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Brief paper

Stochastic sensor activation for distributed state estimation over a sensor network☆

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ABSTRACT

We consider distributed state estimation over a resource-limited wireless sensor network. A stochastic sensor activation scheme is introduced to reduce the sensor energy consumption in communications, under which each sensor is activated with a certain probability. When the sensor is activated, it observes the target state and exchanges its estimate of the target state with its neighbors; otherwise, it only receives the estimates from its neighbors. An optimal estimator is designed for each sensor by minimizing its mean-squared estimation error. An upper and a lower bound of the limiting estimation error covariance are obtained. A method of selecting the consensus gain and a lower bound of the activating probability is also provided.

1. Introduction

A wireless sensor network (WSN) is composed of a large number of geographically distributed sensor nodes, and each sensor is capable of measuring certain parameters of interest such as temperature, humidity or position and velocity of a vehicle. Distributed state estimation using a WSN has attracted increasing attention recently due to its broad applications including battlefield surveillance, intelligent transportation, environment monitoring, health care, etc. A large number of works on consensus-based distributed estimation have been reported since it can drastically reduce the utilization of communication resources, has no requirement on the network topology, and is more flexible for ad-hoc deployment when compared with centralized state estimations (Anderson & Moore, 1979; Iftar, 1993; Rao, Durrant-Whyte, & Sheen, 1993; Sanders, Tacker, & Linton, 1974).

Most of these works (Demetriou, 2010; Federico & Ali, 2010; Kamgarpour & Tomlin, 2008; Li & Ghassan, 2007; Olfati-Saber, 2009; Olfati-Saber & Shamma, 2005; Ren, Beard, & Kingston, 2005; Saber, 2007; Schizas, Ribeiro, & Giannakis, 2008; Shen, Wang, & Hung, 2010; Spanos, Saber, & Murray, 2005a,b; Stanković, Stanković, & Stipanović, 2009; Xi, He, & Liu, 2010; Yu, Chen, Wang, & Yang, 2009) on distributed estimation introduce a consensus scheme to the standard Kalman filter, and they can be broadly classified into two categories: one is to add a consensus term to the Kalman filter update step (Olfati-Saber, 2009; Olfati-Saber & Shamma, 2005; Saber, 2007), and the other is to drive the consensus of the a priori estimate in the Kalman filter prediction step (Federico & Ali, 2010; Stanković et al., 2009). More precisely, those works include distributed weighted average consensus algorithms (Spanos et al., 2005a,b), distributed Kalman filtering (Olfati-Saber, 2009; Olfati-Saber & Shamma, 2005; Saber, 2007), convergence properties of a decentralized Kalman filter (Kamgarpour & Tomlin, 2008), decentralized state estimation with intermittent observations and communication faults (Stanković et al., 2009), diffusion strategies for distributed Kalman filtering and smoothing (Federico & Ali, 2010), distributed estimation of deterministic signals in noisy links (Schizas et al., 2008), distributed parameter estimation over

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a WSN with bit rate constraint (Li & Ghassan, 2007), adaptive consensus filter (Demetriou, 2010; Xi et al., 2010), distributed consensus filtering algorithm with pinning observers (Yu et al., 2009), and distributed H_∞-consensus filtering over a finite-horizon for sensor networks with multiple missing measurements (Shen et al., 2010). More related works can be found from the references therein.

In a typical WSN, communication and computing capabilities of sensors are limited due to various design and implementation considerations such as small battery and finite bandwidth. For example, the energy for data collection and transmission is limited, and sensors may not transmit data all the time. It is an expensive operation to replace or recharge the batteries in many applications. Therefore, proper sensor activation is important and often necessary. It is equivalent to scheduling sensor data transmission. Sensor scheduling for centralized state estimation and control has attracted some attention in the past (Gupta, Chung, Hassibi, & Murray, 2006; Shi, Cheng, & Chen, 2011; Walsh & Ye, 2001). Vitus, Zhang, Abate, Hu, and Tomlin (2012) considered the problem of selecting one from a group of sensors at each time step to minimize a weighted function of the state estimation error. There are few studies, however, on sensor scheduling for distributed state estimation.

In this paper, we consider distributed state estimation over a wireless sensor network and introduce a stochastic sensor activation scheme for a consensus-based distributed estimation algorithm. In real applications, a stochastic sensor scheduling strategy is easy to implement and is computationally tractable. For example, Gupta et al. (2006) proposed a stochastic sensor selection strategy for the case that multiple sensors cannot operate simultaneously and their measurements need to be scheduled. Mo, Ambrosino, and Sinopoli (2011) proposed a stochastic sensor selection strategy to minimize the remote state estimation errors. In our stochastic sensor scheduling strategy, each sensor implements a minimum mean-squared error estimator for estimating the target state, and is activated with a certain probability. When the sensor is activated, it observes the target state and exchanges data with its neighbors; otherwise, it only receives data from its neighbors. It is challenging to analyze the mean-square stability of the state estimators as the estimation errors at different sensors are correlated. Instead, we derive an upper and a lower bound of the steady-state estimation error covariance. In the meantime, we obtain an upper bound of the consensus gain and a lower bound of the activating probability by solving a few linear matrix inequalities (LMIs).

The remainder of the paper is organized as follows. In Section 2, we describe the system model and design an optimal distributed state estimator under stochastic sensor scheduling. In Section 3, we study the stability of the expected estimation error covariance. An upper bound of the consensus gain and a lower bound of the activating probability are given by solving a few LMIs. Differences between our estimator and one existing estimator are compared and discussed. In Section 4, a numerical example is provided. Finally, some concluding remarks and future work are presented in Section 5.

2. Problem statement

Consider the following linear discrete-time system:

$$x(k+1) = Ax(k) + w(k), \tag{1}$$

where $x(k) \in \mathbb{R}^m$ is the state vector, $w(k) \in \mathbb{R}^m$ is the process noise which is assumed to be zero-mean white Gaussian with covariance matrix $Q > 0$. The initial state $x(0)$ is also zero-mean Gaussian with covariance $\Pi_0 \geq 0$, and is independent of $w(k)$ for all $k \geq 0$.

A wireless sensor network consisting of n sensors is used to measure $x(k)$. The measurement equation of the ith sensor is given by

$$y_i(k) = \gamma^i(k)(H^i x(k) + v^i(k)), \tag{2}$$

where $v^i(k) \in \mathbb{R}^m$ is zero-mean white Gaussian with covariance matrix $R_i > 0$ which is independent of x_0, $w(k)$ $\forall k, i$, and is independent of $v^j(s)$ when $i \neq j$ or $k \neq s$, $\gamma^i(k) = 1$ or 0 is a decision variable whether the ith sensor is activated or not, i.e., if $\gamma^i(k) = 1$, the ith sensor is activated. It then measures the system state and exchanges its estimate with its neighbors. If $\gamma^i(k) = 0$, the ith sensor only receives the estimates from its neighbors. In this paper, we adopt a stochastic scheduling approach by setting $\gamma^i(k)$ as i.i.d. Bernoulli random variables with mean $\mathbb{E}[\gamma^i(k) = 1] = q$. Assume that $\gamma^i(k)$, $w(k)$, $v^i(k)$ and the initial state $x(0)$ are mutually independent for all i, k.

We model the sensor network as a directed graph $G = (V, E)$ with the nodes $V = \{1, 2, \dots, n\}$ being the sensors and the edges $E \subset V \times V$ representing the communication links. The existence of edge (i, j) means the ith sensor receives data from the jth sensor. Define the neighboring sensors of the ith sensor by $N_i = \{j : (i, j) \in E\}$. Let $d_i = |N_i|$ be the number of neighboring sensors of the ith sensor. In this paper, we assume that the graph is strongly connected. Recall that the ith sensor cannot send information to its neighbors if $\gamma^i(k) = 0$. Define the Laplacian matrix of G_k, the graph G at step k, as $L_k = [l_{ij}(k)]$, where

$$l_{ij}(k) = \begin{cases} -\gamma^j(k), & \text{if } (i,j) \in E, i \neq j, \\ -\sum_{j \in N_i} l_{ij}(k), & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases}$$

Consider the following distributed state estimator at the ith sensor:

$$\begin{aligned} \hat{x}^i(k+1|k) &= A\hat{x}^i(k|k-1) + K_p^i(k)(y_i(k) - \gamma^i(k)H^i\hat{x}^i(k|k-1)) \\ &\quad - \varepsilon A \sum_{j \in N_i} \gamma^j(k)[\hat{x}^i(k|k-1) - \hat{x}^j(k|k-1)], \end{aligned} \tag{3}$$

with $\hat{x}^i(0|-1) = 0$, $\forall i$. In Eq. (3), ε is the consensus gain and is in the range of $(0, 1/\Delta)$, $\Delta = \max_i(d_i)$, and $K_p^i(k)$ is the estimator gain. In this paper, we design $K_p^i(k)$ by minimizing the mean-squared estimation error

$$E\{[x(k+1) - \hat{x}^i(k+1|k)][x(k+1) - \hat{x}^i(k+1|k)]^T\}, \tag{4}$$

where the expectation is taken over w, v^i and $\gamma^i \forall i$. Define the estimation error at the ith sensor as

$$e^i(k|k-1) = \hat{x}^i(k|k-1) - x(k).$$

To simplify the notations, we write $e^i(k|k-1) = e_k^i$. From Eq. (3), e_k^i evolves as follows:

$$\begin{aligned} e_{k+1}^i &= Ae_k^i - \varepsilon A \sum_{j \in N_i} \gamma^j(k)(e_k^i - e_k^j) \\ &\quad - K_p^i(k) \cdot \gamma^i(k) \cdot H^i e_k^i + K_p^i(k)\gamma^i(k)v^i(k) - w(k). \end{aligned} \tag{5}$$

Let $F_i(k) = A - \gamma^i(k)K_p^i(k)H^i$. Then, it follows from (5) that

$$\begin{aligned} e_{k+1}^i e_{k+1}^{jT} &= F_i(k)e_k^i e_k^{jT} F_j^T(k) - \varepsilon F_i(k) \sum_{s \in N_j} \gamma^s(k)(e_k^i e_k^{jT} - e_k^i e_k^{sT})A^T \\ &\quad + \varepsilon w(k) \sum_{s \in N_j} \gamma^s(k)(e_k^{iT} - e_k^{sT})A^T - \varepsilon A \sum_{r \in N_i} \gamma^r(k)(e_k^i \\ &\quad \times e_k^{jT} - e_k^r e_k^{jT})F_j(k)^T + w(k)w(k)^T \\ &\quad - \varepsilon A \sum_{r \in N_i} \gamma^r(k)(e_k^i - e_k^r)\gamma^j(k)v^j(k)^T K_p^{jT}(k) \\ &\quad - \varepsilon \gamma^i(k)K_p^i(k)v^i(k) \sum_{s \in N_j} \gamma^s(k)(e_k^{iT} - e_k^{sT})A^T \\ &\quad + \varepsilon^2 A \sum_{r \in N_i} \sum_{s \in N_j} \gamma^r(k)\gamma^s(k)[e_k^i e_k^{jT} \end{aligned}$$

$$
\begin{aligned}
& - e_k^i e_k^{sT} - e_k^r e_k^{jT} + e_k^r e_k^{sT}]A^T + \gamma^j(k) F_i(k) e_k^i v^j(k)^T K_p^{jT}(k) \\
& + \gamma^i(k) K_p^i(k) v^i(k) e_k^{jT} F_j(k)^T \\
& - F_i(k) e_k^i w(k)^T - w_k e_k^{jT} F_j(k)^T \\
& + \varepsilon A \sum_{r\in N_i} \gamma^r(k)(e_k^i - e_k^r) w^T(k) \\
& + \gamma^i(k)\gamma^j(k) K_p^i(k) v^i(k) v^j(k) K_p^{jT}(k). \qquad (6)
\end{aligned}
$$

As $P_{i,j}(k) = E\{e_k^i e_k^{jT}\}$, one has

$$
\begin{aligned}
P_{i,j}(k+1) = {} & \bar{F}_i(k) P_{i,j}(k) \bar{F}_j(k)^T + Q + q^2 K_p^i(k) R_{i,j} K_p^j(k)^T \\
& + \varepsilon^2 A \sum_{r\in N_i}\sum_{s\in N_j} q^2 [P_{i,j}(k) \\
& - P_{i,s}(k) - P_{r,j}(k) + P_{r,s}(k)]A^T \\
& - \varepsilon \bar{F}_i(k) \sum_{s\in N_j} q(P_{i,j}(k) - P_{i,s}(k))A^T \\
& - \varepsilon A \sum_{r\in N_i} q(P_{i,j}(k) - P_{r,j}(k)) \bar{F}_j(k)^T, \qquad (7)
\end{aligned}
$$

where

$$\bar{F}_i(k) = A - qK_p^i(k)H^i.$$

Letting $i = j$, and taking expectation with respect to $\gamma^i(k)$, $w(k)$ and $v^i(k)$ on both sides of Eq. (6), one obtains

$$
\begin{aligned}
E\{e_{k+1}^i e_{k+1}^{iT}\} = {} & (1 - d_i q\varepsilon)^2 A P_i(k) A^T \\
& + 2(q\varepsilon - q^2\varepsilon^2) A \sum_{s\in N_i} P_{i,s}(k) A^T + q^2\varepsilon^2 A \\
& \times \sum_{r,s\in N_i} P_{r,s}(k) A^T + Q - q^2 A \Bigg\{ P_i(k) + q\varepsilon \\
& \times \sum_{s\in N_i} [P_{i,s}(k) - P_i(k)] \Bigg\} H^i M_i^{-1}(k) \\
& \cdot H^{iT} \Bigg\{ P_i(k) + \varepsilon \sum_{s\in N_i} [P_{i,s}(k) - P_i(k)] \Bigg\}^T A^T \\
& + [K_p^i(k) - K_p^{i\star}(k)] M_i(k) [K_p^i(k) - K_p^{i\star}(k)]^T, \qquad (8)
\end{aligned}
$$

where

$$P_i(k) = E\{e_k^i e_k^{iT}\}, \qquad P_{i,s}(k) = E\{e_k^i e_k^{sT}\},$$

$$K_p^{i\star}(k) = qA \Bigg\{ P_i(k) + q\varepsilon \sum_{s\in N_i} [P_{i,s}(k) - P_i(k)] \Bigg\} H^{iT} M_i^{-1}(k), \qquad (9)$$

$$M_i(k) = qH^i P_i(k) H^{iT} + qR_i. \qquad (10)$$

Note that $q\varepsilon < 1$ implies $2q\varepsilon - 2q^2\varepsilon^2 > 0$. As a result, $E\{e_{k+1}^i e_{k+1}^{iT}\}$ is minimized when $K_p^i(k) = K_p^{i\star}(k)$.

Remark 1. We can rewrite Eq. (6) as

$$
\begin{aligned}
P_i(k+1) = {} & \bar{F}_i(k) P_i(k) \bar{F}_i^T(k) + q(1-q)(K_p^i(k)H^i) P_i(k) (K_p^i(k)H^i)^T \\
& + Q + qK_p^i(k) R_i K_p^{iT}(k) + \Delta P(k), \qquad (11)
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta P(k) = {} & -q\varepsilon \bar{F}_i(k) \sum_{s\in N_i} [P_i(k) - P_{i,s}(k)]A^T - q\varepsilon A \\
& \times \sum_{r\in N_i} [P_i(k) - P_{r,i}(k)] \bar{F}_i^T(k) + q^2\varepsilon^2 \\
& \times A \sum_{r,s\in N_i} [P_i(k) - P_{i,s}(k) - P_{r,i}(k) + P_{r,s}(k)]A^T.
\end{aligned}
$$

If $\varepsilon = 0$, then

$$
\begin{aligned}
K_p^{i\star}(k) &= qAP_i(k)H^{iT}M_i^{-1}(k), \\
P_i(k+1) &= \bar{F}_i(k)P_i(k)\bar{F}_i^T(k) + Q + qK_p^i(k)R_iK_p^{iT}(k) \\
&\quad + q(1-q)(K_p^i(k)H^i)P_i(k)(K_p^i(k)H^i)^T,
\end{aligned}
$$

which are the sub-optimal Kalman gain and the estimation error covariance, respectively, obtained in Zhang, Song, and Shi (2012).

Note that we designed an optimal estimator gain (9) for the estimator (3). In the remainder of the paper, we search for an upper bound of ε to guarantee a bounded estimation error covariance for any given q. Alternatively, when ε is given, we obtain a lower bound of q to guarantee a bounded estimation error covariance.

3. Convergence analysis

In this section, we analyze the stability of the proposed estimator (3) with the optimal estimator gain (9). Due to the coupling of the estimation errors among the neighboring sensors, it is difficult to prove that the estimation error covariance converges to a unique positive definite matrix such as in the centralized Kalman filter. Instead, we derive an upper bound and a lower bound of the steady-state estimation error covariance.

Define $e_k = [e_k^1, e_k^2, \ldots, e_k^n]^T$ and $v_k = [v_k^1, v_k^2, \ldots, v_k^n]^T$. By stacking all the estimation errors in vector form, one obtains

$$
\begin{aligned}
e_{k+1} &= (I_n \otimes A)e_k - \varepsilon(L_k \otimes A)e_k - \mathrm{diag}\{\gamma^i(k)K_p^i(k)H^i\}e_k \\
&\quad + \gamma^i(k)\mathrm{diag}\{K_p^i(k)\}v_k - 1_n \otimes w_k \\
&= \Gamma(k)e_k + W(k), \qquad (12)
\end{aligned}
$$

where

$$\Gamma(k) = I_n \otimes A - \varepsilon L_k \otimes A - \mathrm{diag}\{\gamma^i(k)K_p^i(k)H^i\},$$

$$W(k) = \mathrm{diag}\{\gamma^i(k)K_p^i(k)\}v_k - 1_n \otimes w_k.$$

Let $P(k) = E\{e_k e_k^T\}$. Then

$$P(k+1) = \bar{\Gamma}(k)P(k)\bar{\Gamma}(k)^T + E\{W(k)W(k)^T\}, \qquad (13)$$

where $\bar{\Gamma}(k) = I_n \otimes A - \varepsilon qL \otimes A - q \cdot \mathrm{diag}\{K_p^i(k)H^i\}$. Note that L is the Laplacian matrix of the physical sensor network. If the ith sensor has a link with the jth sensor, then $l_{ij} = -1$; otherwise, $l_{ij} = 0$.

Similarly to Zhang et al. (2012), we make the following assumptions.

Assumption 2. $(A^T, qH^{iT}, 0, H^{iT})$, $0 < q < 1$ is stabilizable for all i.

Assumption 3. $(A^T, 0, Q^{1/2})$ is exactly observable.

Remark 4. Here, (A^T, H^T, A_0^T, H_0^T) is called stabilizable if there exists a feedback control $u(k) = Kx(k)$, with K being a constant matrix, such that for any $x_0 \in \mathbb{R}^m$, the closed-loop system

$$x(k+1) = [A^T + H^TK]x(k) + [A_0^T + H_0^TK]x(k)\omega(k), \quad x(0) = x_0,$$

is asymptotically mean-square stable. The random sequence $\omega(k)$ is a wide sense stationary, second-order process with $E\{\omega(k)\} = 0$ and $E\{\omega(k)\omega(j)\} = \sigma\delta_{kj}$. One method from Boyd, Ghaoui, Feron, and Balakrishnan (1994) is provided here to verify the mean-square stability of the above system by solving the following Lyapunov equation of P:

$$
\begin{aligned}
& (A^T + H^TK)^T P (A^T + H^TK) - P \\
& \quad + \sigma^2 (A_0^T + H_0^TK)^T P (A_0^T + H_0^TK) + I = 0
\end{aligned}
$$

and by checking whether $P > 0$.

Remark 5 (*Zhang et al., 2012*)**.** Consider the stochastic system

$$
\begin{aligned}
x(k+1) &= A^Tx(k) + A_0^Tx(k)\omega(k), \\
y(k) &= H^Tx(k).
\end{aligned}
$$

Here, (A^T, A_0^T, H^T) is called exactly observable if

$$y(k) \equiv 0 \quad \text{a.s.}\ \forall k \in \{0, 1, \ldots, \} \Rightarrow x_0 = 0.$$

Lemma 6 (Federico & Ali, 2010)**.** *Consider a recursion of the form*

$$X_{k+1} = A(k)X_kA(k)^\star + B(k),$$

where $A(k)$ and $B(k)$ converge uniformly to A and B, respectively, as $k \to \infty$, and A is a stable matrix. Then, X_k converges to X, the solution of the Lyapunov equation

$$X = AXA^\star + B.$$

We now state one of the main results in this paper.

Lemma 7. *Given a fixed q, under Assumptions 2–3, for any $0 < \varepsilon \le \bar{\varepsilon}$, $P(k)$ given by (13) is bounded as $k \to \infty$, where $\bar{\varepsilon}$ is the solution of the following optimization problem:*

$$\bar{\varepsilon} = \operatorname*{argmax}_{\varepsilon} \Phi_\varepsilon(L, A) > 0, \quad 0 < \varepsilon < 1/\Delta,$$

$$\Phi_\varepsilon(L, A) = \begin{pmatrix} I_{nm} & I_{nm} - \varepsilon qL \otimes A \\ (I_{nm} - \varepsilon qL \otimes A)^T & I_{nm} \end{pmatrix}.$$

Proof. Based on Assumption 2, we consider a constant matrix K_c^i such that $A - qK_c^iH^i$ is stable for all i. Its corresponding estimation error covariance is

$$P^c(k+1) = \bar{\Gamma}^c(k)P^c(k)\bar{\Gamma}^{cT}(k) + E\{W^c(k)W^{cT}(k)\},$$

where $\bar{\Gamma}^c(k) = I_n \otimes A - \varepsilon qL \otimes A - q\mathrm{diag}\{K_c^iH^i\}$ and $W^c(k) = \mathrm{diag}\{K_c^iH^i\}v_k - 1_n \otimes w(k)$. Note that one can always find a sufficiently small ε such that $\rho(\bar{\Gamma}^c(k)) < 1$. By Lemma 6, $P^c(k+1)$ converges to a constant block matrix. Since $P(k)$ given by (11) is the least estimation error covariance associated with the optimal estimator, $P^c(k) \ge P(k) \ge 0$, which implies that $P(k)$ is bounded as $k \to \infty$. Note that

$$\bar{\Gamma}^c(k) < I_{nm} - \varepsilon qL \otimes A$$

as $A - qK_c^iH^i$ is stable. Note that the graph is strongly connected. It implies that the Laplacian L has only one zero eigenvalue. Therefore, a sufficient condition for $\rho(\bar{\Gamma}^c(k)) < 1$ is

$$\|I_{nm} - \varepsilon qL \otimes A\|_2 < 1,$$

from which, $\bar{\varepsilon}$, an upper bound of ε, can be found via the stated optimization problem. ■

Lemma 8. *If (A, B) is stable, the solution of the equation*

$$P^e(k+1) = AP^e(k)A^T + BP^e(k)B^T + Q + \Delta P(k)$$

is upper bounded.

Proof. Since $P(k)$ is bounded as $k \to \infty$, there always exists a sufficiently small ε such that $\|\Delta P(k)\| < \kappa$ and $Q - \kappa I_m > 0$ for a positive real number κ. Define a fictitious equation as follows:

$$P(k+1) = AP(k)A^T + BP(k)B^T + Q + \kappa I_m. \tag{14}$$

Let the initial conditions be $P^e(0) = P(0) \ge 0$. For $k = 1$, it is easy to see that

$$P^e(1) < P(1).$$

The inequality follows from the fact that $\|\Delta P(k)\| < \kappa$. Assume that $P^e(k) < P(k)$. Then, it is not difficult to verify that $P^e(k+1) < P(k+1)$. By Lemma 2 in Zhang et al. (2012), Eq. (14) admits a unique solution $P \ge 0$. Hence, $P^e(k)$ is upper bounded for all k. ■

Now, we derive an upper bound and a lower bound for $P_i(k)$ for all k.

Define an algebraic Riccati equation as follows:

$$\begin{aligned}\hat{P}_i(k+1) &= \bar{F}_i^\kappa(k)\hat{P}_i(k)\bar{F}_i^{\kappa T}(k) \\ &\quad + q(1-q)(K_p^{i\kappa}(k)H^i)\hat{P}_i(k)(K_p^{i\kappa}(k)H^i)^T \\ &\quad + Q + qK_p^{i\kappa}(k)R_iK_p^{i\kappa T}(k) + \kappa I_m,\end{aligned} \tag{15}$$

where $\bar{F}_i^\kappa(k) = A - qK_p^{i\kappa}(k)H^i$, $K_p^{i\kappa}(k) = qA\{\hat{P}_i(k) + q\varepsilon\sum_{s\in N_i}[\hat{P}_{i,s}(k) - \hat{P}_i(k)]\}H^{iT}M_i^{-1}(k)$. Then, similar to Zhang et al. (2012), one can prove that the above equation converges to a limit $\hat{P}_i$. We can also prove that the equation

$$\begin{aligned}\check{P}_i(k+1) &= \bar{F}_i^\kappa(k)\check{P}_i(k)\bar{F}_i^{\kappa T}(k) \\ &\quad + q(1-q)(K_p^{i\kappa}(k)H^i)\check{P}_i(k)(K_p^{i\kappa}(k)H^i)^T \\ &\quad + Q + qK_p^{i\kappa}(k)R_iK_p^{i\kappa T}(k) - \kappa I_m\end{aligned} \tag{16}$$

has a unique solution $\check{P}_i$.

Theorem 1. *Under Assumptions 2–3, with the same initial values $P_i(0) = \hat{P}_i(0) = \check{P}_i(0)$ and $P_{i,j}(0) = \hat{P}_{i,j}(0) = \check{P}_{i,j}(0)$ for all $j \in N_i$, the matrix $P_i(k)$ in (11) is upper bounded by $\hat{P}_i$ and lower bounded by $\check{P}_i$ as $k \to \infty$.*

Proof. In the proof of Lemma 7, $(A - qK_c^iH^i, -K_c^iH^i)$ is mean-square stable (Zhang et al., 2012). By Lemma 8, the estimation error covariance is given by

$$\begin{aligned}P_i^c(k+1) &= \bar{F}_i(k)P_i^c(k)\bar{F}_i^T(k) + Q + qK_c^iR_iK_c^{iT} + \\ &\quad + q(1-q)(K_c^iH^i)P_i^c(k)(K_c^iH^i)^T + \Delta P(k)\end{aligned}$$

which is bounded. Since $P(k)$ is the least estimation error covariance associated with the optimal estimator, $P_i^c(k) \ge P_i(k) \ge 0$. Thus, $P_i(k)$ is bounded for all i, k.

Note that $\Delta P(k) \le \kappa I_m$. With the same initial condition, it is easy to see that $P_i(1) < \hat{P}_i(1)$. Assume $P_i(k) < \hat{P}_i(k)$ for $k, k-1, \ldots, 1$. Then

$$\begin{aligned}\hat{P}_i(k+1) &= \bar{F}_i^\kappa(k)\hat{P}_i(k)\bar{F}_i^{\kappa T}(k) + Q + qK_p^{i\kappa}(k)R_iK_p^{i\kappa T}(k) \\ &\quad + q(1-q)(K_p^{i\kappa}(k)H^i)\hat{P}_i(k)(K_p^{i\kappa}(k)H^i)^T + \kappa I_m \\ &\ge \bar{F}_i^\kappa(k)P_i(k)\bar{F}_i^{\kappa T}(k) + Q + qK_p^{i\kappa}(k)R_iK_p^{i\kappa T}(k) \\ &\quad + q(1-q)(K_p^{i\kappa}(k)H^i)P_i(k)(K_p^{i\kappa}(k)H^i)^T + \kappa I_m \\ &\ge \bar{F}_i(k)P_i(k)\bar{F}_i^T(k) + Q + qK_p^i(k)R_iK_p^{iT}(k) \\ &\quad + q(1-q)(K_p^i(k)H^i)P_i(k)(K_p^i(k)H^i)^T + \kappa I_m \\ &\ge \bar{F}_i(k)P_i(k)\bar{F}_i^T(k) + Q + qK_p^i(k)R_iK_p^{iT}(k) \\ &\quad + q(1-q)(K_p^i(k)H^i)P_i(k)(K_p^i(k)H^i)^T + \Delta P(k) \\ &= P_i(k+1).\end{aligned} \tag{17}$$

Thus, as k goes infinity, $P_i(k)$ is upper bounded by $\hat{P}_i$. Note that Eq. (11) is equivalent to

$$P_i(k+1) = AP_i(k)A^T + Q - K_p^i(k)M_i(K)K_p^i(k) + \Delta P(k).$$

Furthermore, by comparing each term of the above equation and that of Eq. (8), one has $\Delta P(k) < 0$ if $\sum_{s\in N_i} P_i(k) - P_{i,s}(k) > 0$. Then, by a similar method as deriving the upper bound, we obtain the lower bound $\check{P}_i$. ■

Lemma 9. *Given a fixed $\varepsilon \le 1/\Delta$, if $(A, Q^{1/2})$ is reachable and (A, H^i) is detectable, then $P(k)$ given by (13) is bounded for all $q > \max_i\{\bar{q}(\hat{P}_i)\}$, where $\bar{q}(\hat{P}_i)$ is the solution of the following optimization*

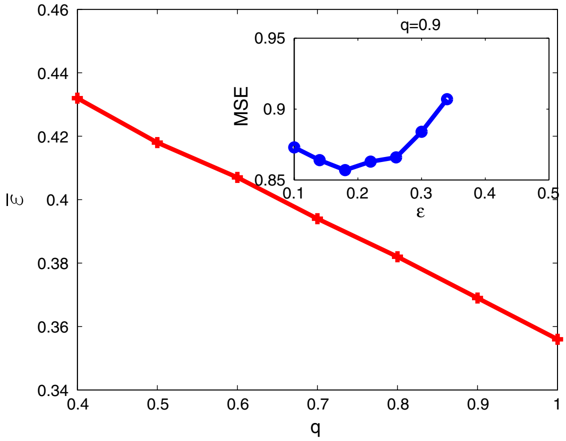

Fig. 1. $\bar{\varepsilon}$ as function of the activating probability q.

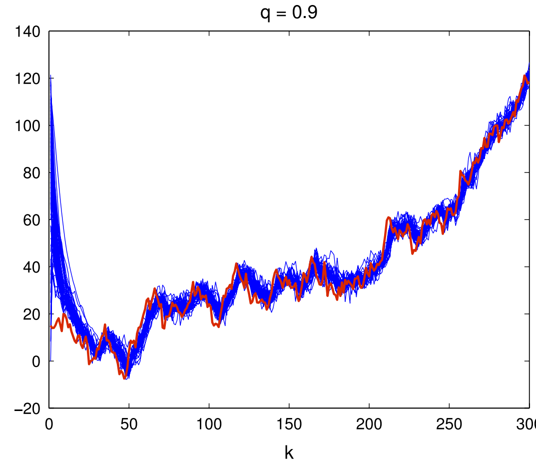

Fig. 2. Tracking performance of the proposed estimator: the blue curves correspond to 30 sensors and the red curve corresponds to the given target.(For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

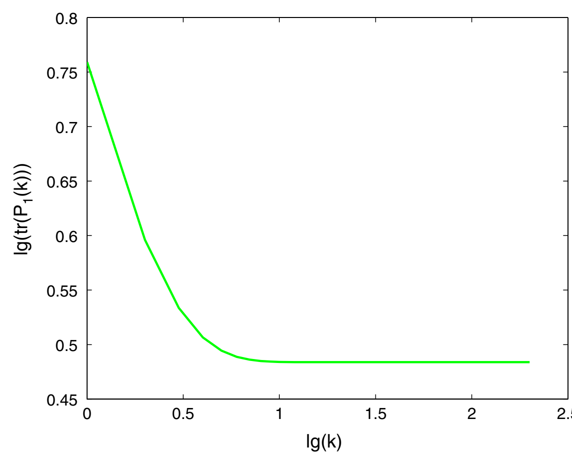

Fig. 3. Trace of covariance of the proposed estimator: node 1.

problem:

$$\bar{q}(\hat{P}_i) = \operatorname*{argmax}_q \psi_q(X^i, Z^i) > 0,$$

$$0 < X^i \le I, \qquad Z^i = X^i K, \quad 0 < q < 1,$$

$$\psi_q(X^i, Z^i) = \begin{pmatrix} X^i & \sqrt{q(1-q)}Z^i H^i & X^i(A - qK^i H^i) \\ \sqrt{q(1-q)}H^{iT} Z^{iT} & X^i & 0 \\ X^i(A - qK^i H^i) & 0 & X^i \end{pmatrix}.$$

Proof. Note that we have proved that $\hat{P}_i(k) > P_i(k)$ for all k. The critical value $\bar{q}(\hat{P}_i)$ which renders $\hat{P}_i$ in Eq. (15) convergence is given by Zhang et al. (2012) as the solution to the above optimization problem. Thus, if $q > \max_i\{\bar{q}(\hat{P}_i)\}$, then the matrix $P_i(k)$ is bounded. ■

To illustrate the optimization procedure, we consider a wireless sensor network consisting of 30 sensors with maximal degree $\Delta = 5$. Other parameters are the same as the example in Section 4. Fig. 1 shows $\bar{\varepsilon}$ for a list of q by solving the optimization problem in Lemma 7. Obviously, each corresponding $\bar{\varepsilon}$ is larger than $1/\Delta$. Intuitively, it is easier to obtain the value of the maximal node degree than the corresponding network topology. Thus, we can choose ε from $(0, 1/\Delta)$, and then find a lower bound for q by solving the optimization problem in Lemma 9. Moreover, we also find that ε which satisfies $\varepsilon \le \bar{\varepsilon}$ has very mild influence on the estimation performance as Fig. 1 shows. In Fig. 1, MSE is the average mean-squared estimation error over last 100 steps as $\sum_{k=\bar{k}-100}^{\bar{k}} \sum_{i=1}^{n} e_k^{iT} e_k^i / 100n$, $\bar{k}$ is a large enough time step at which $P_i(k)$ has converged to a constant for all i.

4. Simulation results

In this section, we illustrate the results derived in Section 3 by numerical simulations. Moreover, we compare the estimation performance of the proposed estimator and the consensus-based distributed Kalman filter (CBDKF) (Federico & Ali, 2010; Stanković et al., 2009).

Consider a wireless sensor network with $n = 30$ sensors. The discrete-time system and sensor parameters are given as follows:

$$A = \begin{pmatrix} 1.01 & 0 \\ 0 & 1.01 \end{pmatrix}, \qquad Q = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix},$$

$$H^i = \begin{pmatrix} 2\delta_i & 0 \\ 0 & 2\delta_i \end{pmatrix}, \qquad R_i = \begin{pmatrix} 2\nu_i & 0 \\ 0 & 2\nu_i \end{pmatrix},$$

where $\delta_i, \nu_i \in (0, 1]$ for all i. We choose an undirected network topology G which has its second eigenvalue $\lambda_2(L) = 1.2483$ and maximal degree $\Delta = 17$. Moreover, $\gamma_i(k)$ is the activating indicator with $\Pr\{\gamma_k^i = 1\} = 0.9$ and $\Pr\{\gamma_k^i = 0\} = 0.1$. We also choose $\varepsilon = 0.01$. As Fig. 2 shows, all the sensors (in blue lines) are able to track the unstable object system state of (1) (in red line). Furthermore, Fig. 3 demonstrates that $P_i(k)$ indeed converges.

Next, we compare the proposed estimator with the CBDKF estimator. Denote the mean-squared estimation error as $\sum_{i=1}^{n} e_k^{iT} e_k^i / n$. The plot in Fig. 4(a) demonstrates that the proposed estimator has smaller estimation error than that of the CBDKF estimator. Due to the introduction of a consensus term, the disagreement of a sensor's estimates also reflects the estimation performance. Similar to Saber (2007), we use $\|\delta(k)\| = (\sum_{i=1}^{n}(\delta^i(k))^2)^{1/2}$ with $\delta^i(k) = \hat{x}^i(k|k-1) - m(k)$, where $m(k) = \frac{1}{n}\sum_i \hat{x}^i(k|k-1)$, to measure the disagreement of the estimates. From Fig. 4(b), we can see that the proposed estimator has more cohesive estimates when compared with the CBDKF estimator.

Note that the activating probability in our proposed stochastic sensor activation strategy is constant. To further utilize the network resources during filtering process, we propose one special type of adaptive activating strategy in which the activating probability of each sensor $q_i(k)$ depends on the past $\gamma_i(k)'s$. Each sensor

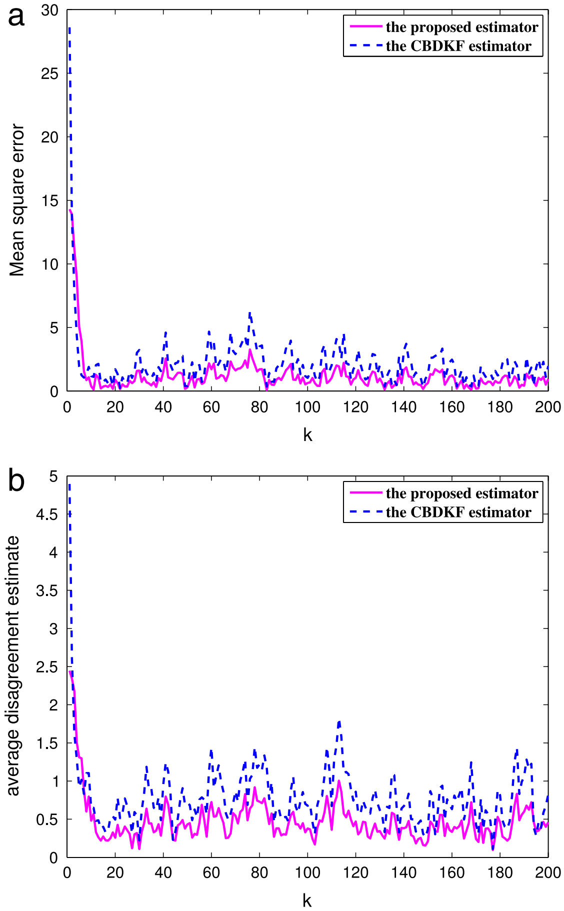

Fig. 4. Comparison of the proposed estimator and the CBDKF estimator.

checks its past l steps' $\gamma_i(k)$ at time step k_s with $\mathrm{mod}(k_s, l) = 0$, and decides what probability to use over the next l steps. Denote the number of activating times in each past l steps as $n^i_{k_s}$. Let $\check{m} < \hat{m}$.

Each sensor updates its activating probability as follows:

$$q_i(k_{s+1}) = \begin{cases} q_i(k_s) + a \cdot (1 - q_i(k_s)) \left(1 - \dfrac{1}{l - n^i_{k_s}}\right)^b, & \text{if } n^i_{k_s} \le \hat{m}, \\ q_i(k_s) - a \cdot q_i(k_s) \left(1 - \dfrac{1}{l - n^i_{k_s}}\right)^b, & \text{if } n^i_{k_s} \ge \check{m}, \\ q_i(k_s), & \text{otherwise.} \end{cases}$$

where $k_{s+1} = k_s + l$, and $q_i(k) = q_i(k_s),\ k \in [k_s, k_{s+1})$.

Remark 10. Intuitively, a small l leads to more frequent variation of the activating probability, which may in turn lead to instability of the networked system. On the other hand, the estimate performance of a large l is close to that of the constant probability strategy. Therefore, a tradeoff exists between the network stability and the estimate performance for different choices of l. In practice, one can run sufficient simulations to determine an appropriate l. Moreover, note that different choices of the parameters a, b, $\hat{m}$, $\check{m}$ lead to different strategies, which may substantially decrease or increase the activating probability over the next ten steps, based on the activating numbers in the past ten steps. A larger a leads to a larger variation of the activating probability. The parameter b can tune the weight to increase or to decrease, depending on the gap between the activating probability and $\check{m}$ or $\hat{m}$. Moreover, larger $\check{m}$ and smaller $\hat{m}$ lead to more frequent variation of the activating probability.

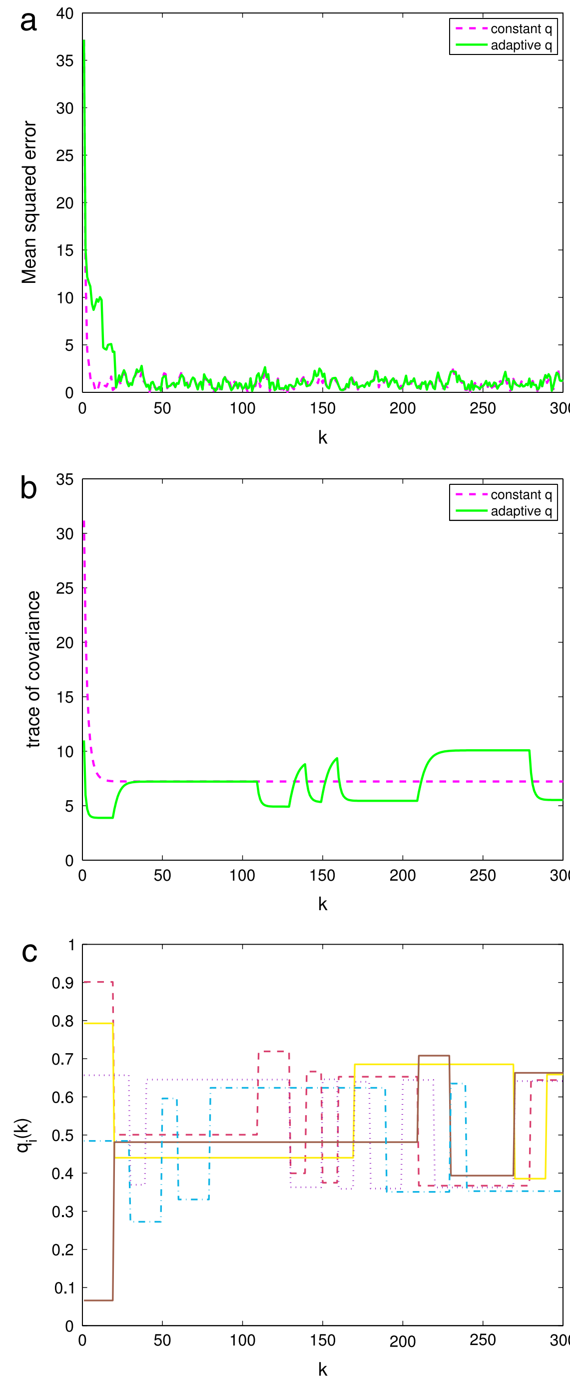

Fig. 5. Comparison of the adaptive probability strategy (adaptive q) and the constant probability strategy (constant q).

Next, we compare the estimation performance of the adaptive probability strategy with our constant probability strategy. In the example, we set $l = 10$. To make a fair comparison with the constant q strategy, i.e., the two strategies have the same average activating numbers in the long run, we choose $\check{m} = 2$, $\hat{m} = 8$, $a = 0.5$, $b = 1$, and set $q = 0.5$ for the constant probability strategy. Intuitively, if over the past ten steps, the activating number is less than 2 (which is smaller than the expected number 5), then over the next ten steps, the activating probability increases; and if over the past ten steps, the activating number is more than 8 (which is more than the expected number 5), then over the next ten steps, the activating probability decreases.

In the simulations, we randomly set the initial activating probability such that the average initial activating probability of all the sensors is equal to 0.5 for the adaptive strategy. As Fig. 5(a) and (b) show, both strategies have very similar estimation performance, and the adaptive probability strategy obtains a slightly better estimate than the constant probability strategy. Fig. 5(c) shows how the randomly selected five sensors' $q_i(k)$ evolve.

5. Conclusion

In this paper, we have considered distributed state estimation over a sensor network and designed an optimal estimator for each sensor with a stochastic scheduling scheme by minimizing its mean-squared estimation error. Under some mild assumptions, we have derived an upper bound and a lower bound for the estimation error covariance. To implement the proposed estimator in real applications, we have provided a method to obtain an upper bound of parameter ε, and a lower bound of probability q by solving a few LMIs. Moreover, we have compared the proposed estimator with the CBDKF estimator. The result shows that our design has smaller estimation errors and more cohesive estimates. Furthermore, we have designed an adaptive probability strategy in which each sensor tuned its activating probability by considering its past ten steps' activating numbers. By comparing with the constant probability strategy, simulation results show that both strategies have similar estimation performance.

Future works along the line of the current research include considering different activating probabilities q_i in a heterogeneous sensor network and finding the optimal q_i, analyzing estimation performance as a function of the network topology, and constructing the optimal topology subject to communication and resource constraints.

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