Automatica 50 (2014) 1235–1242

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Dynamic sensor transmission power scheduling for remote state estimation☆

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ARTICLE INFO

Article history:
Received 9 November 2012
Received in revised form
22 July 2013
Accepted 23 January 2014
Available online 19 March 2014

Keywords:
Kalman filtering
Sensor scheduling
Markov chain
Estimation stability

ABSTRACT

In this paper, we consider the problem of sensor transmission power scheduling for remote state estimation. We assume that the sensor has two transmission energy levels, where the high level corresponds to a high packet reception ratio. By exploiting the feedback information from the remote estimator, we aim to find an optimal transmission power schedule. We formulate the problem as a Markov decision process, and analytically develop a simple and optimal dynamic schedule which minimizes the average estimation error under the energy constraint. Furthermore, we derive the necessary and sufficient condition under which the remote state estimator is stable. It is shown that the estimation stability only depends on the high-energy packet reception ratio and the spectral radius of the system dynamic matrix.

1. Introduction

Networked control systems (NCSs) have attracted great research interest in the past decade, which have a broad range of applications including autonomous vehicles, environmental monitoring, industrial automation, smart grid, etc., (Hespanha, Naghshtabrizi, and Yonggang (2007)). In all these applications, state estimation is an indispensable ingredient. In this paper, we consider the scenario where a sensor is monitoring a system and transmits its local estimation data to a remote state estimator via a wireless communication network.

We assume the sensor has two transmission power levels, and the higher one corresponds to a lower data packet drop rate. To save energy usage (or equivalently to increase lifetime), the sensor tends to use lower transmission power as much as possible. This, however, introduces a large number of data packet drops which in turn deteriorate the estimation quality at the remote estimator. Therefore, when there is a constraint on the sensor energy usage, it is of great importance to optimally schedule the transmission power levels so as to minimize the estimation error at the remote estimator.

We also consider in this scenario that the remote estimator is able to send an acknowledgment packet back (which can be for example achieved by the media access control (MAC) protocol (Tanenbaum (2002))) to the sensor which indicates whether the transmitted packet is received or not. Under this feedback mechanism, the sensor is aware of the packet receptions in the previous times.

Before we present our main contributions and our approach for tackling the power scheduling problem, we briefly go over some related works in the literature. More references can be found from the references therein.

In Baras and Bensoussan (1988), they considered the optimal selection of a schedule of sensors, so as to optimally estimate a function of an underlying parameter. For a number of sensors and actuators, Walsh and Hong (2001) and Walsh, Hong, and Bushnell (2002) investigated when to schedule which process to access the network so that each process can remain absolutely stable. Gupta, Chung, Hassibi, and Murray (2006) proposed a stochastic sensor schedule and gave an optimal probability distribution over the sensors which minimizes an upper bound of the expected estimation

☆ The work is partially supported by NSFC 61222305, 61290321, the SRFDP under Grant 20120101110139, and National Program for Special Support of Top-Notch Young Professionals. The work by L. Shi is supported by an HKUST direct allocation grant FSGRF12EG43. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Giancarlo Ferrari-Trecate under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2014.02.022
0005-1098/

errors. Sandberg, Rabi, Skoglund, and Johansson (2008) considered a heterogeneous sensor network, i.e., low-quality measurement with small cost and high-quality measurement with high cost, and proposed an optimal schedule using time-periodic Kalman filter. Similar problems of sensor scheduling were also considered in Arai, Iwatani, and Hashimoto (2008) and Arai, Iwatani, and Hashimoto (2009). Savage and La Scala (2009) considered an optimal sensor scheduling that minimizes the terminal estimation error covariance for scalar systems. Cao, Chen, Zhang, and Sun (2008) proposed a micro-environmental monitoring and data processing system based on wireless sensor network. In Cao, Cheng, Chen, and Sun (2013), they considered a networked cyber–physical system and developed a joint optimization framework, which consists of communication protocol and online control. Ren, Cheng, Chen, Shi, and Sun (2013) considered an optimal periodic sensor scheduling that minimizes the average estimation error covariance.

Walrand and Varaiya (1983) showed that feedback information is helpful in encoder–decoder design. Bansal and Başar (1989) proposed a simultaneous design of measurement and control strategies for ARMA models. Lipsa and Martins (2011) considered the joint design of pre-processor and estimator, to minimize an objective that combines the expected squared state estimation error and communication cost. They showed that threshold policies at the pre-processor and the estimator are jointly optimal. Both the problems in Walrand and Varaiya (1983) and Lipsa and Martins (2011) are analyzed in the finite time horizon. The most related work to this paper is Shi, Cheng, and Chen (2011), which considered a scheduling problem with two transmission power levels, where high level corresponds to perfect communication (i.e., the packet drop rate is 0) and low level introduces random packet drops. Compared with Shi et al. (2011), we have the following major differences.

1. We aim to find an optimal schedule among the entire schedule space, while Shi et al. (2011) only analyzed periodic schedules.
2. The tools used in this paper is different, which utilizes the communication feedback to improve the remote estimation quality.
3. Each transmission energy level introduces a packet drop rate, which is more realistic, while a higher transmission level leads to perfect communication in Shi et al. (2011).
4. Since the transmitted data can be randomly dropped and is never guaranteed to arrive under any transmission power level, the state estimation error at the remote estimator side may diverge. Thus it is necessary to analyze the stability condition, which is not an issue in Shi et al. (2011).

The main contributions of this paper are summarized as follows.

1. We show how online information can be exploited to minimize the average expected estimation error covariance by an energy-constrained sensor. The problem is formulated as a Markov decision process.
2. We develop a simple and optimal scheduling scheme, and derive an analytical expression of the minimum expected average estimation error covariance.
3. We derive a sufficient and necessary condition under which the stability of the estimator is guaranteed.

The remainder of this paper is organized as follows. In Section 2, we introduce the system models and problem formulation. In Section 3, we give some notations and some preliminaries on Kalman filter. Section 4 shows that we only need to consider stationary schedules. The optimal sensor scheduling scheme with a simple structure is derived in Section 5. Section 6 provides the sufficient and necessary condition for the estimator's stability. Two examples are provided in Section 7 to demonstrate the results. Conclusion is given at the end.

Notations. $\mathbb{Z}$ is the set of integers. $\mathbb{Z}^+$ is the set of positive integers; $k \in \mathbb{Z}^+$ is the time index. $\mathbb{N}$ is the set of nonnegative integers.

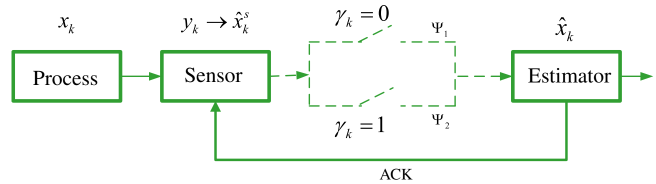

Fig. 1. System block diagram.

$\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbf{0}_n$ is an $1 \times n$ row vector $(0, 0, \ldots, 0)$. $\mathcal{S}_+^n$ is the set of $n \times n$ positive semi-definite matrices. We simply write $X \geq 0$, when $X \in \mathcal{S}_+^n$; and write $X > 0$, when X is positive definite. For functions f, f_1, f_2: $\mathcal{S}_+^n \rightarrow \mathcal{S}_+^n$, $f_1 \circ f_2$ is defined as $f_1 \circ f_2 \triangleq f_1(f_2(X))$, and f^t is defined as $f^t(X) \triangleq \underbrace{f \circ f \circ \cdots \circ f}_{t \text{ times}}(X)$ (particularly, $f^0(X) = X$).

2. System models and problem definition

2.1. System models

Consider the following dynamical model

$$x_{k+1} = Ax_k + \omega_k, \qquad y_k = Cx_k + \nu_k,$$

where $x_k \in \mathbb{R}^n$ is the state of system, $y_k \in \mathbb{R}^m$ is the measurement obtained by the sensor, and A, C are known time-invariant real matrices. $\omega_k \in \mathbb{R}^n$ and $\nu_k \in \mathbb{R}^m$ are both zero-mean Gaussian random noises with covariances $\mathbb{E}[\omega_k \omega_j'] = \Delta_{kj} Q$, $Q \geq 0$, $\mathbb{E}[\nu_k \nu_j'] = \Delta_{kj} R$, $R > 0$, and $\mathbb{E}[\omega_k \nu_j'] = 0\, \forall j, k$, where $\Delta_{kj} = 0$ if $k \neq j$ and $\Delta_{kj} = 1$ otherwise. The initial state x_0 is also a zero-mean Gaussian random vector which is uncorrelated with ω_k or ν_k and has covariance $P_0 \geq 0$. Assume the pair $(A, \sqrt{Q})$ is controllable and (C, A) is observable.

Let $Y_k = \{y_1, \ldots, y_k\}$ be all the measurement data of the system collected by the sensor from time 1 to time k. Based on Y_k, the sensor is able to estimate the system's state as $\hat{x}_k^s$ which is given by

$$\hat{x}_k^s = \mathbb{E}[x_k | Y_k], \qquad P_k^s = \mathbb{E}[(x_k - \hat{x}_k^s)(x_k - \hat{x}_k^s)' | Y_k],$$

where P_k^s is the corresponding estimation error covariance. We assume that the sensor has two energy levels to transmit $\hat{x}_k^s$ to the remote estimator (see Fig. 1). When the sensor uses a low energy Ψ_1 at time k, the data packet can be successfully delivered to the remote estimator with probability (w.p.) $p_1 \in [0, 1)$; when the sensor uses a high energy $\Psi_2 (\Psi_2 > \Psi_1 > 0)$, the data packet can be successfully delivered w.p. $p_2 \in (0, 1]$. From Zuniga and Krishnamachari (2004), the reception rate under transmission power P_t can be approximated by

$$p = \left(1 - \frac{1}{2} exp^{-\frac{P_t - PL - P_n}{2} \frac{1}{0.64}}\right)^{8f}.$$

We can see p is increasing in P_t. Thus it is reasonable to assume $p_2 > p_1$. At time k, sensor will choose one power level to transmit the packet. Let $\gamma_k = 0$ or 1 be the sensor's decision variable at time k whether it chooses the low level or high level to send its current data packet. We use θ to denote sensor's scheduling scheme that assigns the value of γ_k at each k.

In this paper, we assume there is a *communication feedback* between the sensor and the remote estimator (see Fig. 1): when a packet containing $\hat{x}_k^s$ has been transmitted by the sensor, the sensor will be know that whether this packet is successfully received by the estimator after time k. This communication feedback can be achieved by the media access control (MAC) data communication protocol (Rossi, Badia, & Zorzi, 2006; Tanenbaum, 2002). For example, in the popular used CSMA/CA protocol, receiver will send a short acknowledgment frame (ACK) back to the transmitter to signify the receipt. If the sender does not receive an ACK frame, it indicates that the transmission was unsuccessful.

2.2. Problem definition

Under a schedule θ, the estimator will calculate a state estimate $\hat{x}_k(\theta)$ of the system which has an error covariance $P_k(\theta)$. We will write $\hat{x}_k(\theta)$ as $\hat{x}_k$, etc., for short, when the underlying scheduling scheme θ is clear. Let $T \in \mathbb{Z}^+$ be the time horizon. Then the average energy cost is

$$J_a(\theta) \triangleq \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\Big[\sum_{k=1}^{T}(1-\gamma_k)\Psi_1 + \gamma_k\Psi_2\Big]. \tag{1}$$

And the average expected estimation error covariance is

$$P_a(\theta) \triangleq \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\Big[\sum_{k=1}^{T} P_k(\theta)\Big]. \tag{2}$$

Assume the sensor has a limited energy budget δ ($\Psi_1 < \delta < \Psi_2$).[2] In this paper, we are interested in the following problem.

Problem 1.

$$\min_{\theta} P_a(\theta) \quad s.t.\, J_a(\theta) \le \delta.$$

That is, we wish to find a scheduling scheme such that the average expected error covariance at the estimator is minimized under the energy constraint. As in the long run, the estimator's error will fluctuate around the average covariance.

Remark 1. Note that Problem 1 includes two simple scenarios as its special cases. If we set $p_2 = 1$, i.e., the high energy transmission is perfect, and consider no feedback, Problem 1 reduces to the setting in Shi et al. (2011). If we assume $p_1 = 0$ and $p_2 > 0$, Problem 1 reduces to the single power scheduling problem that the sensor decides whether to transmit or drop the packet intentionally for saving energy.

Since p_1 and p_2 may be both less than 1, the transmitted packet is never guaranteed to arrive at the estimator side, P_k may diverges, which leads to the instability of $P_a(\theta)$. In Problem 1, we only consider the schedules where $P_a(\theta)$ is convergent. Thus, unlike Shi et al. (2011), it is necessary to analyze the stability of $P_a(\theta)$.

Problem 2. Find the condition under which P_a converges, when an optimal schedule of Problem 1 is used.

3. Kalman filter preliminaries

In this section, we present some notations and a brief summary of the standard Kalman filter theory, which will be used frequently in the following sections.

First, we define the functions h and g as

$$h(X) \triangleq AXA' + Q,$$
$$g(X) \triangleq X - XC'[CXC' + R]^{-1}CX,$$

where (A, C, Q, R) are as shown in Section 2. It can be proved that if $0 \le X \le Y$, then $h(X) \le h(Y)$, $g(X) \le g(Y)$ and $g(X) \le X$ (see Shi, Epstein, and Murray (2010)). Based on the standard Kalman filter theory, at the sensor's side, $\hat{x}_k^s$ and P_k^s are calculated by a Kalman filter. Denote $\overline{P} = \lim_{k\to\infty} P_{k|k}^s$ as the steady-state error covariance, i.e., the solution of matrix equation $g \circ h(\overline{P}) = \overline{P}$ with $\overline{P} \ge 0$ and $\overline{P} \ne 0$ (Anderson and Moore (1979)). Then $\overline{P}$ has the following property:

Property 1. *Suppose that* $0 \le k_1 \le k_2$. *Then* $h^{k_1}(\overline{P}) \le h^{k_2}(\overline{P})$ *and* $h(\overline{P}) \ne \overline{P}$.

Since P_k^s converges to $\overline{P}$ exponentially fast, we assume the Kalman filter enters steady-state at the sensor side. Then $(\hat{x}_k, P_k)$ at the estimator side is simply given as

$$(\hat{x}_k, P_k) = \begin{cases} (\hat{x}_k^s, \overline{P}), & \text{if } \hat{x}_k^s \text{ is received,} \\ \big(A\hat{x}_{k-1}, h(P_{k-1})\big), & \text{otherwise.} \end{cases}$$

Hence if the latest time that the estimator has received a packet is at time k_1, the estimator's error covariance at time $k_2 (k_2 \ge k_1)$ is $P_{k_2} = h^{k_2-k_1}(\overline{P})$ (note that, if $k_2 = k_1$, $P_{k_2} = h^0(\overline{P}) = \overline{P}$).

4. Stationary schedule

Let $\mathbf{S} \triangleq \{\overline{P}, h(\overline{P}), h^2(\overline{P}), \ldots\}$. Then P_k will take one value in $\mathbf{S}$. Since the estimator has a communication feedback, before the sensor decides to send a packet, the sensor will know whether previous packets have been received by the estimator or not. Then, at the beginning of time k (just before choosing the value of γ_k), the sensor will know the estimator's covariance P_{k-1}. We can design the sensor's decision γ_k based on P_{k-1}, which leads to the following Markov decision process (or called discrete-time stochastic dynamic system (Bertsekas (2000)):

$$P_k = f(P_{k-1}, \gamma_k), \quad k = 1, 2, \ldots \tag{3}$$

where

- k induces discrete time,
- P_k is the state of the Markov decision process and $P_k \in \mathbf{S}$,
- γ_k is the decision variable to be selected at time k and $\gamma_k = 0$ or 1.

Let $\mathbf{Pr}(P_k|P_{k-1}, \gamma_k)$ denote a transition probability that the estimator's covariance goes from P_{k-1} to P_k, if the sensor takes decision γ_k at time k. From the definition of γ_k, we have $\mathbf{Pr}(P_k|P_{k-1}, \gamma_k) = p_1(1-\gamma_k) + p_2\gamma_k$ if $P_k = \overline{P}$, $\mathbf{Pr}(P_k|P_{k-1}, \gamma_k) = q_1(1-\gamma_k) + q_2\gamma_k$ if $P_k = h(P_{k-1})$, where $q_1 \triangleq 1 - p_1$, $q_2 \triangleq 1 - p_2$. This transition process is depicted in Fig. 2, where the dotted lines represent the transitions based on low energy, the solid lines represent those based on high energy, and the values on the lines represent the transition probabilities. We can see Problem 1 is to design the value of γ_k in the process (3) such that P_a is minimized under the energy constraint.

Generally, there are different types of schedules. Under some schedules, γ_k may vary with time, e.g., the periodic schedules. Under some ones, γ_k only depends on the preceding state P_{k-1} and does not depend on time. This type of schedules are called stationary schedules. Under some schedules, γ_k will vary with the time and the state simultaneously. We can see, in the time interval $(0, T]$, $\{\gamma_1, \gamma_2, \ldots, \gamma_T\}$ has 2^T possibilities. Finding the optimal schedule within all the schedules is difficult. Here, we will show that an optimal schedule can be a stationary schedule. Then later we only need to consider the stationary schedules.

Let μ be a general stationary sensor schedule. Then it can be defined as a row vector[3]:

$$\mu = (\mu_0, \mu_1, \mu_2, \ldots), \tag{4}$$

where $\mu_i \in [0, 1]$ represents that, at any time k, $\gamma_k = 1$ w.p. μ_i, and $\gamma_k = 0$ w.p. $1 - \mu_i$, when $P_{k-1} = h^i(\overline{P})$.

Under schedule μ given by (4), if $P_{k-1} = h^i(\overline{P})$ $(i = 0, 1, \ldots)$, we will have $P_k = \overline{P}$ w.p. $(1-\mu_i)p_1 + p_2\mu_i = p_1 + u\mu_i$, and $P_k = h^{i+1}(\overline{P})$ w.p. $q_1 - u\mu_i$, where $u \triangleq p_2 - p_1$. Then schedule

[2] Here, we only analyze the case for $\Psi_1 < \delta < \Psi_2$. This is because, when $\delta = \Psi_1$ ($\delta = \Psi_2$), the sensor only has one schedule: always use low (high) energy to send the packets.

[3] Note that, within vector μ, the first entry is denoted as μ_0, the second entry is μ_1, etc. It is similar to other two vectors π and $\phi(m)$ given later. What is more, in the matrix $\mathbb{T}_\mu$ given later, we also impose that the first row index and the first column index are both 0.

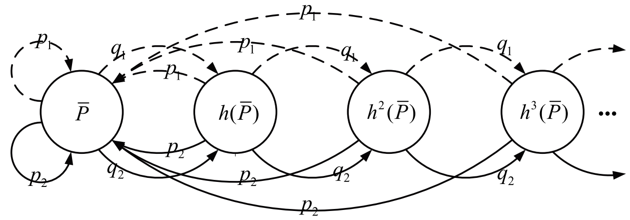

Fig. 2. Transition process.

μ generates a Markov chain with the transition probability matrix $\mathbb{T}_\mu$ given as follows:

$$\mathbb{T}_\mu = \begin{pmatrix} p_1+u\mu_0 & q_1-u\mu_0 & & & \\ p_1+u\mu_1 & & q_1-u\mu_1 & & \\ p_1+u\mu_2 & & & q_1-u\mu_2 & \\ \vdots & & & & \ddots \end{pmatrix},$$

where the (i,j)th entry in the matrix is $\mathbb{T}_\mu(i,j) = \mathbf{Pr}\big(P_k = h^j(\bar{P}) | P_{k-1} = h^i(\bar{P}), \gamma_k = 1 \text{ w.p. } \mu_i\big)$, the missing entries are 0.

Let $\pi = (\pi_0, \pi_1, \pi_2, \ldots)$ be the Markov chain limiting distribution, i.e.,

$$\pi_i = \lim_{T\to\infty} \frac{\mathbb{E}\big[\text{total number of } h^i(\bar{P}) \text{ occurred in } (0,T]\big]}{T}.$$

π can be derived by the following equations (Strook (2005))

$$\pi\mathbb{T}_\mu = \pi, \quad \sum_{i=0}^{\infty} \pi_i = 1. \tag{5}$$

Then using the vector π, the average error covariance is given as (provided $P_a(\mu)$ converges)

$$P_a(\mu) = \sum_{i=0}^{\infty} \pi_i h^i(\bar{P}), \tag{6}$$

and the average energy cost is

$$\begin{aligned} J_a(\mu) &= \sum_{i=0}^{\infty} \pi_i\big[(1-\mu_i)\Psi_1 + \mu_i\Psi_2\big] \\ &= \Psi_1 + (\Psi_2-\Psi_1)\sum_{i=0}^{\infty}\pi_i\mu_i. \end{aligned} \tag{7}$$

Lemma 1. *An optimal schedule for Problem 1 can be achieved by a stationary schedule given by (4).*

Proof. See Appendix A. ■

In the proof, we show that, under the same energy constraint, each scheduling scheme can be achieved by a schedule (4), including time-varying schedules, the widely used periodic ones in Shi et al. (2011). A similar result can be obtained from Altman (1999), but our proof is different from theirs and is derived from the problem's characteristics. Based on this result, we can therefore focus on looking for the optimal stationary schedules which would also be an optimal schedule in the whole schedule set. We introduce some notations that will be used throughout the rest of this paper:

- $\mathbf{U} = \{\mu | J_a(\mu) \le \delta\}$, $\mu^* = \arg\min_{\mu\in\mathbf{U}} P_a(\mu)$,
- $\bar{\mathbf{U}}_\delta = \{\mu | J_a(\mu) = \delta\}$, $\bar{\mu}_\delta^* = \arg\min_{\mu\in\bar{\mathbf{U}}_\delta} P_a(\mu)$.

Hence, we only need to consider the schedules in the set $\mathbf{U}$ and μ^* is an optimal schedule to Problem 1.

5. Optimal sensor schedule

In this section, we will construct an optimal sensor schedule, which utilizes the communication feedback. Unlike periodic schedules in Shi et al. (2011), this schedule is a closed-loop scheme. We can see that this optimal schedule has a simple structure which will be easily implemented by the sensor.

5.1. Necessary condition for optimal schedule

First, we have a result for the schedule $\bar{\mu}_\delta^*$.

Lemma 2. *For any energy level $\Psi_1 < \delta < \Psi_2$, a schedule $\mu = \bar{\mu}_\delta^*$ if and only if it satisfies the following conditions:*

(1) $J_a(\mu) = \delta$, *i.e.,* $\mu \in \bar{\mathbf{U}}_\delta$;
(2) μ *has a structure that* ($m \in \mathbb{N}$ *and* $\mu_m \in (0, 1]$)

$$\mu = (\underbrace{0,\ldots,0}_{m\text{ times}}, \mu_m, 1, 1, \ldots). \tag{8}$$

The proof is presented at Appendix B. We prove it by the following argument: for any schedule $\mu^{(1)} \in \bar{\mathbf{U}}_\delta$ that does not have structure (8), we can always find a new schedule $\mu^{(2)} \in \bar{\mathbf{U}}_\delta$ and $P_a(\mu^{(2)}) < P_a(\mu^{(1)})$. Next, we have a necessary condition of the optimal scheduling scheme.

Theorem 1. *The optimal schedule μ^* to Problem 1 satisfies $J_a(\mu^*) = \delta$.*

For saving space, we only provide an outline of the proof. Let $\delta_1 < \delta$ and $\mu^{(1)} = \bar{\mu}_{\delta_1}^*$, then from Lemma 2, $\mu^{(1)} = (0,\ldots,0, \mu_m^{(1)}, 1, 1, \ldots)$. Define a new schedule $\mu^{(2)}(\epsilon) = (0,\ldots,0,\mu_m^{(1)} + \epsilon, 1, 1, \ldots)$ (if $\mu_m^{(1)} < 1$) or $\mu^{(2)}(\epsilon) = (0,\ldots,0,\epsilon,1,1,\ldots)$ (if $\mu_m^{(1)} = 1$). We can prove that there exists an $\epsilon > 0$ such that $J_a(\mu^{(2)}) \le \delta$ and $P_a(\mu^{(2)}) < P_a(\mu^{(1)})$. Theorem 1 means that an optimal schedule must consume all the energy. Hence we have $\mu^* \in \bar{\mathbf{U}}_\delta$. Since $\bar{\mu}_\delta^* = \arg\min_{\mu\in\bar{\mathbf{U}}_\delta} P_a(\mu)$, we have the following corollary.

Corollary 1. *Optimal schedule to Problem 1 is $\mu^* = \bar{\mu}_\delta^*$.*

5.2. Optimal schedule

Define a schedule $\phi(m) = (\mathbf{0}_m, 1, 1, \ldots)$, i.e., the entries are $\phi_i(m) = 0$ if $0 \le i \le m-1$, $\phi_i(m) = 1$ if $i \ge m$. Then its limiting distribution is $\pi_i = \pi_0 q_1^i$, $i = 0, 1, \ldots, m$, $\pi_i = \pi_0 q_1^m q_2^{i-m}$, $i \ge m+1$, where $\pi_0 = \frac{p_1p_2}{p_2(1-q_1^{m+1})+p_1q_2q_1^m}$. The average energy cost by $\phi(m)$ is $J_a(\phi(m)) = \Psi_1 + (\Psi_2-\Psi_1)\sum_{i=0}^{\infty}\pi_i\phi_i(m) = \Psi_1 + \pi_0(\Psi_2 - \Psi_1)q_1^m/p_2$. As a consequence of Lemma 2 and Corollary 1, we are ready to present the analytical expression of the optimal schedule based on $J_a(\phi(m))$.

Theorem 2. *Given energy level $\Psi_1 < \delta < \Psi_2$, we have the following results:*

(1) *An optimal sensor schedule to Problem 1 is*

$$\mu^* = (\mathbf{0}_{m^*}, \mu_{m^*}^*, 1, 1, \ldots), \tag{9}$$

where m^* is a nonnegative integer such that $J_a(\phi(m^*)) \ge \delta$ and $J_a(\phi(m^*+1)) < \delta$, and $\mu_{m^*}^* = p_2(1-q_1^{m^*+1})/(up_1q_1^{m^*}) + q_1/u - p_2/(\alpha u q_1^{m^*})$, where $\alpha = p_1 + u\frac{\delta-\Psi_1}{\Psi_2-\Psi_1}$;

(2) *Under the optimal scheme μ^*,*

$$\begin{aligned} P_a(\mu^*) &= \alpha\sum_{i=0}^{m^*} q_1^i h^i(\bar{P}) + \alpha q_1^{m^*}(q_1 - u\mu_{m^*}^*) \\ &\quad\times \sum_{i=m^*+1}^{\infty} q_2^{i-m^*-1} h^i(\bar{P}). \end{aligned} \tag{10}$$

Proof. Since $J_a(\phi(m))$ is decreasing with m, the critical value m^* such that $J_a(\phi(m^*)) \ge \delta$ and $J_a(\phi(m^*+1)) < \delta$ is unique. We now prove that the schedule (9) defined above is an optimal solution.

Under the scheme (9), the limiting distribution of μ^* (denoted as π_i^*) is obtained as follows:

$$\pi_i^* = \begin{cases} \pi_0^* q_1^i, & 0 \le i \le m^*, \\ \pi_0^* q_1^{m^*}(q_1 - u\mu_{m^*}^*) q_2^{i-m^*-1}, & i \ge m^*+1, \end{cases}$$

where $\pi_0^* = \alpha$. It can be calculated that $J_a(\mu^*) = \sum_{i=0}^{\infty} \pi_i^* \left[(1-\mu_i^*)\Psi_1 + \mu_i^* \Psi_2\right] = \delta$. Then from Lemma 2, we have $\mu^* = \overline{\mu}_\delta^*$. And from Corollary 1, we have proved that μ^* is an optimal schedule to Problem 1.

Again, using π_i^* and μ_i^*, from $P_a(\mu^*) = \sum_{i=0}^{\infty} \pi_i^* h^i(\overline{P})$, we get the $P_a(\mu^*)$ as in (10). ■

This theorem implies that, at time k, when the estimator's covariance $P_{k-1} < h^{m^*}(\overline{P})$, it is optimal for the sensor to use low energy; when covariance $P_{k-1} > h^{m^*}(\overline{P})$, it is optimal to use high energy; and when $P_{k-1} = h^{m^*}(\overline{P})$, sensor will use high energy w.p. $\mu_{m^*}^*$. Such a clear structured schedule would be quite easy to be implemented in the modern sensor nodes.

6. Stability condition

Owing to the random property of p_1 and p_2, the estimator cannot guarantee a successful transmission within a finite time. In this section, we will study the stability conditions under which the expected error covariance will converge. Assume the matrices A, Q are as defined in Section 2. Let $\rho(A)$ represent the spectral radius of A, i.e., $\rho(A) = max\{|\lambda_i|\}$, where λ_i is an eigenvalue of A. Based on the property of A, Q, we have the following result.

Lemma 3. *If $Q \ge 0$ and $(A, \sqrt{Q})$ is controllable, the series $\sum_{i=0}^{\infty} A^i QA'^i$ converges absolutely if and only if $\rho(A) < 1$.*

Proof. See Appendix C. ■

Theorem 3. *Under scheme μ^*, $P_a(\mu^*)$ converges absolutely, if and only if $p_2 > 1 - \frac{1}{\rho(A)^2}$.*

Proof. Under μ^*, from (10), we can see the convergence of $P(\mu^*)$ is only related to the series $\sum_{i=m^*+1}^{\infty} q_2^{i-m^*-1} h^i(\overline{P})$. Since

$$\sum_{i=m^*+1}^{\infty} q_2^{i-m^*-1} h^i(\overline{P}) = q_2^{-m^*-1} \sum_{i=m^*+1}^{\infty} q_2^i h^i(\overline{P}),$$

we only need to find the convergence condition for the series $\sum_{i=0}^{\infty} q_2^i h^i(\overline{P})$. Now,

$$\begin{aligned}\sum_{i=0}^{\infty} q_2^i h^i(\overline{P}) &= \overline{P} + q_2(A\overline{P}A' + Q) + q_2^2(A^2\overline{P}A'^2 + AQA' + Q) \\ &\quad + q_2^3(A^3\overline{P}A'^3 + A^2QA'^2 + AQA' + Q) + \cdots \\ &= \sum_{i=0}^{\infty} q_2^i A^i \overline{P} A'^i + \sum_{i=1}^{\infty} q_2^i \sum_{j=0}^{i-1} A^j Q A'^j \\ &= \sum_{i=0}^{\infty} q_2^i A^i \overline{P} A'^i + \frac{q_2}{1-q_2} \sum_{i=0}^{\infty} q_2^i A^i Q A'^i \\ &= \sum_{i=0}^{\infty} (\sqrt{q_2}A)^i \overline{P} (\sqrt{q_2}A')^i \\ &\quad + \frac{q_2}{1-q_2} \sum_{i=0}^{\infty} (\sqrt{q_2}A)^i Q (\sqrt{q_2}A')^i.\end{aligned}$$

From Lemma 3, we know this series converges absolutely if and only if $\rho(\sqrt{q_2}A) < 1$, i.e., $p_2 > 1 - \frac{1}{\rho(A)^2}$. Thus, the proof is complete. ■

From this theorem, given the system parameters, the estimator's stability condition is only determined by p_2, the packet reception ratio of high energy. This result can be explained intuitively.

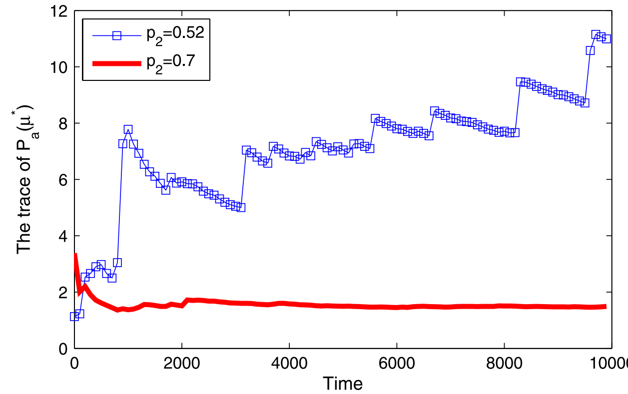

Fig. 3. Performance of $P_a(\mu^*)$ under two p_2's ($p_1 = 0.5$, $\delta = 2$).

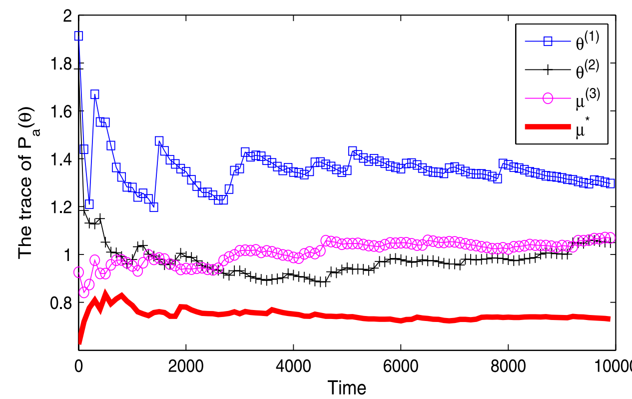

Fig. 4. Performance of different policies ($p_1 = 0.5$, $p_2 = 0.8$, $\delta = 2.5$).

Under the optimal scheduling (9), when $P_{k-1} > h^{m^*}(\overline{P})$, the sensor will always use the high energy to transmit packets. Whenever a successful transmission happens, P_k will recover to $\overline{P}$ immediately. Then, the larger of p_2, the faster will the estimator recover to $\overline{P}$. Thus, the estimator's covariance will not increase too large.

7. Illustrative examples

We give two examples to illustrate our results. Assume the system's parameters are defined as follows:

$$A = \begin{pmatrix} 1 & 0.6 \\ 0.5 & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 2 \end{pmatrix}, \qquad Q = \begin{pmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{pmatrix},$$

and $R = 1$. Suppose that $\Psi_1 = 1$, $\Psi_2 = 4$.

Example 1. Let $p_1 = 0.5$. Then from Theorem 3, when $p_2 > 1 - \frac{1}{\rho(A)^2} = 0.58$, P_a will be stable. In Fig. 3, we assume the energy budget $\delta = 2$, and depict the trace of average covariance P_a conducted by two scheduling schemes with structure (9) which are generated from two different p_2's, respectively. It can be observed that, when $p_2 = 0.52 < 0.58$, $P_a(\mu^*)$ diverges, while $p_2 = 0.7 > 0.58$, $P_a(\mu^*)$ converges rapidly.

Example 2. In Fig. 4, we fix $p_1 = 0.5$, $p_2 = 0.8$ and $\delta = 2.5$. It can be derived that an optimal scheme is $\mu^* = (0.23, 1, 1, \ldots)$. We will compare μ^* with another three schemes:

(1) periodic scheme $\theta^{(1)} = \{1\,1\,0\,0\}$: $\gamma_k = 1$ when $k = 4n+1$ or $k = 4n+2$ ($n \in \mathbb{Z}$); $\gamma_k = 0$ when $k = 4n+3$ or $k = 4(n+1)$.

(2) periodic scheme $\theta^{(2)} = \{1\,0\,1\,0\}$: $\gamma_k = 1$ when $k = 4n+1$ or $k = 4n+3$; $\gamma_k = 0$ when $k = 4n+2$ or $k = 4(n+1)$.

(3) $\mu^{(3)} = (0.6, 0, 0.7, 1, 1, \ldots)$: a stationary scheme in $\overline{\mathbf{U}}_\delta$.

Fig. 4 plots the trace of P_a under four above schemes, which shows that the trace of μ^* outperforms all the others.

8. Conclusion

In this paper, we consider the sensor scheduling problem with transmission energy constraint. Based on the communication feedback from the remote estimator, the sensor dynamically decides the optimal transmission energy level for minimizing the average expected estimation error. We formulate the problem as a Markov decision process, and first prove that the optimal solution can be achieved by a stationary schedule. Then we derive a necessary condition for a schedule to be optimal. Based on the necessary condition, we analytically develop an optimal dynamic schedule with a simple structure. We further provide the necessary and sufficient condition for the estimation stability under the optimal schedule. Examples and simulation are provided to demonstrate the results.

Appendix A. Proof of Lemma 1

Let θ^{ar} be an arbitrary sensor schedule. We will show that, there exists a stationary schedule $\mu^s = (\mu_i^s)$ given by (4) such that $J_a(\mu^s) \le J_a(\theta^{ar})$ and $P_a(\mu^s) \le P_a(\theta^{ar})$.

Define sequences $f_T^{P,ar} = \frac{1}{T}\mathbb{E}\Big[\sum_{k=1}^T P_k(\theta^{ar})\Big]$, $f_T^{\gamma,ar} = \frac{1}{T}\mathbb{E}\Big[\sum_{k=1}^T \gamma_k(\theta^{ar})\Big]$, $f_{T,i}^{ar} = \frac{1}{T}\mathbb{E}\Big[\sum_{k=1}^T I_{k,i}^{ar}\Big]$ $(i \in \mathbb{N})$, $T = 1, 2, \ldots$, where $I_{k,i}^{ar}$ denotes an indicator and is defined as $I_{k,i}^{ar} = 1$ if $P_k(\theta^{ar}) = h^i(\overline{P})$, $I_{k,i}^{ar} = 0$, otherwise. Then $\sum_{i=0}^{\infty} f_{T,i}^{ar} = 1, \forall T$.

Let $T_n \in \mathbb{Z}^+$ be some increasing sequence of times such that all the limits $\lim_{n\to\infty} f_{T_n}^{P,ar}$, $r^{H,ar} \triangleq \lim_{n\to\infty} f_{T_n}^{\gamma,ar}$, $\pi_i^{ar} \triangleq \lim_{n\to\infty} f_{T_n,i}^{ar}(i \in \mathbb{N})$ exist. Then $\pi_i^{ar} = \lim_{n\to\infty}\mathbb{E}\big[\text{total number of } h^i(\overline{P}) \text{ occurred in } (0, T_n]\big]/T_n$. It can be proved that $\sum_{i=0}^{\infty}\pi_i^{ar} = 1$. To see this, since $f_{T_n,i}^{ar}$ is nonnegative, from Fatou Lemma (Royden and Fitzpatrick (2010)), we have

$$1 = \liminf_{n\to\infty}\sum_{i=0}^{\infty} f_{T_n,i}^{ar} \ge \sum_{i=0}^{\infty}\pi_i^{ar}.$$

Since when n is sufficiently large, $f_{T_n,i}^{ar} \le 2\pi_i^{ar}$, it follows that $\sum_{i=0}^{\infty}\pi_i^{ar} = 1$. Then vector (π_i^{ar}) can be seen as a probability distribution.

Now, we will design the stationary schedule $\mu^s = (\mu_i^s)$. Let $\pi^s = (\pi_i^s)$ be π^s's limiting distribution and $\mathbb{T}_{\mu^s}$ be the transition probability matrix incurred by μ^s. Set $\pi_i^s = \pi_i^{ar}, \forall i$. Since μ^s and π^s satisfy equations $\pi^s\mathbb{T}_{\mu^s} = \pi^s$ and $\sum_{i=0}^{\infty}\pi_i^s = 1$, all the values of μ_i^s can be calculated from π^s. Then μ^s is obtained. Under this design, we have $P_a(\mu^s) = \sum_{i=0}^{\infty}\pi_i^s h^i(\overline{P}) = \sum_{i=0}^{\infty}\pi_i^{ar}h^i(\overline{P})$. Compared with $P_a(\theta^{ar})$,

$$\begin{aligned} P_a(\theta^{ar}) &= \limsup_{T\to\infty}\frac{1}{T}\mathbb{E}\Big[\sum_{k=1}^{T}P_k(\theta^{ar})\Big] \ge \lim_{n\to\infty}\frac{1}{T_n}\mathbb{E}\Big[\sum_{k=1}^{T_n}P_k(\theta^{ar})\Big] \\ &= \lim_{n\to\infty}\sum_{i=0}^{\infty} f_{T_n,i}^{ar}h^i(\overline{P}) \ge \sum_{i=0}^{\infty}\pi_i^{ar}h^i(\overline{P}) = P_a(\mu^s). \end{aligned}$$

It is observed that the average estimation error of schedule μ^s is less than θ^{ar}.

On the other hand, let index $I_k^{L,ar} = 1$ if $\gamma_k = 0$ and $P_k = \overline{P}$; $I_k^{L,ar} = 0$, otherwise. Let $I_k^{H,ar} = 1$ if $\gamma_k = 1$ and $P_k = \overline{P}$; $I_k^{L,ar} = 0$, otherwise. From the definition of π_0^{ar} and $r^{H,ar}$, we get

$$\begin{aligned} \pi_0^{ar} &= \lim_{n\to\infty}\tfrac{1}{T_n}\mathbb{E}\Big[\sum_{k=1}^{T_n}I_k^{L,ar} + \sum_{k=1}^{T_n}I_k^{H,ar}\Big] \\ &= \lim_{n\to\infty}\tfrac{1}{T_n}\Big[\sum_{k=1}^{T_n}\mathbb{E}(I_k^{L,ar}) + \sum_{k=1}^{T_n}\mathbb{E}(I_k^{H,ar})\Big] \\ &= \lim_{n\to\infty}\tfrac{1}{T_n}\Big[\sum_{k=1}^{T_n}p_1\mathbb{E}(1-\gamma_k) + \sum_{k=1}^{T_n}p_2\mathbb{E}(\gamma_k)\Big] \\ &= p_1(1-r^{H,ar}) + p_2 r^{H,ar}, \end{aligned} \tag{A.1}$$

i.e., there is a relation between π_0^{ar} and $r^{H,ar}$. In addition, there also exists a relation between $J_a(\theta^{ar})$ and $r^{H,ar}$ as shown in the following.

$$\begin{aligned} J_a(\theta^{ar}) &= \limsup_{T\to\infty}\frac{1}{T}\mathbb{E}\Big[\sum_{k=1}^{T}\Psi_1(1-\gamma_k(\theta^{ar})) + \Psi_2\gamma_k(\theta^{ar})\Big] \\ &\ge \lim_{n\to\infty}\frac{1}{T_n}\mathbb{E}\Big[\sum_{k=1}^{T_n}\Psi_1(1-\gamma_k(\theta^{ar})) + \Psi_2\gamma_k(\theta^{ar})\Big] \\ &= \Psi_1(1-r^{H,ar}) + \Psi_2 r^{H,ar}. \end{aligned} \tag{A.2}$$

Replacing $r^{H,ar}$ in (A.2) by π_0^{ar} from (A.1), $J_a(\theta^{ar}) \ge \Psi_1 + (\Psi_2 - \Psi_1)\frac{\pi_0^{ar}-p_1}{p_2-p_1}$ is obtained. To $J_a(\mu^s)$, we have

$$\begin{aligned} J_a(\mu^s) &= \sum_{i=0}^{\infty}\pi_i^s\big[(1-\mu_i^s)\Psi_1 + \mu_i^s\Psi_2\big] \\ &= \Psi_1 + (\Psi_2-\Psi_1)\sum_{i=0}^{\infty}\pi_i^s\mu_i^s. \end{aligned}$$

From equation $\pi^s\mathbb{T}_{\mu^s} = \pi^s$, $\pi_0^s = p_1 + u\sum_{i=0}^{\infty}\mu_i^s\pi_i^s$ which leads to $J_a(\mu^s) = \Psi_1 + (\Psi_2-\Psi_1)\frac{\pi_0^s-p_1}{p_2-p_1} \le J_a(\theta^{ar})$. Therefore the designed stationary schedule is better than θ^{ar}, which completes the proof.

Appendix B. Proof of Lemma 2

Since the statement of condition (1) is clear, we mainly focus on condition (2). Assume vector $\mu^{(1)} = (\mu_i^{(1)}) \in \overline{\mathbf{U}}_\delta$ but does not possess the structure (8). Then in $\mu^{(1)}$, we can find an integer t such that the entry $\mu_t^{(1)} > 0$ and $\mu_{t+1}^{(1)} < 1$. Define a new schedule: $\mu^{(2)} = (\mu_0^{(1)}, \ldots, \mu_{t-1}^{(1)}, \mu_t^{(1)} - \epsilon_1, \mu_{t+1}^{(1)} + \epsilon_2, \mu_{t+2}^{(1)}, \ldots)$, where $\epsilon_1, \epsilon_2 > 0$. First, we will prove that, there exist $\epsilon_1, \epsilon_2 > 0$ such that $\mu_t^{(1)} - \epsilon_1 \ge 0$, $\mu_{t+1}^{(1)} + \epsilon_2 \le 1$ and $J_a(\mu^{(2)}) = \delta$. Note that, if $\mu_t^{(1)} - \epsilon_1 < 0$, $\mu_{t+1}^{(1)} + \epsilon_2 > 1$, $\mu^{(2)}$ will not be a feasible schedule.

Since any stationary policy μ must satisfy equation $\pi\mathbb{T}_\mu = \pi$, $\sum_{i=0}^{\infty}\pi_i = 1$, where π is μ's limiting distribution. It follows that $\pi_i = \prod_{j=0}^{i-1}(q_1 - u\mu_j)\pi_0$, $i \ge 1$. Thus

$$\begin{aligned} J_a(\mu) &= \sum_{i=0}^{\infty}\Big[(1-\mu_i)\Psi_1 + \mu_i\Psi_2\Big]\pi_i = \Psi_1 + (\Psi_2-\Psi_1) \\ &\quad\times\Big[\pi_0\mu_0 + \sum_{i=1}^{\infty}\pi_0\mu_i\prod_{j=0}^{i-1}(q_1-u\mu_j)\Big]. \end{aligned} \tag{B.1}$$

$$P_a(\mu) = \pi_0\overline{P} + \sum_{i=1}^{\infty}\pi_0 h^i(\overline{P})\prod_{j=0}^{i-1}(q_1-u\mu_j). \tag{B.2}$$

Let $L(\mu) \triangleq 1 + \sum_{i=1}^{\infty}\prod_{j=0}^{i-1}(q_1-u\mu_j)$. Then from $\sum_{i=0}^{\infty}\pi_i = 1$, we have $\pi_0 = 1/L(\mu)$. In addition, from the proof in Lemma 1, we have $J_a(\mu) = \Psi_1 + (\Psi_2-\Psi_1)\frac{\pi_0-p_1}{p_2-p_1}$, which means $J_a(\mu)$ is only determined by π_0.

Let vectors $\pi^{(1)} = (\pi_i^{(1)})$ and $\pi^{(2)} = (\pi_i^{(2)})$ be the limiting distribution for schedule $\mu^{(1)}$ and $\mu^{(2)}$, respectively. We begin to calculate the values of ϵ_1, ϵ_2 by solving the equation $L(\mu^{(1)}) = L(\mu^{(2)})$, which is equivalent to

$$\begin{aligned} &-u\epsilon_1\prod_{j=0}^{t-1}(q_1-u\mu_j^{(1)}) + \sum_{i=t+1}^{\infty}\prod_{j\in\Lambda_{t,i}}(q_1-u\mu_j^{(1)}) \\ &\quad\times\big[\beta_1\epsilon_1 + \beta_2(\epsilon_1)\epsilon_2\big] = 0, \end{aligned} \tag{B.3}$$

where $\beta_1 = -u(q_1 - u\mu_{t+1}^{(1)})$, $\beta_2(\epsilon_1) = u(q_1 - u\mu_t^{(1)}) + u^2\epsilon_1$, $\Lambda_{t,i} = \{0, 1, \dots, t-1, t+2, \dots, i\}$. Assume ϵ_1 is given. Let $\gamma_1(\epsilon_1) = u\epsilon_1 \prod_{i=1}^{t-1}(q_1 - u\mu_j^{(1)}) - \beta_1\epsilon_1 \sum_{i=t+1}^{\infty}\prod_{j\in\Lambda_{t,i}}(q_1 - u\mu_j^{(1)})$, $\gamma_2(\epsilon_1) = \beta_2(\epsilon_1)\sum_{i=t+1}^{\infty}\prod_{j\in\Lambda_{t,i}}(q_1 - u\mu_j^{(1)})$, then Eq. (B.3) implies $\epsilon_2 = \gamma_1(\epsilon_1)/\gamma_2(\epsilon_1) > 0$. Observe that, when $\epsilon_1 = 0$, $\gamma_1(0) = 0$ and $\gamma_2(0) > 0$. Thus $\epsilon_2 = \gamma_1(\epsilon_1)/\gamma_2(\epsilon_1)$ and ϵ_1 both can be arbitrarily small. When ϵ_1 is small enough, we will have $\epsilon_1, \epsilon_2 > 0$ such that $\mu_t^{(1)} - \epsilon_1 \geq 0$, $\mu_{t+1}^{(1)} + \epsilon_2 \leq 1$.

Now using the ϵ_1, ϵ_2 obtained above, we have $L(\mu^{(1)}) = L(\mu^{(2)})$, then $\pi_0^{(1)} = \pi_0^{(2)}$. Then from (B.1) and (B.3) (or since for any μ, $J_a(\mu)$ is only determined by π_0), we get $J_a(\mu^{(1)}) - J_a(\mu^{(2)}) = 0$, i.e., $J_a(\mu^{(2)}) = \delta$.

Next, we establish that $P_a(\mu^{(1)}) > P_a(\mu^{(2)})$. Since $(p_1 - p_2)\epsilon_1 < 0$ and $q_1 - u\mu_j^{(1)} > 0$, from (B.3), we have $\beta_1\epsilon_1 + \beta_2(\epsilon_1)\epsilon_2 > 0$. Then, using Eq. (B.2),

$$
\begin{aligned}
P_a(\mu^{(1)}) - P_a(\mu^{(2)}) &= \pi_0^{(1)}h^{t+1}(\overline{P})\prod_{j=1}^{t-1}(q_1 - u\mu_j^{(1)})(p_1 - p_2)\epsilon_1 \\
&\quad + \pi_0^{(1)}\left[\beta_1\epsilon_1 + \beta_2(\epsilon_1)\epsilon_2\right]\sum_{i=t+1}^{\infty} h^{i+1}(\overline{P})\prod_{j\in\Lambda_{t,i}}(q_1 - u\mu_j^{(1)}) \\
&> \pi_0^{(1)}h^{t+1}(\overline{P})\Big[\prod_{j=0}^{t-1}(q_1 - u\mu_j^{(1)})(p_1 - p_2)\epsilon_1 \\
&\quad + \left[\beta_1\epsilon_1 + \beta_2(\epsilon_1)\epsilon_2\right] \times \sum_{i=t+1}^{\infty}\prod_{j\in\Lambda_{t,i}}(q_1 - u\mu_j^{(1)})\Big] = 0.
\end{aligned}
$$

Hence $P_a(\mu^{(1)}) > P_a(\mu^{(2)})$, i.e., the new policy $\mu^{(2)}$ is better than $\mu^{(1)}$. Therefore, proceeding this modification, at last, the optimal schedule must possess the structure (8), which proves the statement.

Appendix C. Proof of Lemma 3

"$\Leftarrow$": We can see $\|A^iQA'^i\| \leq \|A^i\| \times \|A'^i\| \times \|Q\|$, where $\|\cdot\|$ is a compatible norm. When $\rho(A) < 1$, $\|Q\|\sum_{i=0}^{\infty}(\|A^i\| \times \|A'^i\|)$ converges. Thus $\sum_{i=0}^{\infty}A^iQ(A')^i$ converges absolutely.

"$\Rightarrow$": Let $T^{-1}AT = J$ be a Jordan decomposition of A, where T is invertible and $J = diag(J_1, J_2, \dots, J_s)$ is the Jordan normal form of A with Jordan blocks, $1 \leq i \leq s$,

$$
J_i = \begin{pmatrix} \lambda_i & 1 & & \\ & \ddots & \ddots & \\ & & \lambda_i & 1 \\ & & & \lambda_i \end{pmatrix}. \tag{C.1}
$$

We assume the Jordan blocks are ordered in increasing order of the absolute values of the eigenvalues. Then $|\lambda_s| = \rho(A)$. With (C.1), the last row of J is $(0, \dots, 0, \lambda_s)$. Since

$$
\begin{aligned}
\sum_{i=0}^{\infty}A^iQA'^i &= \sum_{i=0}^{\infty}TJ^iT^{-1}\sqrt{Q}(\sqrt{Q})'(TJ^iT^{-1})' \\
&= T\left(\sum_{i=0}^{\infty}(J^iT^{-1}\sqrt{Q})(J^iT^{-1}\sqrt{Q})'\right)T',
\end{aligned} \tag{C.2}
$$

the convergence of $\sum_{i=0}^{\infty}A^iQA'^i$ is equivalent to (C.2). Then, when $\sum_{i=0}^{\infty}A^iQA'^i$ converges absolutely, we have

$$
\lim_{i\to\infty}(J^iT^{-1}\sqrt{Q})(J^iT^{-1}\sqrt{Q})' = 0. \tag{C.3}
$$

Let $M_c = \left[\sqrt{Q}, A\sqrt{Q}, A^2\sqrt{Q}, \dots, A^{n-1}\sqrt{Q}\right]$ denote the controllability matrix. Since $(A, \sqrt{Q})$ is controllable, we have rank$M_c = n$. This implies every row of M_c will not be all zeros. Recall that the last row of J is $(0, \dots, 0, \lambda_s)$. With

$$
\begin{aligned}
M_c &= \left[\sqrt{Q}, TJT^{-1}\sqrt{Q}, TJ^2T^{-1}\sqrt{Q}, \dots, TJ^{n-1}T^{-1}\sqrt{Q}\right] \\
&= T\left[T^{-1}\sqrt{Q}, JT^{-1}\sqrt{Q}, J^2T^{-1}\sqrt{Q}, \dots, J^{n-1}T^{-1}\sqrt{Q}\right],
\end{aligned}
$$

there is one $J^jT^{-1}\sqrt{Q}(0 \leq j \leq n-1)$ having form

$$
J^jT^{-1}\sqrt{Q} = \begin{pmatrix} * & * & * \\ * & \lambda_s^j r_{n,k} & * \end{pmatrix},
$$

where $r_{n,k} \in \mathbb{R}$ and one of them, say r_{n,k_0}, is nonzero in the last row of $T^{-1}\sqrt{Q}$. Therefore, when $i > j$,

$$
\begin{aligned}
(J^iT^{-1}\sqrt{Q})(J^iT^{-1}\sqrt{Q})' &= \begin{pmatrix} * & * & * \\ * & \lambda_s^i r_{n,k} & * \end{pmatrix}\begin{pmatrix} * & * & * \\ * & \lambda_s^i r_{n,k} & * \end{pmatrix}' \\
&= \begin{pmatrix} * & * \\ * & \lambda_s^{2i} r_{n,k_0}^2 + \lambda_s^{2i}\sum_{k\neq k_0} r_{n,k}^2 \end{pmatrix}.
\end{aligned}
$$

Hence, from (C.3), we have $|\lambda_s| < 1$, i.e., $\rho(A) < 1$.

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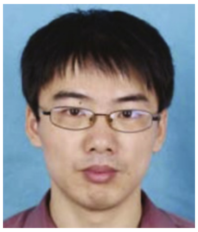

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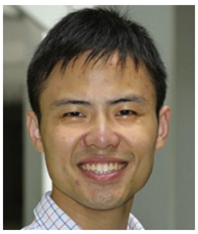

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