^{-1} + HT_kIT_kH')^{-1}A' + Q \\ &= AP_kA' + Q \\ &\quad - AP_kH'T_k(T_kHP_kH'T_k + T_kIT_k)^\dagger T_kHP_kA'. \end{aligned}$$

Then

$$P_{k+1} = \Phi(\bar{K}_k, P_k, T_k),$$

where

$$\bar{K}_k = AP_kHT_k'(T_kHP_kHT_k' + T_kIT_k)^\dagger.$$

Moreover,

$$P_{k+1} = \min_L \Phi(L, P_k, T_k).$$

Define

$$L_k \triangleq AV_kH'(W \odot (HV_kH' + I))^{-1}\Lambda^{-1}.$$

We use mathematical induction to prove the argument. When $k = 1$, the argument holds. Assume when $k = l, \boldsymbol{E}P_l \leq V_l$ holds. For time $k = l + 1$, we have

$$P_{l+1} \leq \Phi(L_l, P_l, T_l).$$

Take expectation of both sides with respect to τ_k's,

$$\begin{aligned} &\boldsymbol{E}P_{l+1} \\ &\quad \leq \boldsymbol{E}\Phi(L_l, P_l, T_l) \\ &\quad = \boldsymbol{E}\left[(A - L_lT_lH)P_l(A - L_lT_lH)' + Q + L_lT_lIT_lL_l'\right] \\ &\quad = \boldsymbol{E}\left[(A - L_lT_lH)\boldsymbol{E}P_l(A - L_lT_lH)' + Q + L_lT_lIT_lL_l'\right] \\ &\quad \leq \boldsymbol{E}\left[(A - L_lT_lH)V_l(A - L_lT_lH)' + Q + L_lT_lIT_lL_l'\right] \\ &\quad = AV_lA' + Q + \boldsymbol{E}[L_lT_lHV_lHT_l'L_l'] + \boldsymbol{E}[L_lT_lIT_lL_l'] \\ &\qquad - \boldsymbol{E}[AV_lHT_l'L_l'] - \boldsymbol{E}[L_lT_lHV_lA'] \\ &\quad = AV_lA' + Q + L_l\left(W \odot (\Lambda(HV_lH' + I)\Lambda)\right)L_l' \\ &\qquad - AV_lH'\Lambda L_l' - L_l\Lambda HV_lA' \\ &\quad = g_\Lambda(V_l) \\ &\quad = V_{l+1}. \end{aligned}$$

From mathematical induction, $\boldsymbol{E}P_k \leq V_k$ holds for all $k > 0$.

C. Proof of Lemma 4

We make the following statement:

$(\star)$ $\exists X > 0$ and $L(\sigma)$, such that $L_\infty \triangleq \lim_{\sigma\to\infty} L(\sigma)$ exists and $X \geq \lim_{\sigma\to\infty} \psi_{\boldsymbol{\lambda}}(L(\sigma), X, \Gamma)$ holds, where $\psi_{\boldsymbol{\lambda}}(L(\sigma), X, \Gamma)$ is given by (28).

We prove that (1), (2), and $(\star)$ are equivalent.

$(\mathbf{1}) \Rightarrow (\star)$*:* When $\exists X > 0$ such that $X \geq \tilde{g}_{\boldsymbol{\lambda}}(X; \Gamma)$, consider $L^\star(\sigma)$ given by (29). Lemma 6 shows

$$\tilde{g}_{\boldsymbol{\lambda}}(X; \Gamma) = \lim_{k\to\infty} \psi_{\boldsymbol{\lambda}}(L^\star(\sigma), X, \Gamma)$$

and $\lim_{\sigma\to\infty} L^\star(\sigma)$ exists. Hence, X and $L^\star(\sigma)$ satisfy $X \geq \lim_{k\to\infty} \psi_{\boldsymbol{\lambda}}(L^\star(\sigma), X, \Gamma)$.

$(\star) \Rightarrow (1)$*:* Assume $\exists X>0$ and $L_0(\sigma), X \geq \lim_{\sigma\to\infty} \psi_{\boldsymbol{\lambda}}(L_0(\sigma), X, \Gamma)$ holds. Notice that

$$\psi_{\boldsymbol{\lambda}}(L^\star(\sigma), X, \Gamma) = \min_{L(\sigma)} \psi_{\boldsymbol{\lambda}}(L(\sigma), X, \Gamma). \tag{35}$$

Then we have

$$\psi_{\boldsymbol{\lambda}}(L_0(\sigma), X, \Gamma) \geq \psi_{\boldsymbol{\lambda}}(L^\star(\sigma), X, \Gamma).$$

Hence, we have

$$\begin{aligned} X &> \lim_{\sigma\to\infty} \psi_{\boldsymbol{\lambda}}(L_0(\sigma), X, \Gamma) \\ &\geq \lim_{\sigma\to\infty} \psi_{\boldsymbol{\lambda}}(L^\star(\sigma), X, \Gamma) \\ &= \tilde{g}_{\boldsymbol{\lambda}}(X; \Gamma). \end{aligned}$$

$(\star) \Rightarrow (\mathbf{2})$*:* Assume $\exists X > 0$ and $L(\sigma), X \geq \lim_{\sigma\to\infty} \psi_{\boldsymbol{\lambda}}(L(\sigma), X, \Gamma)$ holds. Then $\exists\Delta > 0$, when $\sigma > \Delta$, we have $X \geq \psi_{\boldsymbol{\lambda}}(L(\sigma), X, \Gamma)$ according to the property of limits. $\psi_{\boldsymbol{\lambda}}(L(\sigma), X, \Gamma)$ can be extended as follows:

$$\begin{aligned} &\psi_{\boldsymbol{\lambda}}(L, X, \Gamma) \\ &\quad = AXA' + Q + L\Lambda(HXH' + \tilde{\Gamma})\Lambda L' - AXH'\Lambda L' \\ &\qquad - L\Lambda HXA' + L\Lambda\left((D_{SNR}^{-1}\mathcal{I}) \odot (HXH' + \tilde{\Gamma})\right)\Lambda L' \\ &\quad = (A - L\Lambda H)X(A - L\Lambda H)' + L\Lambda F\tilde{\Gamma}F\Lambda L' + Q \\ &\qquad + L\Lambda G\left(\sum_{i=1}^N \overline{H}_iX\overline{H}_i'\right)G\Lambda L', \end{aligned}$$

where $L(\sigma)$ is simplified as L, assuming no misunderstanding is caused. From the inequality $X \geq \psi_{\boldsymbol{\lambda}}(L, X, \Gamma)$, we have

$$\begin{aligned} X - L\Lambda F\tilde{\Gamma}F\Lambda L' - Q - L\Lambda G\left(\sum_{i=1}^{N}\overline{H}_i X\overline{H}_i'\right)G\Lambda L' \\ \geq (A - L\Lambda H)X(A - L\Lambda H)' \\ \geq 0. \end{aligned}$$

Then by Schur complement decomposition it is equivalent to

$$\begin{bmatrix} \star\star\star & A - L\Lambda H \\ A' - H'\Lambda L' & X^{-1} \end{bmatrix} \geq 0,$$

where the first entry marked by $\star\star\star$ is the left hand side term of the last inequality. Using several more times the Schur complement decomposition on the first element of the matrix we have

$$\Psi = \begin{bmatrix} \Psi_1 & \Psi_2 \\ \Psi_2' & \Psi_3 \end{bmatrix} \geq 0, \tag{36}$$

where

$$\Psi_1 = \begin{bmatrix} X & A - L\Lambda H & I & L\Lambda F \\ A' - H'\Lambda L' & X^{-1} & 0 & 0 \\ I & 0 & Q^{-1} & 0 \\ F\Lambda L' & 0 & 0 & \tilde{\Gamma}^{-1} \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} L\Lambda G\overline{H}_1 & \cdots & L\Lambda G\overline{H}_N \\ & \mathbf{0} & \end{bmatrix},$$

and

$$\Psi_3 = \begin{bmatrix} X^{-1} & & \\ & \ddots & \\ & & X^{-1} \end{bmatrix}.$$

Let

$$\Xi = \begin{bmatrix} X^{-1} & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \Psi \begin{bmatrix} X^{-1} & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix},$$

then (36) is equivalent to

$$\Xi \geq 0.$$

Let

$$\Xi = \begin{bmatrix} \Xi_1 & \Xi_2 \\ \Xi_2' & \Xi_3 \end{bmatrix}.$$

Then

$$\Xi_1 = \begin{bmatrix} X^{-1} & X^{-1}(A - L\Lambda H) & X^{-1} & X^{-1}L\Lambda F \\ \star & X^{-1} & 0 & 0 \\ \star & 0 & Q^{-1} & 0 \\ \star & 0 & 0 & \tilde{\Gamma}^{-1} \end{bmatrix}$$

where the entries marked by $\star$ can be deduced from the symmetry of the matrix. Moreover, we have

$$\Xi_2 = \begin{bmatrix} X^{-1}L\Lambda G\overline{H}_1 & \cdots & X^{-1}L\Lambda G\overline{H}_N \\ & \mathbf{0} & \end{bmatrix},$$

and

$$\Xi_3 = \Psi_3.$$

Let $Y \triangleq X^{-1}$, and $Z(\sigma) \triangleq X^{-1}L(\sigma)\Lambda$. Define

$$\Theta(\sigma) = \begin{bmatrix} \Theta_1(\sigma) & \Theta_2 \\ \Theta_2' & \Theta_3 \end{bmatrix},$$

where

$$\Theta_1(\sigma) = \begin{bmatrix} Y & YA - Z(\sigma)H & Y & Z(\sigma)F \\ \star & 0 & 0 & \\ \star & 0 & Q^{-1} & 0 \\ \star & 0 & 0 & \tilde{\Gamma}^{-1} \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} ZG\overline{H}_1 & \cdots & ZG\overline{H}_N \\ & \mathbf{0} & \end{bmatrix},$$

and

$$\Theta_3 = \begin{bmatrix} Y & & \\ & \ddots & \\ & & Y \end{bmatrix}.$$

Then $\Xi \geq 0$ is equivalent to $\bar{\Theta}(\sigma) \geq 0$. Hence, we have shown that $X \geq \psi_{\boldsymbol{\lambda}}(L, X, \Gamma)$ is equivalent to $\Theta(\sigma) \geq 0$. Consequently, when $\sigma > \Delta, \bar{\Theta}(\sigma) \geq 0$ also holds, which leads to that $\lim_{\sigma\to\infty} \bar{\Theta}(\sigma) \geq 0$ according to the property of limits. Let $\Theta \triangleq \lim_{\sigma\to\infty} \Theta(\sigma)$ and $\Theta_1 \triangleq \lim_{\sigma\to\infty} \Theta_1(\sigma)$. Then

$$\Theta = \begin{bmatrix} \Theta_1 & \Theta_2 \\ \Theta_2' & \Theta_3 \end{bmatrix},$$

and

$$\Theta_1 = \begin{bmatrix} Y & YA - ZH & Y & ZF \\ AY - Z'H' & Y & 0 & 0 \\ Y & 0 & Q^{-1} & 0 \\ FZ' & 0 & 0 & \Gamma \end{bmatrix},$$

where $Z = X^{-1}\lim_{\sigma\to\infty} L(\sigma)\Lambda = X^{-1}L_\infty\Lambda$. Hence, there exist $Y = X^{-1}$ and $Z = X^{-1}L_\infty\Lambda$, such that $\Theta \geq 0$. (2) is verified.

(2) $\Rightarrow (\star)$*:* The proof is similar to that of $(\star) \Rightarrow (2)$.

From the discussion above, the argument is verified.

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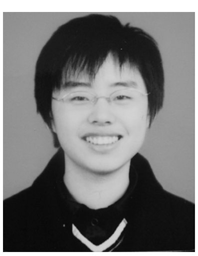

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