Automatica 50 (2014) 1641–1648

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

An event-triggered approach to state estimation with multiple point- and set-valued measurements[☆]

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ARTICLE INFO

Article history:
Received 19 April 2013
Received in revised form
30 November 2013
Accepted 10 March 2014
Available online 18 April 2014

Keywords:
Event-based estimation
Sensor fusion
Kalman filters
Wireless sensor networks

ABSTRACT

In this work, we consider state estimation based on the information from multiple sensors that provide their measurement updates according to separate event-triggering conditions. An optimal sensor fusion problem based on the hybrid measurement information (namely, point- and set-valued measurements) is formulated and explored. We show that under a commonly-accepted Gaussian assumption, the optimal estimator depends on the conditional mean and covariance of the measurement innovations, which applies to general event-triggering schemes. For the case that each channel of the sensors has its own event-triggering condition, closed-form representations are derived for the optimal estimate and the corresponding error covariance matrix, and it is proved that the exploration of the set-valued information provided by the event-triggering sets guarantees the improvement of estimation performance. The effectiveness of the proposed event-based estimator is demonstrated by extensive Monte Carlo simulation experiments for different categories of systems and comparative simulation with the classical Kalman filter.

1. Introduction

Event-based estimation strategy provides the possibility to maintain estimation performance under limited communication resources (Åström & Bernhardsson, 2002) and has attracted considerable attention in the control community for the past few years. For scalar linear systems, Imer and Basar (2005) and Rabi, Moustakides, and Baras (2006) studied the optimal event-based finite-horizon sensor transmission scheduling problems in continuous and discrete time, respectively. Li, Lemmon, and Wang (2010) extended the results to vector linear systems by relaxing the zero mean initial conditions and considering measurement noises. In Li and Lemmon (2011), the tradeoff between performance and average sampling period was analyzed, where a sub-optimal event-triggering scheme with a guaranteed least average sampling period was proposed. Rabi, Moustakides, and Baras (2012) considered the adaptive sampling for state estimation of linear continuous-time systems. In Wu, Jia, Johansson, and Shi (2013), the Minimum Mean Squared Error (MMSE) estimator was derived, and the tradeoff between communication rate and performance was analyzed. Shi, Chen, and Shi (2014) studied the likelihood estimation problem for a level-based event-triggering scheme, and the evaluation of upper and lower bounds on communication rates was discussed. Sijs and Lazar (2012) formulated a general description of event sampling, and a state estimator with a hybrid update was proposed to reduce the computational complexity.

The above results consider the scenario that only one event detector is used to process the measured state information from the sensor. There also exist many applications (e.g., in the context of wireless sensor networks) where multiple sensors with multiple event detectors are equipped to measure the state of the process. These invariably lead to sensor scheduling/fusion issues, which have been extensively studied for the case of periodic sampled systems (Alriksson & Rantzer, 2005; Mo, Ambrosinob, & Sinopoli, 2011; Shi & Chen, 2013). However, the effect of multiple event detectors on the MMSE estimates still remains unexplored, which is the basic motivation of our research. In this work, we consider the scenario that the process is measured by a network of sensors and that each sensor chooses to provide its latest measurement update according to its own event-triggering condition. In

☆ This work was supported by Natural Sciences and Engineering Research Council of Canada; in addition, the work by L. Shi was supported by an HK RGC GRF grant 618612. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Martin Enqvist under the direction of Editor Torsten Söderström.

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http://dx.doi.org/10.1016/j.automatica.2014.04.004
0005-1098/

this case, the hybrid information is provided by the whole group of sensors as well as the event-triggering sets. For the sensors whose event-triggering conditions are satisfied, the exact values of the sensor outputs are known, providing "point-valued measurement information" to the estimator; for sensors that the event-triggering conditions are not satisfied, some information contained in the event-triggering sets is known to the estimator as well, to which we refer as "set-valued measurement information" in this paper. The basic goal is to find the MMSE estimate given the hybrid measurement information. As will be addressed later, the main issues arise from the computational aspect, due to the non-Gaussianity of the *a posteriori* distributions. Therefore we focus on the derivation of an approximate (due to the Gaussian assumption) MMSE estimate that possesses a simple structure but still inherits the important properties of the exact optimal estimate. In Sijs and Lazar (2012), a sum of Gaussians approach was utilized to solve the MMSE problem under a uniform distribution assumption; for the single-channel case, an alternative approach was proposed by Nguyen and Suh (2007), where an adaptive scheduling algorithm was developed to adjust the virtual moments of the measurement noises to achieve the improved estimation performance. The difference is that the aforementioned results would add an additional covariance matrix to the measurement noise covariance, while the present approach introduces a scalar weight when updating the estimation error covariance matrix (see Theorem 7). The main contributions are summarized as follows:

(1) An approximate MMSE estimate induced by the hybrid measurement information provided by a sequence of sensors has been derived. We show that the estimate is determined by the conditional mean and covariance of the innovations. The results are valid for general event-triggering schemes and reduce to the results obtained in Wu et al. (2013) if only one sensor and the level-based event-triggering conditions are considered.

(2) Insights on the optimal estimate when each sensor has only one channel are provided. In this case, closed-form recursive state estimate update equations are obtained. Utilizing the recent results on the partial order of uncertainty and information (Chen, 2011), we show that the exploration of the set-valued information guarantees the improved estimation performance in terms of smaller estimation error covariance. The results are equally applicable to multiple-channel sensors with uncorrelated/correlated measurement noises but separate event-triggering conditions on each channel.

(3) Extensive Monte Carlo experiments are performed to test the effectiveness of the proposed estimator. Compared with the Kalman filter that only exploits the received point-valued measurements, the proposed estimator provides almost-guaranteed improved performance, which is not sensitive to the sensor sequence used.

The rest of the paper is organized as follows: Section 2 presents the system description and problem setup. Section 3 presents the main results. Experimental verification using Monte Carlo simulation is provided in Section 4, followed by the concluding remarks in Section 5.

2. System description and problem setup

Consider a linear time-invariant process that evolves in discrete time driven by white noise:

$$x_{k+1} = Ax_k + w_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state, and $w_k \in \mathbb{R}^n$ is the process noise, which is zero-mean Gaussian with covariance $Q \geq 0$. The initial value x_0 of the state is Gaussian with $\mathbf{E}(x_0) = \mu_0$, and covariance P_0. The state information is measured by a number of battery-powered sensors, which communicate with the state estimator through a wireless channel, and the output equations are

$$y_k^i = C^i x_k + v_k^i, \tag{2}$$

where $v_k^i \in \mathbb{R}^m$ is zero-mean Gaussian with covariance $R^i > 0$. In addition, x_0, w_k and v_k^i are uncorrelated with each other. We assume that the number of sensors equals M. Considering limitation in sensor battery capacity and the communication costs, an event-based data scheduler is equipped with each sensor i. At each time instant k, sensor i produces a measurement y_k^i, and the scheduler of sensor i tests the event-triggering condition

$$\gamma_k^i = \begin{cases} 0, & \text{if } y_k^i \in \Xi_k^i \\ 1, & \text{otherwise} \end{cases} \tag{3}$$

where Ξ_k^i denotes the event-triggering set of sensor i at time k and decides whether to allow a data transmission. If $\gamma_k^i = 1$, sensor i sends y_k^i to the estimator through the wireless channel. Notice that the event-triggering scheme in (3) is fairly general and covers most schemes considered in the literature and industrial applications, e.g., the "send on delta" strategy and the level-based triggering conditions (not necessarily being symmetric). For many previously considered event-triggering schemes (e.g., the level-based event-triggering conditions in Shi et al. (2014) and Wu et al. (2013)), feedback communication from the estimator to the sensor is needed at certain time instants as the event is related to the innovation; however, since the event-triggering sets Ξ_k^i can be designed offline, the remote estimator will have full knowledge of them without communication. In this way, the proposed results are applicable to battery-powered wireless sensor networks, where it is normally too costly to use feedback communication.

Since the main task is to study event-based estimation and sensor fusion, we assume that the capacity of the channel is greater than M so that it is possible for the sensors to communicate with the estimator at the same time.

Let $\hat{x}_k^i$ denote the optimal estimate of x_k after updating the measurement of the ith sensor and denote P_k^i as the corresponding covariance matrix.[2] Denote $\mathbb{S}_+^n$ as the set of symmetric positive semidefinite matrices. Define the functions $h(\cdot)\colon \mathbb{S}_+^n \to \mathbb{S}_+^n$ and $\tilde{g}_i(\cdot,\cdot)\colon \mathbb{S}_+^n \times \mathbb{R} \to \mathbb{S}_+^n$ as follows:

$$\begin{aligned} h(X) &:= AXA^\top + Q, \\ \tilde{g}_i(X, \vartheta) &:= X - \vartheta X (C^i)^\top [C^i X (C^i)^\top + R^i]^{-1} C^i X. \end{aligned} \tag{4}$$

For brevity, we denote $\tilde{g}_i(X,1)$ as $\tilde{g}_i(X)$. Denote $\mathcal{Y}_k := \{\mathcal{Y}_k^1, \mathcal{Y}_k^2, \ldots, \mathcal{Y}_k^M\}$ as the collection of measurement information received by the estimator. Notice that if $\gamma_k^i = 1$, $\mathcal{Y}_k^i = \{y_k^i\}$; otherwise, $\mathcal{Y}_k^i = \{y_k^i | y_k^i \in \Xi_k^i\}$. In the latter case, although y_k^i is unknown, it is still jointly Gaussian with x_k. Further define

$$\mathcal{I}_k^i := \left\{\mathcal{Y}_1, \mathcal{Y}_2, \ldots, \mathcal{Y}_{k-1}, \{\mathcal{Y}_k^1, \mathcal{Y}_k^2, \ldots, \mathcal{Y}_k^i\}\right\} \tag{5}$$

for $i \in \mathbb{N}_{1:M}$, and in this way, we are able to summarize all the information we have in $\mathcal{I}_k^i$ before considering the additional information $\mathcal{Y}_k^{i+1}$ from sensor $i+1$ at time k. The objective of our work is to explore the MMSE estimate of the process state (namely, $\mathbf{E}(x_k|\mathcal{I}_k^M)$) by taking into account all given information, namely, the set- and point-valued measurements provided by the sensor network as well as the event-triggering schemes.

When the state information is contained in combined point- and set-valued measurements, following a standard Bayesian

[2] Here we denote the 0th sensor as the case that no sensor information has been fused, namely, the prediction case.

argument, the exact MMSE estimate is the mean of the distribution of x_k conditioned on $\mathcal{I}_k^M$,

$$\mathbf{E}(x_k|\mathcal{I}_k^M) = \int_{\mathbb{R}^n} x f_{x_k}(x|\mathcal{I}_k^M)\mathrm{d}x. \tag{6}$$

The major problem of this estimate arises from the computational aspect, due to the fact that the conditional distribution of x_k in (6) is no longer Gaussian when set-valued measurements are provided. This conditional distribution can be updated recursively by fusing the information sequentially

$$f_{x_k}(x|\mathcal{I}_k^i) = \frac{f_{x_k}(x|\mathcal{I}_k^{i-1}) \int_{\Xi_k^i} f_{y_k^i}(y|\mathcal{I}_k^{i-1}, x_k = x)\mathrm{d}y}{\int_{\mathbb{R}^n} f_{x_k}(x|\mathcal{I}_k^{i-1}) \int_{\Xi_k^i} f_{y_k^i}(y|\mathcal{I}_k^{i-1}, x_k = x)\mathrm{d}y\mathrm{d}x}, \tag{7}$$

and the final result does not depend on the sensor sequence utilized during the fusion procedure (since the distribution is unique). However, analytical solutions to the integrations in (7) rarely exist and the only method to implement this estimate is numerical integration, which is inevitably expensive in computation.

On the other hand, one notices that

$$f_{x_k}(x|\mathcal{I}_k^i) = \frac{\int_{\mathcal{Y}_k^i} f_{x_k}(x|y_k^i = y, \mathcal{I}_k^{i-1}) f_{y_k^i}(y|\mathcal{I}_k^{i-1})\mathrm{d}y}{\int_{\mathcal{Y}_k^i} f_{y_k^i}(y|\mathcal{I}_k^{i-1})\mathrm{d}y} \tag{8}$$

$$= \int_{\mathbb{R}^m} f_{x_k}(x|y_k^i = y, \mathcal{I}_k^{i-1}) f_{y_k^i}(y|y \in \mathcal{Y}_k^i, \mathcal{I}_k^{i-1})\mathrm{d}y, \tag{9}$$

where $f_{y_k^i}(y|y \in \mathcal{Y}_k^i, \mathcal{I}_k^{i-1})$ satisfies $f_{y_k^i}(y|y \in \mathcal{Y}_k^i, \mathcal{I}_k^{i-1}) = 0,\ y \notin \mathcal{Y}_k^i$ and $\int_{\mathbb{R}^m} f_{y_k^i}(y|y \in \mathcal{Y}_k^i, \mathcal{I}_k^{i-1})\mathrm{d}y = 1$, and behaves similarly as the Dirac function $\delta(y)$, which equals to 0 except for $y = 0$ and satisfies $\int_{\mathbb{R}^m} \delta(y)\mathrm{d}y = 1$. If point-valued measurements are always available, Eq. (9) becomes $\int_{\mathbb{R}^m} f_{x_k}(x|y_k^i = y, \mathcal{I}_k^{i-1})\delta(y_k^i - y)\mathrm{d}y = f_{x_k}(x|y_k^i = y, \mathcal{I}_k^{i-1})$, which maintains Gaussianity. Motivated by these observations, we introduce the following assumption:

Assumption 1. The conditional distribution of x_k given $\mathcal{I}_k^i$ can be approximated by a Gaussian distribution with the same mean and covariance.

This assumption is also a commonly used technique in designing nonlinear Gaussian filters (Anderson & Moore, 1979; Arasaratnam & Haykin, 2009; Ito & Xiong, 2000; Julier, Uhlmann, & Durrant-Whyte, 2000). To further illustrate the above assumption in the context of event-based estimation, we present the following numerical example.

Example 1. Consider a linear system measured by one sensor and assume x_{k-1} is Gaussian with

$$A = \begin{bmatrix} 1.5 & 0.7 \\ 0.8 & 1.6 \end{bmatrix}, \qquad Q = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.6 \end{bmatrix},$$

$$\mathbf{Cov}(x_{k-1}) = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.8 \end{bmatrix},$$

$C = [1.2\ 0.3]$, $R = 0.3$, and $\mathbf{E}(x_{k-1}) = [0.5\ 0.5]^\top$ respectively. We study the distribution of x_k conditioned on set-valued measurement information. We perform Monte Carlo simulation and collect the realizations of x_k's such that $y_k \in \mathcal{Y}_k := [CA\mathbf{E}(x_{k-1}) - \delta, CA\mathbf{E}(x_{k-1}) + \delta]$, and estimate the resulting distribution. Different values of δ are considered, and 10 million realizations of x_k satisfying $y_k \in \mathcal{Y}_k$ are used to estimate the conditional pdf $f_{x_k}(x|y_k \in \mathcal{Y}_k)$ for each δ. The pdf of Gaussian distributions $\hat{f}_{x_k}(x|y_k \in \mathcal{Y}_k)$ with equal first two moments are also included for comparison in the plots (see also the KL-divergences $D_{KL}(f\|\hat{f})$ and $D_{KL}(\hat{f}\|f)$ of the distributions). From Fig. 1, it is reasonable to approximate the conditional distributions as Gaussian distributions with acceptable approximation errors. □

Now we are in a position to state the main problem considered in this paper:

Problem 2. At time k, given a sequence of measurement information $\{\mathcal{Y}_k^i | i \in \mathbb{N}_{1:M}\}$ of x_k and under Assumption 1, is it possible to find a simple approximate MMSE estimator in the recursive form? Does the exploration of the set-valued information lead to the improved estimation performance in terms of estimation error covariance?

Meanwhile, since the exact MMSE estimate is the same for all fusion sequences under the Bayesian decision framework (by the uniqueness of the conditional distribution), when an approximate solution of a simple form is obtained, an additional question to ask is whether the estimation performance is sensitive to the fusion sequence (due to the Gaussian assumption); we will further address this issue in the experimental verification section, where we test the performance of the proposed results extensively by Monte Carlo simulations.

3. Optimal fusion of sequential event-triggered measurement information

In this section, Problem 2 is studied in detail. Define $z_k^i = y_k^i - C^i\hat{x}_k^0$. Since $\hat{x}_k^0$ is known at time k by the estimator, this relationship maps the set Ξ_k^i to a unique set $\Omega_k^i := \{z_k^i : z_k^i = y_k^i - C^i\hat{x}_k^0, y_k^i \in \Xi_k^i\}$. Define $L_k^{i+1} := P_k^i(C^{i+1})^\top[C^{i+1}P_k^i(C^{i+1})^\top + R^{i+1}]^{-1}$, and $e_k^i := x_k - \hat{x}_k^i$. We have the following result:

Theorem 3. (1) *The optimal prediction $\hat{x}_k^0$ of the state x_k and the corresponding covariance P_k^0 are given by*

$$\hat{x}_k^0 = A\hat{x}_{k-1}^M,$$
$$P_k^0 = h(P_{k-1}^M).$$

(2) *For $i \in \mathbb{N}_{0:M-1}$, the fusion of information from the $(i+1)$th sensor leads to the following recursive state estimation equations:*

(a) *If $\gamma_k^{i+1} = 1$,*

$$\hat{x}_k^{i+1} = \hat{x}_k^i + L_k^{i+1}(z_k^{i+1} - \bar{z}_k^{i+1|i}), \tag{10}$$

$$P_k^{i+1} = \tilde{g}_{i+1}(P_k^i); \tag{11}$$

(b) *If $\gamma_k^{i+1} = 0$,*

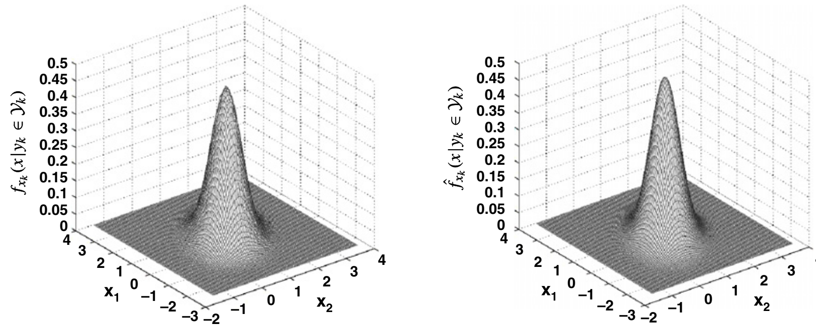
$$\hat{x}_k^{i+1} = \hat{x}_k^i + L_k^{i+1}(\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i}), \tag{12}$$

$$P_k^{i+1} = \tilde{g}_{i+1}(P_k^i) + L_k^{i+1}\mathbf{Cov}(z_k^{i+1}|\mathcal{I}_k^{i+1})(L_k^{i+1})^\top, \tag{13}$$

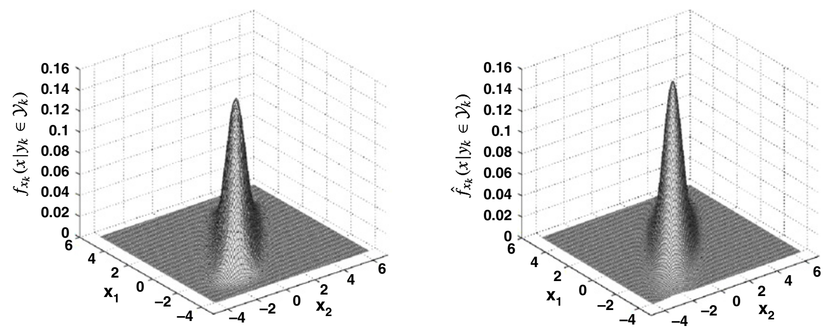
where $\bar{z}_k^{i+1|i} := C^{i+1}(\hat{x}_k^i - \hat{x}_k^0)$, and $\bar{z}_k^{i+1|i+1} := \mathbf{E}(z_k^{i+1}|\mathcal{I}_k^{i+1})$.

Proof. See Appendix. □

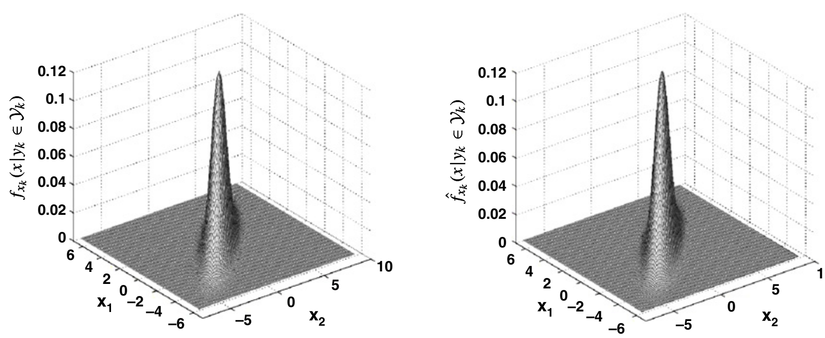
From the above result, the first and second moments of the truncated Gaussian distributions, namely, $\mathbf{E}(z_k^{i+1}|\mathcal{I}_k^{i+1})$ and $\mathbf{Cov}(z_k^{i+1}|\mathcal{I}_k^{i+1})$ need to be calculated to implement the event-based estimator. Fortunately, the moment evaluation problems of truncated Gaussian distributions have been extensively studied in the literature of statistical analysis; explicit formulae and efficient implementation methods have been proposed for a variety of truncation sets. See Manjunath and Wilhelm (2012), Tallis (1961, 1963) and the references therein. Also, the estimate in (10) and (12) can be written in terms of the sum of series of random variables with Gaussian and non-Gaussian distributions. According to the asymptotic distribution theory for state estimate from a Kalman



(a) $\delta = 1$. $D_{KL}(\hat{f}\|f) = 0.0032$, $D_{KL}(f\|\hat{f}) = 0.0033$.



(b) $\delta = 5$. $D_{KL}(\hat{f}\|f) = 0.0267$, $D_{KL}(f\|\hat{f}) = 0.0319$.



(c) $\delta = 10$. $D_{KL}(\hat{f}\|f) = 0.0006$, $D_{KL}(f\|\hat{f}) = 0.0004$.

Fig. 1. Plot of the conditional distributions.

filter in the absence of Gaussian assumptions (Aliev & Ozbek, 1999; Spall & Wall, 1984), the central limit theorem for the estimates is still valid, which helps explain the rationality of Assumption 1.

The above result provides an acceptable answer to the first part of Problem 2. The second part of the problem, however, is difficult to answer for general event-triggering schemes. In the following, we consider $m = 1$, namely, when each sensor has only one channel. Notice that this scenario is equivalent to that the sensors have multiple channels, but each channel has uncorrelated measurement noise and separate event-triggering conditions, which is easy to implement in most prevailing embedded systems. Furthermore, the results can be equally applied to the case of multiple-channel sensors with correlated measurement noise but separate event-triggering conditions. To do this, it suffices to first transform each sensor measurement y_k^i to $\hat{y}_k^i = U^i y_k^i$ (where U^i is an orthogonal matrix satisfying $R^i = (U^i)^\top \Lambda^i U^i$, Λ^i being a diagonal matrix containing the eigenvalues of R^i), and then design the event-triggering conditions for each channel of $\hat{y}_k^i$.

When $m = 1$, without loss of generality, the event-triggering sets can be parameterized as $\Omega_k^i = \{z_k^i | a_k^i \le z_k^i \le b_k^i\}$, for $i \in \mathbb{N}_{1:M}$. For this type of sets, we have the following well-known result (Johnson, Kotz, & Balakrishnan, 1994).

Lemma 4. *For a univariate Gaussian random variable $z_k^{i+1}|\mathcal{I}_k^i \sim \mathcal{N}(\bar{z}_k^{i+1|i}, Q_{z_k^{i+1}})$, its truncated mean and variance over $\Omega_k^{i+1} = \{z_k^{i+1} | a_k^{i+1} \le z_k^{i+1} \le b_k^{i+1}\}$ satisfy*

$$\mathbf{E}(z_k^{i+1}|\mathcal{I}_k^{i+1}) = \bar{z}_k^{i+1|i} + \hat{z}_k^{i+1}, \tag{14}$$

$$\mathbf{Cov}[z_k^{i+1}|\mathcal{I}_k^{i+1}] = (1 - \vartheta_k^{i+1}) Q_{z_k^{i+1}}, \tag{15}$$

where $\phi(z) := \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}z^2)$,

$$\hat{z}_k^{i+1} = \frac{\phi\left(\frac{a_k^{i+1} - \bar{z}_k^{i+1|i}}{Q^{1/2}_{z_k^{i+1}}}\right) - \phi\left(\frac{b_k^{i+1} - \bar{z}_k^{i+1|i}}{Q^{1/2}_{z_k^{i+1}}}\right)}{\mathcal{Q}\left(\frac{a_k^{i+1} - \bar{z}_k^{i+1|i}}{Q^{1/2}_{z_k^{i+1}}}\right) - \mathcal{Q}\left(\frac{b_k^{i+1} - \bar{z}_k^{i+1|i}}{Q^{1/2}_{z_k^{i+1}}}\right)} Q^{1/2}_{z_k^{i+1}}, \tag{16}$$

$$\vartheta_k^{i+1} = \left[\frac{\phi\left(\frac{a_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\right) - \phi\left(\frac{b_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\right)}{\mathcal{Q}\left(\frac{a_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\right) - \mathcal{Q}\left(\frac{b_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\right)} \right]^2 - \frac{\frac{a_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\phi\left(\frac{a_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\right) - \frac{b_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\phi\left(\frac{b_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\right)}{\mathcal{Q}\left(\frac{a_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\right) - \mathcal{Q}\left(\frac{b_k^{i+1}-\bar{z}_k^{i+1|i}}{Q_{z_k^{i+1}}^{1/2}}\right)}, \tag{17}$$

$\mathcal{Q}(\cdot)$ denotes the standard Q function.

Based on this result, we will show that the optimal estimate subject to a given sequence of measurement information reduces to a simple closed form and that the exploration of set-valued information could lead to guaranteed enhanced performance. To do this, we introduce the following lemmas.

Lemma 5 (*Theorem 2 (Chen, 2011)*)**.** *Let z be an absolutely continuous random variable with cumulative distribution function $F(z)$. The conditional variance* $\mathbf{Cov}(z|a \le z \le b)$ *is increasing in b if and only if*

$$\int_{a\le z_1\le z_2\le b} \{F(z_1) - F(a)\}\, dz_1 dz_2 \tag{18}$$

is log-concave in b, and it is decreasing in a if and only if

$$\int_{a\le z_1\le z_2\le b} \{F(b) - F(z_1)\}\, dz_1 dz_2 \tag{19}$$

is log-concave in a. When both conditions in (18) and (19) are satisfied for all $a, b \in \mathcal{C}$ for some convex set $\mathcal{C}$, then $\mathbf{Cov}(z|z \in \mathcal{A})$ *is partially monotonic in an interval $\mathcal{A}$ such that $\mathcal{A} \subset \mathcal{C}$.*

Lemma 6 (*Lemma 1 (Chen, 2011)*)**.** *If a function $f(z)$ is log-concave for $z \in (a, b)$, then the antiderivative $F(x) = \int_a^z f(t)dt$ is also log-concave for $z \in (a, b)$ whenever it is well defined.*

Now we are ready to present the following result.

Theorem 7. (1) *The optimal prediction $\hat{x}_k^0$ of the state x_k and the corresponding covariance P_k^0 are given by*

$$\hat{x}_k^0 = A\hat{x}_{k-1}^M,\quad P_k^0 = h(P_{k-1}^M). \tag{20}$$

(2) *For $i \in \mathbb{N}_{0:M-1}$, the fusion of information from the $(i+1)$th sensor leads to the following recursive state estimation equations:*

(a) *If $\gamma_k^{i+1} = 1$,*

$$\hat{x}_k^{i+1} = \hat{x}_k^i + L_k^{i+1}(z_k^{i+1} - \bar{z}_k^{i+1|i}), \tag{21}$$

$$P_k^{i+1} = \tilde{g}_{s_{i+1}}(P_k^i). \tag{22}$$

(b) *If $\gamma_k^{i+1} = 0$,*

$$\hat{x}_k^{i+1} = \hat{x}_k^i + L_k^{i+1}\hat{z}_k^{i+1}, \tag{23}$$

$$P_k^{i+1} = \tilde{g}_{s_{i+1}}(P_k^i, \vartheta_k^{i+1}), \tag{24}$$

where $\hat{z}_k^{i+1}$ is given in (16), and ϑ_k^{i+1} is given in (17) and in particular, satisfies $\vartheta_k^{i+1} \in (0, 1)$.

Proof. It suffices to prove Eqs. (23) and (24). Eq. (23) follows from (14) and (12). From (13),

$$\begin{aligned}P_k^{i+1} &= \tilde{g}_{s_{i+1}}(P_k^i) + L_k^{i+1}\mathbf{Cov}(z_k^{i+1}|\mathcal{I}_k^{i+1})(L_k^{i+1})^\top \\ &= \tilde{g}_{s_{i+1}}(P_k^i) + (1-\vartheta_k^{i+1})L_k^{i+1} \\ &\quad [C^{i+1}P_k^i(C^{i+1})^\top + R^{i+1}](L_k^{i+1})^\top \\ &= \tilde{g}_{s_{i+1}}(P_k^i, \vartheta_k^{i+1}).\end{aligned}$$

Finally we show $\vartheta_k^{i+1} \in (0, 1)$. Since $\mathbf{Cov}[z_k^{i+1}|\mathcal{I}_k^{i+1}] > 0$, we have $\vartheta_k^{i+1} < 1$. We consider the case $\bar{z}_k^{i+1|i} \in [a_k^{i+1}, b_k^{i+1}]$. In this case, $(a_k^{i+1} - \bar{z}_k^{i+1|i})/Q_{z_k^{i+1}}^{1/2} \le 0$ and $(b_k^{i+1} - \bar{z}_k^{i+1|i})/Q_{z_k^{i+1}}^{1/2} \ge 0$ hold. From (17), we have $\vartheta_k^{i+1} > 0$. This implies that a pair (a_k^{i+1}, b_k^{i+1}) such that $a_k^{i+1} \le \bar{z}_k^{i+1|i} \le b_k^{i+1}$ will lead to $\mathbf{Cov}[z_k^{i+1}|\mathcal{I}_k^{i+1}] < Q_{z_k^{i+1}}$. Now consider the case that $\bar{z}_k^{i+1|i} \notin [a_k^{i+1}, b_k^{i+1}]$. There always exists a pair $(\underline{a}_k^{i+1}, \bar{b}_k^{i+1})$ such that $[a_k^{i+1}, b_k^{i+1}] \subset [\underline{a}_k^{i+1}, \bar{b}_k^{i+1}]$ and $\bar{z}_k^{i+1|i} \in [\underline{a}_k^{i+1}, \bar{b}_k^{i+1}]$. Since $\phi(z)$ is a logarithmically concave function, from Lemmas 5 and 6, we have $\mathbf{Cov}[z_k^{i+1}|\mathcal{I}_k^{i+1}] \le \mathbf{Cov}\big[z_k^{i+1}|\mathcal{I}_k^i, z_k^{i+1} \in [\underline{a}_k^{i+1}, \bar{b}_k^{i+1}]\big] < Q_{z_k^{i+1}}$. Thus we have $\vartheta_k^{i+1} > 0$, which completes the proof. □

Since $\vartheta_k^i \in (0, 1)$ is guaranteed when $\gamma_k^i = 0$, smaller estimation error covariance can be obtained by exploiting the set-valued measurement information, which implies the improved estimation performance. Also, we know that for a given sensor information sequence s, the resultant optimal estimate evolves according to (21) and (23). The calculation of ϑ_k^i mainly requires the calculation of the standard Q-functions, which is easy to implement. Therefore, theoretically, the derived event-based estimator enjoys both potentially improved performance and a simple closed form with low computational complexity. The actual effectiveness of the estimator will be further verified in the following section.

4. Experimental verification of the proposed results based on Monte Carlo simulations

In this section, we test the efficiency of the proposed results by Monte Carlo simulation. Specifically, we consider the practical "send on delta" communication strategy (Miskowicz, 2006), namely, at time k, sensor i decides whether to send new measurement updates to the remote estimator according to the following condition:

$$\gamma_k^i = \begin{cases} 1 & \text{if } |y_k^i - y_{\tau_k^i}^i| \ge \delta^i, \\ 0 & \text{otherwise}, \end{cases} \tag{25}$$

where τ_k^i denotes the last instance when the measurement of sensor i is transmitted. To study the applicability of the results, we consider three categories of systems:

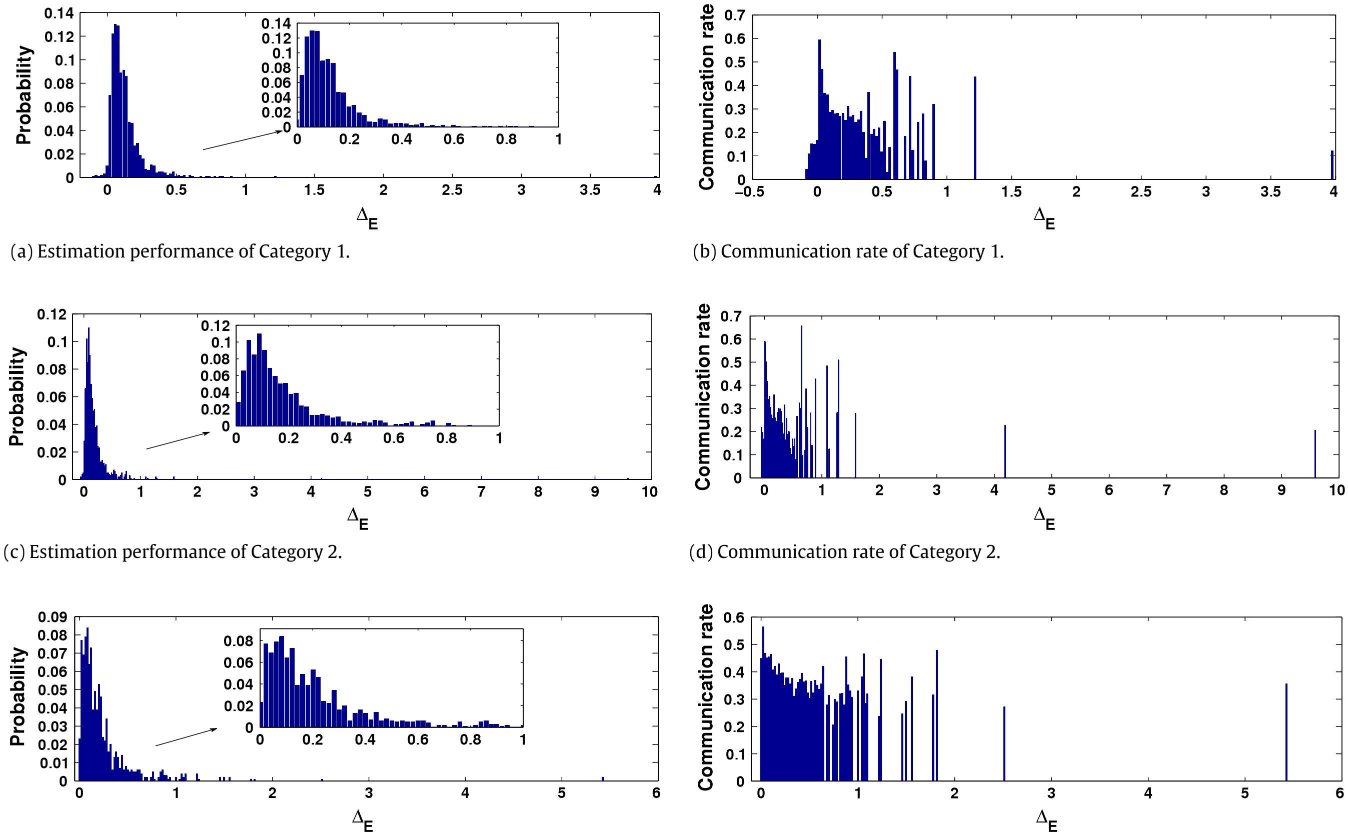
(1) Category 1: $\mathrm{trace}\{Q\}/n \gg \mathrm{trace}\{R^i\}/m$.
(2) Category 2: $\mathrm{trace}\{Q\}/n \sim \mathrm{trace}\{R^i\}/m$.
(3) Category 3: $\mathrm{trace}\{Q\}/n \ll \mathrm{trace}\{R^i\}/m$.

For each category, we randomly generate 1000 third-order stable discrete-time systems,[3] the eigenvalues of which lie uniformly in $[-0.95, 0.95]$, and measure each system by 5 sensors with $m = 1$ and randomly generated parameters.[4] For each system, we perform the simulation for 1000 time instants and evaluate the performance of the proposed event-based estimator from two aspects:

(1) To study the possible performance improvement induced by exploring the set-valued information, comparison is made with

[3] We do not consider unstable eigenvalues here to avoid errors introduced by the unbounded state trajectories.

[4] The Q and R^i matrices are obtained by first enumerating a set of positive real numbers satisfying the same uniform distributions, and then decreasing (increasing) those corresponding to R^i's by one magnitude for Category 1 (Category 3); the δ^i's are also randomly generated positive real numbers to allow for different communication rates.



(a) Estimation performance of Category 1.
(b) Communication rate of Category 1.
(c) Estimation performance of Category 2.
(d) Communication rate of Category 2.
(e) Estimation performance of Category 3.
(f) Communication rate of Category 3.

Fig. 2. Performance validation of the proposed event-based estimator.

the Kalman filter with intermittent observations exploring only the received point-valued measurement information. To quantify the performance difference, the estimation errors are normalized by the averaged norm of the original state:

$$\Delta_E := \frac{e_K - e_E}{\sqrt{\sum_{t=1}^{1000} \|x_t\|^2/1000}}, \tag{26}$$

where e_K denotes the root average squared estimation error of the Kalman filter with intermittent observations, e_E denotes the root average squared estimation error of the proposed event-based estimator, and x_t denotes the random generated state trajectory of the system. The distributions of Δ_E's for different categories as well as the corresponding average communication rates[5] are plotted in Fig. 2. From this figure, it is observed that the proposed event-based estimator obtained almost guaranteed improved performance compared with the Kalman filter with intermittent observations, indicating the efficient exploitation of the set-valued information. The only few cases that the event-based estimator slightly deteriorates the estimation performance belong to Category 1 (see Fig. 2(a)), and from Fig. 2(b). It is observed that these cases have very low communication rates, which correspond to large δ^i's; intuitively, the Gaussian assumptions sometimes may not be accurate enough to provide effective description of the *a priori* distributions for this case, thus resulting in less effective estimates.

(2) To test the sensitivity of the estimation performance to sensor fusion sequences, comparison is made between the estimates that are obtained according to different sequences of sensors. The first one is obtained by the sequences that minimize the estimation error variances at each time instant, while the second one is obtained by sequences that maximize the estimation error variance at each time instant. To quantify the performance difference, define the normalized performance difference as

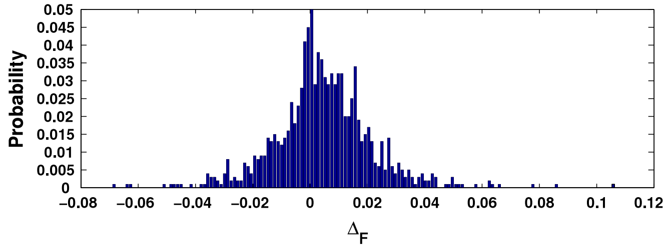
$$\Delta_F := \frac{e_W - e_B}{e_B}, \tag{27}$$

where e_B and e_W denote the root average squared estimation errors of the fusion sequences obtained by minimizing and maximizing the error variance, respectively. The distribution of Δ_F's and the corresponding communication rates are shown in Fig. 3. It is observed that the difference is always relatively small, and becomes smaller as the system becomes more measurement-noise dominant. Since the difference should be zero for the MMSE estimate without the Gaussian assumption, the results indicate that the proposed estimator represents the exact MMSE estimator to a satisfactory extent.

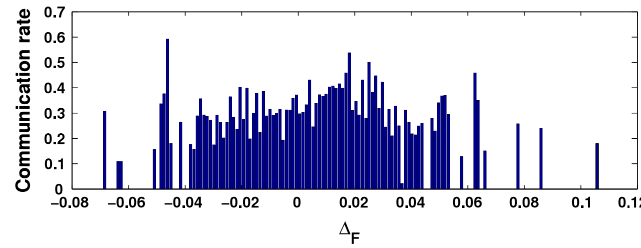
5. Conclusion

In this work, the problem of optimal fusion of hybrid measurement information for event-based estimation is studied. For a fixed sensor sequence, we show that the optimal MMSE estimate depends on the conditional mean and variance of the innovations. When each sensor has only one channel, a closed-form representation for the MMSE estimate is developed, and it is proved that exploring the set-valued information always improves estimation performance. The results are equally applicable to multiple-channel sensors with separate event-triggering conditions. Extensive simulation results show that the proposed

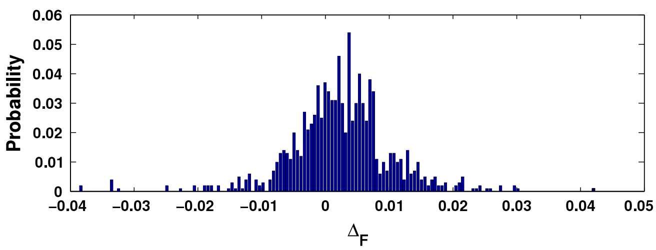
5 The average communication rates are calculated as $\frac{1}{5\cdot 1000}\sum_{i=1}^{5}\sum_{k=1}^{1000}\gamma_k^i$, which are nonnegative by definition.



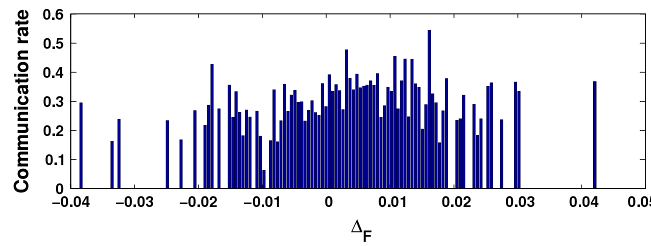
(a) Performance comparison of Category 1.



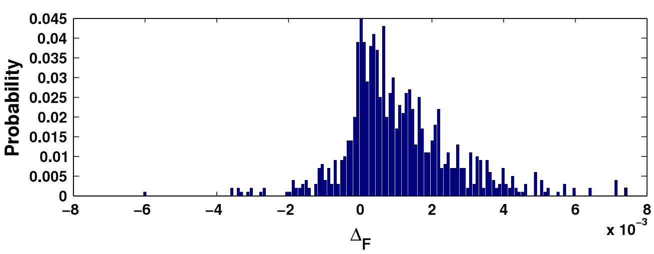
(b) Communication rate of Category 1.



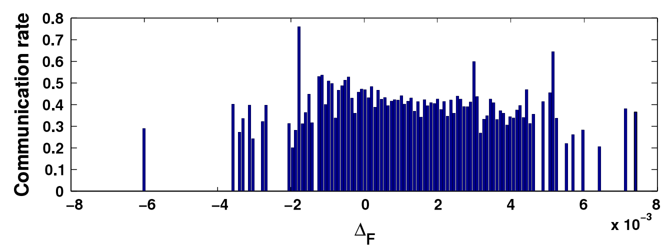
(c) Performance comparison of Category 2.



(d) Communication rate of Category 2.



(e) Performance comparison of Category 3.



(f) Communication rate of Category 3.

Fig. 3. Performance comparison between different fusion sequences.

estimator provides improved performance for most cases and is not sensitive to the fusion sequence. Future work includes the exploration of Cramér–Rao lower bound of the event-based estimation problem as well as the exploration of other nonlinear filtering techniques to remove the Gaussian assumptions.

Acknowledgments

The authors would like to thank the Associate Editor and the anonymous reviewers for their suggestions which have improved the quality of the work.

Appendix. Proof of Theorem 3

The first part of the result follows from Assumption 1. The proof of the second part is given in two steps.

(1) Proof of a few instrumental equalities:

$$\mathbf{E}[e_k^i(z_k^{i+1}-\bar{z}_k^{i+1|i})^\top|\mathcal{I}_k^{i+1}] = L_k^{i+1}\mathbf{E}[(z_k^{i+1}-\bar{z}_k^{i+1|i})(z_k^{i+1}-\bar{z}_k^{i+1|i})^\top|\mathcal{I}_k^{i+1}],$$

$$\mathbf{E}[(e_k^i-L_k^{i+1}(z_k^{i+1}-\bar{z}_k^{i+1|i}))(z_k^{i+1}-\bar{z}_k^{i+1|i})^\top|\mathcal{I}_k^{i+1}]=0,$$

$$\mathbf{E}[(e_k^i-L_k^{i+1}(z_k^{i+1}-\bar{z}_k^{i+1|i})) \times (e_k^i-L_k^{i+1}(z_k^{i+1}-\bar{z}_k^{i+1|i}))^\top|\mathcal{I}_k^i, z_k^{i+1}=z]=\tilde{g}_{i+1}(P_k^i),$$

$$\mathbf{E}[(e_k^i-L_k^{i+1}(z_k^{i+1}-\bar{z}_k^{i+1|i})) \times (e_k^i-L_k^{i+1}(z_k^{i+1}-\bar{z}_k^{i+1|i}))^\top|\mathcal{I}_k^{i+1}]=\tilde{g}_{i+1}(P_k^i).$$

Since $y_k^{i+1}=C^{i+1}x_k+v_k^{i+1}$, we have

$$\mathbf{E}(y_k^{i+1}|\mathcal{I}_k^i)=C^{i+1}\mathbf{E}(x_k|\mathcal{I}_k^i)=C^{i+1}\hat{x}_k^i. \tag{A.1}$$

$$\begin{aligned}\mathbf{Cov}[y_k^{i+1}|\mathcal{I}_k^i] &= \mathbf{E}[(y_k^{i+1}-\mathbf{E}(y_k^{i+1}|\mathcal{I}_k^i))(y_k^{i+1}-\mathbf{E}(y_k^{i+1}|\mathcal{I}_k^i))^\top|\mathcal{I}_k^i],\\ &= \mathbf{E}[(C^{i+1}e_k^i+v_k^{i+1})(C^{i+1}e_k^i+v_k^{i+1})^\top|\mathcal{I}_k^i]\\ &= C^{i+1}P_k^i(C^{i+1})^\top+R^{i+1},\end{aligned} \tag{A.2}$$

where $P_k^i=\mathbf{Cov}[x_k|\mathcal{I}_k^i]$. Since $z_k^{i+1}=y_k^{i+1}-C^{i+1}\hat{x}_k^0$,

$$\mathbf{E}(z_k^{i+1}|\mathcal{I}_k^i)=C^{i+1}\hat{x}_k^i-C^{i+1}\hat{x}_k^0. \tag{A.3}$$

$$\begin{aligned}\mathbf{Cov}[z_k^{i+1}|\mathcal{I}_k^i] &= \mathbf{E}[(z_k^{i+1}-\mathbf{E}(z_k^{i+1}|\mathcal{I}_k^i))(z_k^{i+1}-\mathbf{E}(z_k^{i+1}|\mathcal{I}_k^i))^\top|\mathcal{I}_k^i],\\ &= \mathbf{E}[(C^{i+1}e_k^i+v_k^{i+1})(C^{i+1}e_k^i+v_k^{i+1})^\top|\mathcal{I}_k^i]\\ &= C^{i+1}P_k^i(C^{i+1})^\top+R^{i+1}.\end{aligned} \tag{A.4}$$

Similarly, we have

$$\mathbf{Cov}[y_k^{i+1}x_k^\top|\mathcal{I}_k^i]=C^{i+1}P_k^i. \tag{A.5}$$

Thus

$$\mathbf{Cov}[x_k|\mathcal{I}_k^i, y_k^{i+1}=y]=\tilde{g}_{i+1}(P_k^i), \tag{A.6}$$

$$\mathbf{E}[x_k|\mathcal{I}_k^i, y_k^{i+1}=y]=\hat{x}_k^i+L_k^{i+1}(y_k^{i+1}-C^{i+1}\hat{x}_k^i). \tag{A.7}$$

By Assumption 1, z_k^{i+1} conditioned on $\mathcal{I}_{k-1}^M$ is zero-mean Gaussian; however, based on the above calculation, z_k^{i+1} conditioned on $\mathcal{I}_k^i$ is Gaussian with nonzero mean. Define $p_k^{i+1}:=\mathbf{Pr}[z_k^{i+1}\in\Omega_k^{i+1}|\mathcal{I}_k^i]=\int_{z\in\Omega_k^{i+1}}f_{z_k^{i+1}}(z|\mathcal{I}_k^i)\mathrm{d}z$. We have the conditional pdf

$$f_{z_k^{i+1}}(z|\mathcal{I}_k^{i+1})=\begin{cases}f_{z_k^{i+1}}(z|\mathcal{I}_k^i)/p_k^{i+1}, & \text{if } z\in\Omega_k^{i+1};\\ 0, & \text{otherwise}.\end{cases} \tag{A.8}$$

The rest of the proof follows from similar arguments as those below Eq. 27 of Wu et al. (2013).

(2) Proof of the theorem: the case of $\gamma_k^{i+1} = 1$ follows from (A.6) and (A.7). Now we focus on the case of $\gamma_k^{i+1} = 0$. If the information provided by sensor $i+1$ is given as a set $\mathcal{Y}_k^{i+1}$, $\hat{x}_k^{i+1}$ should evolve according to

$$\begin{aligned}\hat{x}_k^{i+1} &= \mathbf{E}[x_k|\mathcal{I}_k^{i+1}] \\ &= \int_{z\in\Omega_k^{i+1}} \mathbf{E}[x_k|\mathcal{I}_k^i, z_k^{i+1} = z] f_{z_k^{i+1}}(z|\mathcal{I}_k^i)\mathrm{d}z / p_k^{i+1} \\ &= \frac{1}{p_k^{i+1}} \int_{z\in\Omega_k^{i+1}} [\hat{x}_k^i + L_k^{i+1} z + L_k^{i+1} C^{i+1}(\hat{x}_k^0 - \hat{x}_k^i)] \\ &\quad f_{z_k^{i+1}}(z|\mathcal{I}_k^i)\mathrm{d}z \\ &= \hat{x}_k^i - L_k^{i+1}\bar{z}_k^{i+1|i} + L_k^{i+1}\bar{z}_k^{i+1|i+1},\end{aligned} \tag{A.9}$$

where $\bar{z}_k^{i+1|i+1} := \frac{1}{p_k^{i+1}} \int_{z\in\Omega_k^{i+1}} z f_{z_k^{i+1}}(z|\mathcal{I}_k^i)\mathrm{d}z = \mathbf{E}(z_k^{i+1}|\mathcal{I}_k^{i+1})$.
Finally we calculate the covariance of x_k conditioned on $\mathcal{I}_k^{i+1}$:

$$\begin{aligned}P_k^{i+1} &= \mathbf{E}[(x_k - \hat{x}_k^{i+1})(x_k - \hat{x}_k^{i+1})^\top|\mathcal{I}_k^{i+1}] \\ &= \mathbf{E}[(e_k^i - L_k^{i+1}(\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i})) \\ &\quad (e_k^i - L_k^{i+1}(\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i}))^\top|\mathcal{I}_k^{i+1}] \\ &= \tilde{g}_{i+1}(P_k^i) \\ &\quad + L_k^{i+1}\mathbf{E}[(z_k^{i+1} - \bar{z}_k^{i+1|i})(z_k^{i+1} - \bar{z}_k^{i+1|i})^\top|\mathcal{I}_k^{i+1}](L_k^{i+1})^\top \\ &\quad - L_k^{i+1}(\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i})(\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i})^\top(L_k^{i+1})^\top \end{aligned} \tag{A.10}$$

$$= \tilde{g}_{i+1}(P_k^i) + L_k^{i+1}\mathbf{Cov}(z_k^{i+1}|\mathcal{I}_k^{i+1})(L_k^{i+1})^\top, \tag{A.11}$$

where Eq. (A.10) follows from the instrumental equalities as well as the equation

$$\begin{aligned}\mathbf{E}[e_k^i|\mathcal{I}_k^{i+1}] &= \mathbf{E}[x_k - \hat{x}_k^i|\mathcal{I}_k^{i+1}] \\ &= \int_{z\in\Omega_k^{i+1}} \mathbf{E}[x_k - \hat{x}_k^i|\mathcal{I}_k^i, z_k^i = z] f_{z_k^{i+1}}(z|\mathcal{I}_k^i)\mathrm{d}z / p_k^{i+1} \\ &= \int_{z\in\Omega_k^{i+1}} L_k^{i+1}(z - \bar{z}_k^{i+1|i}) f_{z_k^{i+1}}(z|\mathcal{I}_k^i)\mathrm{d}z / p_k^{i+1} \\ &= L_k^{i+1}(\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i}),\end{aligned} \tag{A.12}$$

and Eq. (A.11) follows from the relation

$$\begin{aligned}&\mathbf{E}[(z_k^{i+1} - \bar{z}_k^{i+1|i})(z_k^{i+1} - \bar{z}_k^{i+1|i})^\top|\mathcal{I}_k^{i+1}] \\ &= \mathbf{E}[((z_k^{i+1} - \bar{z}_k^{i+1|i+1}) - (\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i})) \\ &\quad ((z_k^{i+1} - \bar{z}_k^{i+1|i+1}) - (\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i}))^\top|\mathcal{I}_k^{i+1}] \\ &= \mathbf{Cov}[z_k^{i+1}|\mathcal{I}_k^{i+1}] \\ &\quad + (\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i})(\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i})^\top. \quad \square\end{aligned}$$

References

Aliev, F. A., & Ozbek, L. (1999). Evaluation of convergence rate in the central limit theorem for the Kalman filter. *IEEE Transactions on Automatic Control*, *44*(10), 1905–1909.

Alriksson, P., & Rantzer, A. (2005). Sub-optimal sensor scheduling with error bounds. In *Proceedings of the 16th IFAC world congress*, Prague.

Anderson, B. D. O., & Moore, J. B. (1979). *Optimal filtering*. Englewood Cliffs, NJ: Prentice-Hall.

Arasaratnam, I., & Haykin, S. (2009). Cubature Kalman filters. *IEEE Transactions on Automatic Control*, *54*(6), 1254–1269.

Åström, K.J., & Bernhardsson, B.M. (2002). Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In *Proceedings of the 41st IEEE conference on decision and control, vol. 2* (pp. 2011–2016) Dec.

Chen, J. (2011). A partial order on uncertainty and information. *Journal of Theoretical Probability*, http://dx.doi.org/10.1007/s10959-011-0375-2.

Imer, O.C., & Basar, T. (2005). Optimal estimation with limited measurements. In *Proceedings of the 44th IEEE conference on decision and control, and the European control conference* (pp. 1029–1034) Dec.

Ito, K., & Xiong, K. (2000). Gaussian filters for nonlinear filtering problems. *IEEE Transactions on Automatic Control*, *45*(5), 910–927.

Johnson, N. L., Kotz, S., & Balakrishnan, N. (1994). *Continuous univariate distributions: Vol. 1* (2nd ed.). Wiley.

Julier, S., Uhlmann, J., & Durrant-Whyte, H. F. (2000). A new method for the nonlinear transformation of means and covariances in filters and estimators. *IEEE Transactions on Automatic Control*, *45*(3), 477–482.

Li, L., & Lemmon, M. (2011). Performance and average sampling period of sub-optimal triggering event in event triggered state estimation. In *Proceedings of the 50th IEEE conference on decision and control and European control conference* (pp. 1656–1661) Dec.

Li, L., Lemmon, M., & Wang, X. (2010). Event-triggered state estimation in vector linear processes. In *Proceedings of the 2010 American control conference* (pp. 2138–2143) July.

Manjunath, B.G., & Wilhelm, S. (2012). Moments calculation for the doubly truncated multivariate normal density. arXiv:1206.5387.

Miskowicz, M. (2006). Send-on-delta concept: an event-based data reporting strategy. *Sensors*, (6), 49–63.

Mo, Y., Ambrosinob, R., & Sinopoli, B. (2011). Sensor selection strategies for state estimation in energy constrained wireless sensor networks. *Automatica*, *47*, 1330–1338.

Nguyen, V. H., & Suh, Y. S. (2007). Improving estimation performance in networked control systems applying the send-on-delta transmission method. *Sensors*, *7*, 2128–2138.

Rabi, M., Moustakides, G.V., & Baras, J.S. (2006). Multiple sampling for estimation on a finite horizon. In *Proceedings of the 45th IEEE conference on decision and control* (pp. 1351–1357).

Rabi, M., Moustakides, G., & Baras, J. (2012). Adaptive sampling for linear state estimation. *SIAM Journal on Control and Optimization*, *50*(2), 672–702.

Shi, D., & Chen, T. (2013). Approximate optimal periodic scheduling of multiple sensors with constraints. *Automatica*, *49*(4), 993–1000.

Shi, D., Chen, T., & Shi, L. (2014). Event-triggered maximum likelihood state estimation. *Automatica*, *50*(1), 247–254.

Sijs, J., & Lazar, M. (2012). Event based state estimation with time synchronous updates. *IEEE Transactions on Automatic Control*, *57*(10), 2650–2655.

Spall, J. C., & Wall, K. D. (1984). Asymptotic distribution theory for the Kalman filter state estimator. *Communications in Statistics—Theory and Methods*, *13*(16), 1981–2003.

Tallis, G. M. (1961). The moment generating function of the truncated multinormal distribution. *Journal of the Royal Statistical Society. Series B. Statistical Methodology*, *23*(1), 223–229.

Tallis, G. M. (1963). Elliptical and radial truncation in normal populations. *The Annals of Mathematical Statistics*, *34*(3), 940–944.

Wu, J., Jia, Q., Johansson, K., & Shi, L. (2013). Event-based sensor data scheduling: trade-off between communication rate and estimation quality. *IEEE Transactions on Automatic Control*, *58*(4), 1041–1045.

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