An Online Sensor Power Schedule for Remote State Estimation With Communication Energy Constraint

Duo Han, Peng Cheng, Jiming Chen, and Ling Shi

***Abstract*—We consider sensor transmission power scheduling for remote state estimation with limited communication energy. A sensor needs to decide when to switch between different transmission energy levels in order to minimize the average expected estimation error covariance subject to the available energy budget. In the existing work the sensor only exploits the prior knowledge of the system parameters, the noise covariance and the channel characteristics but neglects the realtime information the estimator can provide. Thanks to the power asymmetry between the sensor and the estimator, we propose an online scheduling scheme which makes a choice based on the acknowledgement sequence at the remote estimator side and show that the scheme outperforms the optimal offline schedule under the same energy constraint.**

***Index Terms*—Kalman filter, power schedule, sensor scheduling.**

I. Introduction

Networked control systems (NCSs) have many applications in different areas, including aerospace, health care, manufacturing, public transportation, etc. [1]. In many such applications, local sensors transmit their data packets to the remote estimators over an imperfect communication channel, which might be bandwidth-limited or could induce transmission delay and even packet dropout [2]. As the major factor causing the deterioration of the estimation performance, only packet dropout is considered in this technical note. Sinopoli *et al.* [3] showed that beyond a critical value, the dropout rate will lead to unbounded estimation error covariance of a Kalman filter with intermittent observation. There are several ways to reduce the impact of packet dropout on the system estimation performance, such as better sensor location or network topology or multiple transmission for every single packets. The experimental study in [4] revealed that the impact of variable transmit power on link quality. The authors found that the larger the transmit power is, the higher the packet reception rate is. Thus an alternative approach to compensate the loss of the packets is that the sensors use the higher transmit power to ensure good estimation performance. However, most of battery-powered sensors are not able to send packets using high transmit power all the time because recharging or replacing the battery of the sensor is not economical or even impossible in some situations. Therefore a desired tradeoff between limited transmit energy of the local sensor and the remote estimation performance is wanted, which requires an appropriate power scheduling scheme.

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A lot of research has been done on power control and scheduling. Xiao *et al.* [5] suggested that in the decentralized estimation, sensors with bad channels or poor observation qualities should decrease quantization resolution or become inactive while those remaining active sensors should determine their quantization and transmit power levels based on path loss, observation noise and target performance. Wimalajeewa *et al.* [6] considered the optimal power scheduling problem for distributed detection in a sensor network with independent and correlated observation. They obtained a closed-form solution for independent observation case and a computation method for correlated observation case. Sengupta *et al.* [7] applied game theory to solve the power control problem in a CDMA based distributed sensor network. They presented that the system is power stable only if the nodes comply with certain transmit power threshold and evaluated the power level each node should transmit at to maximize the utility. Shi *et al.* [8] proposed an optimal offline sensor power scheduling scheme in terms of the estimation error covariance under some energy budget constraint. They make the best use of all offline information and design an optimal periodic power schedule.

The aforementioned works fully exploit the prior knowledge of the system parameters, the channel characteristics and the noise covariance from the point view of the sensor. However, the capability asymmetry between the sensor and the remote estimator is often neglected. Compared to the battery-powered sensor, the remote estimator or the base station usually has much larger capacity of power and computation [9]. Thanks to the power asymmetry, the estimator or the base station is able to render some feedback information to the local sensor with high reliability. The practical example could be remote state estimation based on IEEE 802.15.4/ZigBee protocol [10] in which the sensor is the network device and the estimator is the coordinator. Xiao *et al.* [11] studied dynamic transmit power control for longer usage of the body-wearable sensors used in continuous health monitoring. They found that the wireless link quality in body area varies rapidly and adjusting the transmit power in real time based on the feedback information of the link quality from the receiver is effective to achieve the desired tradeoff between energy savings and reliability. Inspired by the TCP-like structure considered in Garone *et al.* [12], we develop an online scheduling scheme which satisfies the energy constraint and further reduces the estimation error covariance compared to the optimal offline schedule. In the proposed scheme, the sensor decides whether to use high energy level based on the packets arrival feedback information. The main contributions of this technical note and comparison with existing work from the literature are summarized as follows:

1) We consider the interaction between the sensor and the estimator rather than the passive reception mode at the estimator side. The information exchange is able to further improve the estimation performance.
2) We propose an online sensor power schedule under energy constraint and compare analytically the estimation performance with the optimal offline scheduling scheme in [8].

The remainder of the technical note is organized as follows. In Section II, the mathematical model of the considered problem is given. The optimal offline schedule is introduced in Section III. An online sensor power schedule is then proposed in Section IV and a performance comparison is conducted in Section V. Some concluding remarks are provided in the end.

Notations: $\mathbb{Z}$ is the set of non-negative integers. $\mathbb{N}$ is the set of natural numbers. $k \in \mathbb{Z}$ is the time index. $\mathbb{E}[\cdot]$ is the expectation of a random variable and $\mathbb{E}[\cdot|\cdot]$ is the conditional expectation. $\Pr(\cdot)$ is the probability of a random event. $\mathrm{Tr}(\cdot)$ is the trace of a matrix and $\|\cdot\|_1$

is the 1-norm of a vector. $\mathbb{S}_+^n$ is the set of n by n positive semi-definite matrices. When $X \in \mathbb{S}_+^n$, it is written as $X \geq 0$.

For functions $f, f_1, f_2 : \mathbb{S}_+^n \to \mathbb{S}_+^n$, $f_1 \circ f_2$ is defined as $f_1 \circ f_2(X) \triangleq f_1\big(f_2(X)\big)$ and f^t is defined as $f^t(X) \triangleq \underbrace{f \circ f \circ \cdots \circ f}_{t \text{ times}}(X)$ with $f^0(X) = X$. MSB is the abbreviation of the most significant bit of a binary string. LSB is the abbreviation of the least significant bit of a binary string.

II. Problem Setup

Consider the following discrete linear time-invariant system:

$$x_{k+1} = Ax_k + w_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the process state vector of the system at time k, $y_k \in \mathbb{R}^n$ is the is the observation vector at k, w_k's and v_k's are zero-mean Gaussian noises with $\mathbb{E}[w_k w_j'] = \delta_{kj} Q (Q \geq 0), \mathbb{E}[v_k v_j'] = \delta_{kj} R (R \geq 0), \mathbb{E}[w_k v_j'] = 0 \,\forall j, k$, where the Kronecker delta $\delta_{kj} = 1$ if $k = j$, otherwise $\delta_{kj} = 0$. The initial state x_0 is also zero-mean Gaussian with covariance $\Pi_0 > 0$, which is uncorrelated with w_k and v_k. Assume $(A, \sqrt{Q})$ is controllable and (A, C) is observable. The local sensor's state estimate $\hat{x}_k^s$ and its corresponding error covariance are

$$\hat{x}_k^s = \mathbb{E}[x_k | y_1, \ldots, y_k],$$
$$P_k^s = \mathbb{E}[(x_k - \hat{x}_k^s)(x_k - \hat{x}_k^s)' | y_1, \ldots, y_k].$$

Assume that the local sensor preprocesses the measurements up to time k and sends the local estimate $\hat{x}_k^s$ to the remote estimator over a packet-dropping channel. Reliable transmission is essentially obtained using larger transmission power. However, the limited energy budget prevents the sensor using high transmission power at each k. This requests that the local sensor reduces the transmission power at some time, which inevitably sacrifices the remote estimation performance. In practice, there are many commercial sensors with different transmission energy levels built in nowadays [5]. For simplicity, we assume the sensor has two operation modes: if the sensor uses Δ energy, the sensor data is guaranteed to arrive at the estimator; if the sensor spends δ energy, the data packet arrives at the remote estimator only with probability $\lambda \in (0, 1)$[1]. Assume both Δ and δ are rational numbers. Denote γ_k as the energy choice at time k, i.e., $\gamma_k = 1$ means the sensor sends data using Δ while $\gamma_k = 0$ means the sensor uses δ. When δ energy is used, let $\lambda_k = 1$ or 0 indicate whether the data packet arrives successfully or not. Assume $\lambda_k' s$ are i.i.d Bernoulli random variables with mean λ.

A power schedule θ is represented as $\{\gamma_1, \gamma_2, \ldots, \gamma_k, \ldots\}$. Denote $G_k(\theta)$ as the data packets received by the remote estimator up to time k, i.e.

$$G_k(\theta) = \{\gamma_1 y_1, \ldots, \gamma_k y_k\}.$$

Apparently, $G_k(\theta)$ depends on the underlying sensor power schedule θ and the random packets dropping over the channel. As a result, the remote estimate and the estimation error covariance

$$\hat{x}_k = \mathbb{E}[x_k | G_k(\theta)]$$
$$P_k = \mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)' | G_k(\theta)]$$

depends on θ and the unreliable communication channel[2]

We define $J(\theta)$ as the average expected energy cost over a infinite time horizon and $\Psi(\theta)$ as the trace of the average expected estimation error covariance. Let ϕ be the energy budget. In this technical note, we consider the following problem [8].

Problem 2.1:

$$\min_\theta \Psi(\theta) = \limsup_{T \to \infty} \frac{1}{T} \sum_{k=1}^{T} \mathrm{Tr}\left(\mathbb{E}\left[P_k(\theta)\right]\right),$$

$$\text{s.t. } J(\theta) = \limsup_{T \to \infty} \frac{1}{T} \sum_{k=1}^{T} \mathbb{E}\left[\gamma_k \Delta + (1 - \gamma_k)\delta\right] \leq \phi$$

where $\delta \leq \phi \leq \Delta$ and ϕ is a rational number and represents the available energy budget at the sensor.

III. Preliminaries

Recall from the standard Kalman filter [14], $\hat{x}_k^s$ and P_k^s are computed recursively as

$$\hat{x}_{k|k-1}^s = A\hat{x}_{k-1}^s \tag{3}$$

$$P_{k|k-1}^s = AP_{k-1}^s A' + Q \tag{4}$$

$$K_k = P_{k|k-1}^s C' [CP_{k|k-1}^s C' + R]^{-1} \tag{5}$$

$$\hat{x}_k^s = A\hat{x}_{k-1}^s + K_k(y_k - CA\hat{x}_{k-1}^s) \tag{6}$$

$$P_k^s = (I - K_k C) P_{k|k-1}^s \tag{7}$$

where the recursion starts from $\hat{x}_0^s = 0$ and $P_0^s = \Pi_0$. Denote the steady-state error covariance of the local Kalman filter as $\overline{P}$. To facilitate our analysis, we define the function $h : \mathbb{S}_+^n \to \mathbb{S}_+^n$ as

$$h(X) = AXA' + Q.$$

The following property [8] is useful in subsequent analysis.

Lemma 3.1: For $1 \leq t_1 \leq t_2, h^{t_1}(\overline{P}) \leq h^{t_2}(\overline{P})$ and $h(\overline{P}) \neq \overline{P}$. In addition, if $t_1 < t_2$, then $\mathrm{Tr}\left(h^{t_1}(\overline{P})\right) < \mathrm{Tr}\left(h^{t_2}(\overline{P})\right)$.

Since we consider infinite-time horizon, without loss of generality, we ignore the transient period of local Kalman filter at the sensor and assume that $P_k^s = \overline{P}$ for all k. Then at the estimator side, it is straightforward to show that the optimal state estimate and its estimation error covariance is given by

$$(\hat{x}_k, P_k) = \begin{cases} (A\hat{x}_{k-1}, h(P_{k-1})), & \text{if } \lambda_k = 0, \\ (\hat{x}_k^s, \overline{P}), & \text{otherwise.} \end{cases} \tag{8}$$

Shi *et al.* [8] introduced the optimal offline schedule[3] to Problem 2.1 as shown in the following proposition.

Proposition 3.2: The optimal offline schedule $\theta_{off}^\star$ to problem 2.1 over a period of q in terms of γ_k is constructed as follows:

$$\underbrace{(1\underbrace{0\ldots0}_{d_0+1})\ldots(1\underbrace{0\ldots0}_{d_0+1})}_{m}\underbrace{(1\underbrace{0\ldots0}_{d_0})\ldots(1\underbrace{0\ldots0}_{d_0})}_{n}$$

where $\frac{p}{q} = \frac{\phi - \delta}{\Delta - \delta}$ for two co-prime integers p and q, $m = q - p(d_0 + 1)$, $n = p(d_0 + 2) - q$, d_0 is the largest integer such that $d_0 \leq \frac{q}{p} - 1$. Under $\theta_{off}^\star$, the corresponding average expected energy cost is

$$J(\theta_{off}^\star) = \frac{(m + n)\Delta + (md_0 + nd_0 + m)\delta}{md_0 + nd_0 + 2m + n}$$

[1]In MQAM (Multiple Quadrature Amplitude Modulation), the symbol SNR is a increasing function of the transmit power P, i.e., $\mathrm{SNR} = f(P)$. The SER (Symbol Error Rate) $\approx e^{-\beta f(P)}$ for sufficiently large symbol SNR, where β is a constant. The symbol reception rate is $1 - e^{-\beta f(P)}$ which is a increasing function of P[13].

[2]Note that the P_k defined here is different from the usual error covariance matrix in the standard Kalman filtering due to the different conditioning. Here P_k is conditioned on the available data set $G_k(\theta)$ while the error covariance matrix in the standard Kalman filtering (e.g., P_k^s) is conditioned on all measurement data $y_1, \ldots, y_k$. This is also illustrated from the different recursive calculation of P_k and P_k^s in (7) and (8).

[3]Here offline schedule only depends on time and the system parameters, but does not utilize the realtime information of the packet arrivals.

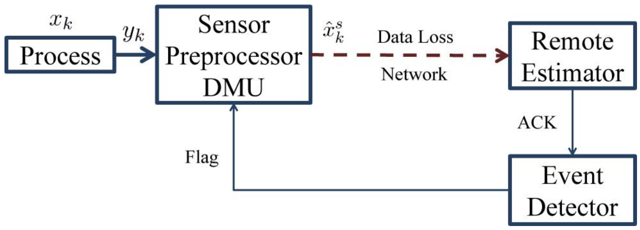

Fig. 1. Proposed online power schedule architecture.

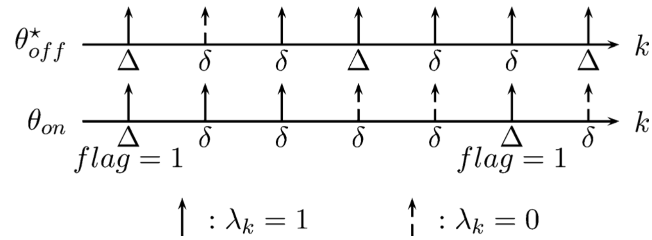

Fig. 2. Realization of $\theta_{off}^\star$ and θ_{on}.

and the trace of average expected estimation error covariance is

$$\begin{aligned}\Psi(\theta_{off}^\star) = & \frac{1}{m(d_0+2)+n(d_0+1)}\mathrm{Tr}\{m\,[1+\lambda(d_0+1)]\,\overline{P} \\ & + m\sum_{i=1}^{d_0+1}[1+\lambda(d_0+1-i)]\,(1-\lambda)^i h^i(\overline{P})+n(1 \\ & +\lambda d_0)\overline{P}+n\sum_{i=1}^{d_0}[1+\lambda(d_0-i)](1-\lambda)^i h^i(\overline{P})\}.\end{aligned}$$

The optimal offline power schedule can be determined before the system runs, which gives the optimal scheduling solution based on the prior knowledge of the entire system. Fixed periodic high-energy transmission ensures the estimation error covariance will not increase too much in each short period and thus minimizes the average expected estimation error covariance over infinite time horizon. However, in practical application if most or all of the packets in one period arrives successfully, the high-energy transmission in the next period is wasted. This motivates us to consider using the realtime feedback arrival information to further reduce the energy usage.

IV. An Online Power Schedule

In this section, we propose an online power schedule using the realtime feedback information from the remote estimator to the sensor. We construct an online sensor power schedule on the top of the optimal offline sensor power schedule $\theta_{off}^\star$ (Fig. 1).

First, we introduce the structure of the proposed scheduling scheme. Note that the remote estimator can feed the packet arrival information back to the sensor. The remote estimator has a 1-bit Acknowledgement (ACK) function which indicates whether the data packet at time k arrived or not. At the estimator side, an event detector is a unit that analyzes the ACKs and outputs bits representing different events. The event detector collects the ACKs from the estimator and store them in its L-bit memory. To save the bandwidth of the feedback channel, only 1-bit flag is used as the output of the event detector. The working principle of the memory and flag will be introduced later. Assume that the communication between the event detector and the Decision Making Unit (DMU) at the sensor side consumes Δ_f energy over a different channel and there is no bit loss during the communication. We first introduce this online schedule θ_{on} and give an explicit expression of $J(\theta_{on})$ and $\Psi(\theta_{on})$.

The operation principle is as follows. Without loss of generality, we use Δ to send the first packet. The memory is set to $11,\ldots,11$ and the flag is set to 0 initially. The detector chooses to activate a z-bit memory with a probability of μ or a $(z+1)$-bit memory with probability $\eta = 1-\mu$, where $\mu, \eta \in [0,1]$, $z+1 \le L$. When an ACK is generated and sent to the detector at each time step, the memory shifts all bits towards the MSB direction, and the new-coming ACK becomes the LSB, while the previous MSB is abandoned. Once the activated memory pattern is $00,\ldots,00$, the flag is set to 1 and then sent to DMU. The proposed online power schedule is

$$\gamma_k = \begin{cases} 1, & \text{if } flag = 1, \\ 0, & \text{if } flag = 0. \end{cases} \tag{9}$$

Meanwhile, the memory and the flag are reset to $11,\ldots,11$ and 0, respectively, and the detector needs to choose the memory length again. Otherwise, the flag remains 0 and unsent.

Remark 4.1: According to the necessary condition for optimal scheduling schemes [8], our proposed power schedule with this TCP-like architecture also satisfies $J(\theta_{on}) = \phi$. The design of activating either memory is to guarantee an exact mapping from a particular θ_{on} to the continuous rational constraint variable ϕ, by tuning z and μ. The functional block in the detector can be practically implemented by a binary series shift register, a random number generator and a binary comparator.

An example realization of $\theta_{off}^\star$ and θ_{on} is given in Fig. 2. To show the difference of the two scheduling mechanism, $\theta_{off}^\star$ is given for $\lambda = 0.5$, $\Phi = \frac{1}{3}\Delta + \frac{2}{3}\delta$ and the period is 2. Every 2 time steps the sensor will transmit the packet using high power Δ no matter whether the last two packets are dropped or not. On the contrary, the instances for the sensors in θ_{on} use high energy Δ to send packets to the remote estimator are stochastic due to the random data packet dropouts. Fig. 2 shows an episode of a specific realization of θ_{on} for $\lambda = 0.5, \mu = 1$ and $z = 2$, where μ and z are determined based on the same energy budget as that of $\theta_{off}^\star$. At time step 6 in the figure, the last two packets are observed to be dropped and the flag turns 1. Intuitively, the sensor using θ_{on} spends the high power in transmission only when the error covariance is too large. The strategy that the energy resource is distributed according to the needs turns out to be better than the offline strategy, which will be shown in Section V.

The following well-known Ergodic Theorem for Markov chain [15] is useful to derive the main results of this section.

Theorem 4.2: Let $i \in I$ be a state, I be the state space, and a transition matrix T be irreducible and positive recurrent. Let π_0 be any distribution. Suppose $X_n \sim \mathrm{Markov}(\pi_0, T)$, for any bounded function $f\colon I \to U$

$$\Pr\left(\frac{1}{n}\sum_{k=0}^{n-1} f(X_k) \to \overline{f} \text{ as } n \to \infty\right) = 1$$

where

$$\overline{f} = \sum_{i\in I}\pi(i)f(i)$$

and the vector π is the unique stationary distribution of the Markov chain.

Now we are ready to present our results. Given a pair of memory length choices z_0 and z_0+1, the selection probability μ and packets arrival rate λ, the theorems below provide closed-form solutions to $J(\theta_{on})$ and $\Psi(\theta_{on})$.

Theorem 4.3: For the system (1)–(2) and the proposed online power schedule in (9), the average expected energy cost at the sensor under θ_{on} is given by

$$J(\theta_{on}) = \frac{\mu\lambda(1-\lambda)^{z_0} + \eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1} - \eta(1-\lambda)^{z_0+2}}\Delta$$

$$+\frac{1-\mu(1-\lambda)^{z_0}-\eta(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\delta \tag{10}$$

and the induced average expected energy cost at the estimator is given by

$$J_e(\theta_{on}) = \frac{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\Delta_f$$

$for\ z_0 \in \mathbb{N}, \lambda \in (0,1), \mu, \eta \in [0,1]$.

Proof: Define τ and Φ as

$$\tau \triangleq \sup\{t|\lambda_t = 1, t < k\},$$
$$\Phi \triangleq \{\lambda_\tau, \ldots, \lambda_{k-1}, z\}$$

where Φ describes all accessible Markovian states at time k which are defined as follows:

$$\begin{aligned}
S_0(k) &\triangleq \{\Phi|\tau = k-1, z = z_0\ or\ z_0+1\};\\
S_1(k) &\triangleq \{\Phi|\tau = k-2, z = z_0\};\\
&\vdots\\
S_{z_0}(k) &\triangleq \{\Phi|\tau = k-z_0-1, z = z_0\};\\
S_{z_0+1}(k) &\triangleq \{\Phi|\tau = k-2, z = z_0+1\};\\
&\vdots\\
S_{2z_0+1}(k) &\triangleq \{\Phi|\tau = k-z_0, z = z_0+1\}.
\end{aligned}$$

The states above form a state space of a Markov chain. Denote the state transition matrix T_0 as

$$\begin{bmatrix}
\varrho_{00} & \varrho_{01} & \cdots & \varrho_{0(2z_0)} & \varrho_{0(2z_0+1)}\\
\varrho_{10} & \varrho_{11} & \cdots & \varrho_{1(2z_0)} & \varrho_{1(2z_0+1)}\\
\vdots & \vdots & \ddots & \vdots & \vdots\\
\vdots & \vdots & \ddots & \vdots & \vdots\\
\varrho_{(2z_0)0} & \varrho_{(2z_0)1} & \cdots & \varrho_{(2z_0)(z-1)} & \varrho_{(2z_0)(z)}\\
\varrho_{(2z_0+1)0} & \varrho_{(2z_0+1)1} & \cdots & \varrho_{(2z_0+1)(z-1)} & \varrho_{(2z_0+1)(z)}
\end{bmatrix}$$

where $\varrho_{ij} \triangleq \Pr(S_j(k+1)|S_i(k))$. Then one can easily obtain that

$$T_0 = \begin{bmatrix}
\lambda & \mu(1-\lambda) & \ldots & \eta(1-\lambda) & 0 & \ldots & 0\\
\lambda & 0 & \ldots & 0 & 0 & \ldots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
1 & 0 & \ldots & 0 & 0 & \ldots & 0\\
\lambda & 0 & \ldots & 0 & (1-\lambda) & \ldots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
\lambda & 0 & \ldots & 0 & 0 & \ldots & 1-\lambda\\
1 & 0 & \ldots & 0 & 0 & \ldots & 0
\end{bmatrix}.$$

Let $\overrightarrow{\Pr}$ contains the probability of each state at time k, i.e.

$$\overrightarrow{\Pr}_{(\Phi)} = [\Pr(S_0(k))\ \Pr(S_1(k))\ \ldots\ \Pr(S_{2z_0+1}(k))]'.$$

At the steady state, we have $\begin{cases} T' \cdot \overrightarrow{\Pr}_{(\Phi)} = \overrightarrow{\Pr}_{(\Phi)} \\ \|\overrightarrow{\Pr}_{(\Phi)}\|_1 = 1 \end{cases}$

Solving the above linear equations, one obtains

$$\overrightarrow{\Pr}_{(\Phi)} = \begin{pmatrix}
\frac{\lambda}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\\
\frac{\mu\lambda(1-\lambda)}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\\
\vdots\\
\frac{\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}
\end{pmatrix}. \tag{11}$$

Since the Markov chain is ergodic, the average expected energy cost is equivalent to the conditional expectation of $\gamma_k\Delta + (1-\gamma_k)\delta$ on an infinite time horizon according to the Ergodic Theorem. Thus

$$\begin{aligned}
J(\theta_{on}) &= \lim_{T\to\infty}\frac{1}{T}\sum_{k=1}^{T}[\gamma_k\Delta + (1-\gamma_k)\,\delta]\\
&= \mathbb{E}[\gamma_k\Delta + (1-\gamma_k)\delta|\Phi]\\
&= \Delta\cdot[\Pr(S_{z_0}(k)) + \Pr(S_{2z_0+1}(k))]\\
&\quad + \delta\cdot[1-\Pr(S_{z_0}(k)) - \Pr(S_{2z_0+1}(k))]\\
&= \frac{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\Delta\\
&\quad + \frac{1-\mu(1-\lambda)^{z_0}-\eta(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\delta.
\end{aligned}$$

The induced average expected energy cost is

$$\begin{aligned}
J_e(\theta_{on}) &= \lim_{T\to\infty}\frac{1}{T}\sum_{k=1}^{T}[\gamma_k\Delta_f] = \mathbb{E}[\gamma_k\Delta_f|\Phi]\\
&= \frac{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\Delta_f.
\end{aligned}$$

■

Remark 4.4: The extra energy and bandwidth cost is very limited practically for the following three reasons:

1) The estimator uses only 1-bit flag transmission to inform the sensor to make a decision, where $\Delta_f \ll \Delta$ and the required bandwidth is negligible compared to the data packets transmit bandwidth. When the data lengths vary significantly, however, the energy cost can also be very different. When at the receiving/transmitting mode, it is quite common to assume the energy cost is linear with the activation time of the receiving/transmitting mode, which is equivalent to the amount of received/sent data when the data rate is stable [16]. Therefore when the incoming data is only 1-bit (which is encapsulated in a small ACK packet), the total activation time is negligible, which implies that the additional energy cost is negligible;
2) The 1-bit flag is sent stochastically and sparsely with a probability of $\frac{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}$;
3) The fusion center or the base station for estimation generally is supplied with sufficiently large power.

From Theorem 4.3 we can determine the parameters z_0, μ and η to design a specified detector which satisfies the energy budget ϕ in Problem 2.1. The following steps 1) to 3) are for searching z_0, μ and η: We can also obtain the closed-form expression on the trace of the average expected estimation error covariance from (11).

1).Compute $\alpha = \frac{\phi-\delta}{\Delta-\delta}$.

2).Search z_0 such that $\frac{\lambda(1-\lambda)^{z_0+1}}{1-(1-\lambda)^{z_0+2}} \le \alpha \le \frac{\lambda(1-\lambda)^{z_0}}{1-(1-\lambda)^{z_0+1}}$ using the bisection method.

3).Solve μ and $\eta = 1-\mu$ such that

$$\phi = \frac{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\Delta + \frac{1-\mu(1-\lambda)^{z_0}-\eta(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\delta$$

using any numerical root searching method.

Theorem 4.5: For the system (1)–(2) and the proposed online power schedule in (9), the trace of the average expected estimation error covariance under θ_{on} is given by

$$\Psi(\theta_{on}) = \frac{\lambda}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}} \cdot \mathrm{Tr}\left[\sum_{i=0}^{z_0}(1-\lambda)^i h^i(\overline{P}) + \eta(1-\lambda)^{z_0+1}h^{z_0+1}(\overline{P})\right] \quad (12)$$

for $z_0 \in \mathbb{N}, \lambda, \mu, \eta \in (0,1)$.

Proof: Denote the estimation error covariance at time k conditioned on a specified packet arrival sequence $S_i(k)$ as $P_{k|S_i(k)}$

$$\begin{aligned}\Psi(\theta_{on}) &= \lim_{T\to\infty}\frac{1}{T}\sum_{k=1}^{T}\mathrm{Tr}\left(\mathbb{E}\left[P_k(\theta)\right]\right) = \mathrm{Tr}\left(\mathbb{E}[P_k|\Phi]\right)\\ &= \mathrm{Tr}(\Pr(S_0(k))P_{k|S_0(k)} + \cdots \\ &\quad + \Pr(S_{2z_0})P_{k|S_{2z_0}(k)} + \Pr(S_{2z_0+1})P_{k|S_{2z_0+1}(k)}\\ &= \frac{\lambda}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}} \cdot \\ &\quad \mathrm{Tr}\left[\sum_{i=0}^{z_0}(1-\lambda)^i h^i(\overline{P}) + \eta(1-\lambda)^{z_0+1}h^{z_0+1}(\overline{P})\right].\end{aligned}$$

■

V. Performance Analysis

Before we present the performance analysis of the proposed schedule, we first give a brief stability result of the remote state estimator.

Proposition 5.1: Consider the system in Fig. 1 with the sensor power schedule 9 for a nonzero energy budget Φ. The error covariance of remote estimator is upper bounded by $P_k \leq h^{z_0+1}(\overline{P})$.

Proof: The upper bound is due to the worst case where a sequence of $z_0 + 1$ packets is dropped with probability $\frac{\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}$. ■

In the rest of this section we will compare the performance of θ_{on} and $\theta^\star_{off}$. To compare the average expected error covariance of θ_{on} with $\theta^\star_{off}$, we assume $J(\theta^\star_{off}) = J(\theta_{on}) = \phi$, i.e., both schedules use the same energy budget. It is straightforward to see that it is equivalent to

$$d_0 + \frac{m}{m+n} = \frac{1-\mu(1-\lambda)^{z_0}-\eta(1-\lambda)^{z_0+1}}{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}. \quad (13)$$

Lemma 5.2: $d_0 \geq z_0$, for $z_0, d_0 \in \mathbb{N}, \lambda \in (0,1), \mu, \eta \in [0,1]$.

Proof: To prove the inequality above, first we prove the following inequality by mathematical induction:

$$\frac{1-\mu(1-\lambda)^{z_0}-\eta(1-\lambda)^{z_0+1}}{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}} > z_0.$$

When $z = 1$

$$\frac{1-\mu(1-\lambda)-\eta(1-\lambda)^2}{\mu\lambda(1-\lambda)+\eta\lambda(1-\lambda)^2} - 1 > 0.$$

Assume $\frac{1-\mu(1-\lambda)^k-\eta(1-\lambda)^{k+1}}{\mu\lambda(1-\lambda)^k+\eta\lambda(1-\lambda)^{k+1}} > k$ holds, where $k \in \mathbb{N}$. Then

$$\begin{aligned}&\frac{1-\mu(1-\lambda)^{k+1}-\eta(1-\lambda)^{k+2}}{\mu\lambda(1-\lambda)^{k+1}+\eta\lambda(1-\lambda)^{k+2}} - k - 1\\ &= \frac{1-\mu(1-\lambda)^{k}-\eta(1-\lambda)^{k+1}}{\mu\lambda(1-\lambda)^{k}+\eta\lambda(1-\lambda)^{k+1}} - k\\ &\quad + \frac{1}{(1-\mu\lambda)(1-\lambda)^{k+1}} - 1\\ &> 0\end{aligned}$$

also holds. The inequality holds for all $z_0 \in \mathbb{N}$. Then it is straightforward to see $d_0 \geq z_0$, since $d_0 \in \mathbb{N}$ and $0 \leq \frac{m}{m+n} < 1$. ■

In the next theorem we show that θ_{on} outperforms $\theta^\star_{off}$ using the same energy budget.

Theorem 5.3: Consider the two scheduling scheme $\theta^\star_{off}$ and θ_{on} with the same average expected energy cost $J(\theta^\star_{off}) = J(\theta_{on})$. Then

$$\Psi(\theta^\star_{off}) > \Psi(\theta_{on}).$$

Proof: First recall that $\Psi(\theta^\star_{off})$ in Proposition 3.2. Let $d = d_0 + \frac{m}{m+n} = \frac{1-\mu(1-\lambda)^{z_0}-\eta(1-\lambda)^{z_0+1}}{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}$. From Lemma 5.2, when $d_0 > z_0$

$$\begin{aligned}\Psi(\theta^\star_{off}) &= \frac{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\mathrm{Tr}\{\sum_{i=0}^{d_0}[1+\lambda(d\\ &\quad -i)](1-\lambda)^i h^i(\overline{P}) + \frac{m}{m+n}(1-\lambda)^{d_0+1}h^{d_0+1}(\overline{P})\}.\end{aligned}$$

Therefore,

$$\begin{aligned}&\Psi(\theta^\star_{off}) - \Psi(\theta_{on})\\ &= \frac{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\mathrm{Tr}\{\sum_{i=0}^{z_0}(-\lambda i)\cdot\\ &\quad (1-\lambda)^i h^i(\overline{P}) - \eta(1+\lambda d)(1-\lambda)^{z_0+1}h^{z_0+1}(\overline{P})\\ &\quad + \sum_{i=z_0+1}^{d_0}[1+\lambda(d-i)](1-\lambda)^i h^i(\overline{P}) + \frac{m}{m+n}\cdot\\ &\quad (1-\lambda)^{d_0+1}h^{d_0+1}(\overline{P})\}\\ &> \frac{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\mathrm{Tr}\{\sum_{i=0}^{z_0}(-\lambda i)\cdot\\ &\quad (1-\lambda)^i h^{z_0+1}(\overline{P}) - \eta(1+\lambda d)(1-\lambda)^{z_0+1}h^{z_0+1}(\overline{P})\\ &\quad + \sum_{i=z_0+1}^{d_0}[1+\lambda(d-i)](1-\lambda)^i h^{z_0+1}(\overline{P}) + \frac{m}{m+n}\cdot\\ &\quad (1-\lambda)^{d_0+1}h^{z_0+1}(\overline{P})\}\\ &> \frac{\mu\lambda(1-\lambda)^{z_0}+\eta\lambda(1-\lambda)^{z_0+1}}{1-\mu(1-\lambda)^{z_0+1}-\eta(1-\lambda)^{z_0+2}}\mathrm{Tr}\{0\cdot h^{z_0+1}(\overline{P})\}\\ &= 0.\end{aligned}$$

The last inequality is from Lemma 3.1. In the case that $d_0 = z_0$, it follows a similar proof. ■

It can be seen that θ_{on} performs strictly better than $\theta^\star_{off}$ using the same energy budget. Note that the memory length of θ_{on} and λ determine how much θ_{on} outperforms $\theta^\star_{off}$. When z_0 is large for a fixed λ or λ is high for a fixed z_0, i.e., $(1-\lambda)^{z_0} \to 0$, which makes $J(\theta_{on}) \to \delta$ and $J(\theta^\star_{off}) \to \delta$, consequently $\Psi(\theta^\star_{off}) \to \Psi(\theta_{on})$ as the estimate error covariance under each power schedule only depends on the randomness of packets arrival sequence.

To illustrate how effective the proposed schedule is, we consider the following parameters for the system (1)–(2): $A = 1.35, C = 0.2, Q = 0.5, R = 0.5$. For simplicity, assume the relationship between the packet arrival rate and the transmit power ρ is $\lambda = 1 - 0.01^\rho$, and thus $\lambda \approx 1$ when $\Delta = 1$ and $\lambda = 0.6$ when $\delta = 0.2$. We run both offline and online schedule under the same energy constraint ranging from 0.2 to 0.5 with the step size 3×10^{-5}, to illustrate how much the proposed power schedule excels the optimal online schedule. We further compare the performance of θ_{on} and $\theta^\star_{off}$ with a randomized schedule θ_{rand}, which chooses the Δ transmission energy with probability $\frac{\phi-\delta}{\Delta-\delta}$ and chooses the δ transmission energy with probability $\frac{\Delta-\phi}{\Delta-\delta}$ at each k so that the average expected transmission energy is ϕ, i.e., the same as that under θ_{on} and $\theta^\star_{off}$. From Fig. 3 we can see a significant performance improvement when the energy budget is low, which indicates the proposed scheme is more suitable for the sensors with small power storage. The qualitative analysis such as finding the global maxima for the performance difference function, i.e., $\Psi(\theta^\star_{off}) - \Psi(\theta_{on})$, will be done in the future work.

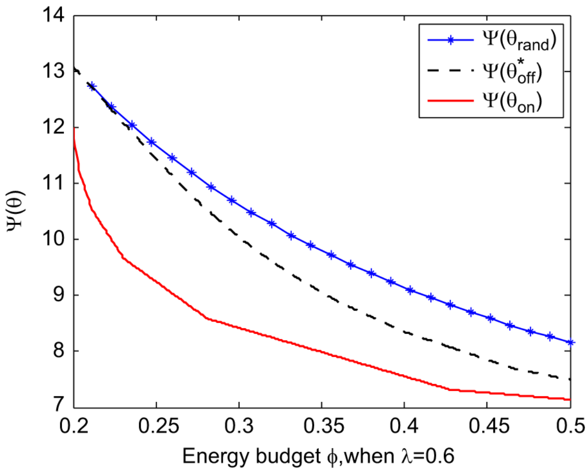

Fig. 3. Comparison of three power scheduling strategy: (1) randomized schedule, (2) optimal offline schedule, and (3) proposed online schedule under different energy constraint.

VI. Conclusion

We considered sensor power scheduling under energy constraint over a lossy channel. We proposed an online scheduling scheme which utilizes the feedback of the packet arrivals from the remote estimator. Compared with the optimal offline scheduling scheme $\theta_{off}^{\star}$, the proposed scheme θ_{on} outperforms $\theta_{off}^{\star}$ in terms of the average expected estimation error covariance under the same energy budget with negligible energy and bandwidth cost at the estimator. Future work includes power scheduling with multiple energy choices and other types of online power schedules using tools from Markov decision processes.

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