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Brief paper

Event-triggered maximum likelihood state estimation ☆

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ABSTRACT

The event-triggered state estimation problem for linear time-invariant systems is considered in the framework of Maximum Likelihood (ML) estimation in this paper. We show that the optimal estimate is parameterized by a special time-varying Riccati equation, and the computational complexity increases exponentially with respect to the time horizon. For ease in implementation, a one-step event-based ML estimation problem is further formulated and solved, and the solution behaves like a Kalman filter with intermittent observations. For the one-step problem, the calculation of upper and lower bounds of the communication rates from the process side is also briefly analyzed. An application example to sensorless event-based estimation of a DC motor system is presented and the benefits of the obtained one-step event-based estimator are demonstrated by comparative simulations.

1. Introduction

In wireless sensor networks, smart sensors and actuators are normally powered by batteries with limited capacity (Akyildiz, Su, Sankarasubramaniam, & Cayirci, 2002) and usually perform two types of tasks (Culler, Estrin, & Srivastava, 2004; Sveda & Vrba, 2003): simple calculation (including data acquisition) and data transmission via the wireless channel. The comparison between standard ZigBee chips designed according to IEEE 802.15.4 (2006) (e.g., CC2530 by Texas Instruments, 2011) and analog to digital converters (e.g., AD7988, 16-digit ADC from ANALOG DEVICES, 2012) indicates that the energy consumption of wireless transmission is at least one magnitude greater than that of data acquisition and basic calculation. Consequently, less communication between the sensor and the remote state estimator (or actuator) can significantly prolong the lifetime of the sensors. Event-based sensor data schedules provide an inspiring opportunity for reducing the sensor-to-estimator communications.

Pioneered by the work of Åström and Bernhardsson (2002) on Lebesgue sampling, event-based data scheduling for state estimation has received considerable attention during the last few years. The optimal event-based finite-horizon sensor transmission scheduling problems were studied in Imer and Basar (2005), and Rabi, Moustakides, and Baras (2006) for continuous-time and discrete-time scalar linear systems, respectively. The results were extended to vector linear systems in Li, Lemmon, and Wang (2010) by relaxing the zero mean initial conditions and considering measurement noises. The tradeoff between the performance and the average sampling period was analyzed in Li and Lemmon (2011), and a sup-optimal event-triggering scheme with a guaranteed least average sampling period was proposed. Adaptive sampling for state estimation of continuous-time linear systems was considered in Rabi, Moustakides, and Baras (2012). Shi, Johansson, and Qiu (2011) proposed a hybrid sensor data scheduling method by combining time and event-based methods with reduced computational complexity. In Weimer, Araújo, and Johansson (2012), a distributed event-triggered estimation problem was considered and a global event-triggered communication policy for state estimation was proposed by minimizing a weighted function of network energy consumption and communication cost while considering estimation performance constraints. The joint design of event-trigger and estimator for first-order stochastic systems with arbitrary distributions was considered in Molin and Hirche (2012), where a game-theoretic framework was utilized to analyze the optimal trade-off between the mean squared estimation error and the expected transmission rate.

In addition to the scheduling issues, another important problem is to find the optimal estimate for a specified event-triggering scheme, which provides additional information to the estimator

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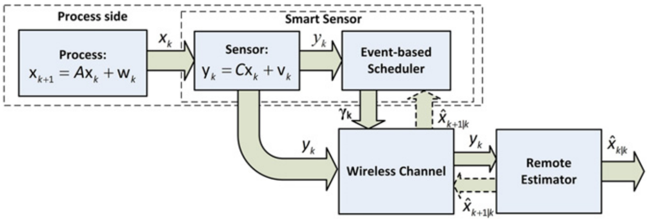

Fig. 1. Block diagram of the overall system. Note that for the proposed one-step event-triggered ML estimator, the feedback communication from the remote estimator to the smart sensor (see the dotted arrow) is only required when an event occurs at the smart sensor.

even when no measurement is transmitted from the sensor. In Wu, Jia, Johansson, and Shi (2013), the Minimum Mean Squared Error (MMSE) estimator was derived based on the Gaussian assumption of the prediction error, and the tradeoff between the sensor-to-estimator communication rate and the performance was analytically characterized. In Sijs and Lazar (2012), a general description of event sampling was proposed, and a state estimator with a hybrid update was derived using a sum of Gaussians approach to reduce the computational complexity. Trimpe and D'Andrea (2012) considered the variance-based event-triggering conditions, and the convergence of the resultant variance iterations to a periodic solution was proved. For linear Gaussian systems with periodic sensor measurements, the MMSE estimate, namely, the Kalman filter, coincides with the Maximum Likelihood (ML) estimate (Rauch, Striebel, & Tung, 1965). However, this equivalence no longer holds when the sensor measurements are updated according to an event-triggered scheme, due to the non-Gaussianity of the conditional Probability Distribution Functions (PDFs).

In this paper, the event-based state estimation problem is considered under the maximum-likelihood estimation framework. We study the remote state estimation of a process based on the measurements taken by a battery-powered smart-sensor on the process side, the output of which is transmitted to the remote estimator through a wireless channel. Foreshadowed by the discussions above, we assume that wireless transmission consumes more energy than basic calculation, and thus an event-based data-scheduler is proposed on the process side to prolong the battery life (utilizing the limited calculation capacity of the smart sensor). The main contributions of this paper are two folds:

(1) The structure of the event-based ML state estimator is provided. We show that the optimal estimate is parameterized by a special time-varying Riccati equation, and the computational complexity increases exponentially with the time horizon. Note that the solution to the Riccati equation is not necessarily the covariance matrix of the estimation error for event-based ML state estimation problems, due to event-based data updating.

(2) For ease in implementation of the event-based ML estimator, a one-step event-based ML estimation problem is formulated, and its solution is shown to behave like the Kalman filter with intermittent observations (Sinopoli et al., 2004) and only requires feedback communication when an event occurs at the smart sensor. This is different from the results in Wu et al. (2013), where feedback communication is always needed. Also, discussions on the communication rates are provided from the process side.

The rest of the paper is organized as follows. Section 2 provides the system description and problem formulation. The structure of the solution to the event-based ML state estimation problem is derived in Section 3, where the implementation issues are also discussed. In Section 4, the one-step event-based ML estimation problem is solved and the communication rate is briefly analyzed. Section 5 presents a numerical example to illustrate the efficiency of the proposed results, followed by some concluding remarks in Section 6.

Notations: $\mathbb{N}$ and $\mathbb{N}^+$ denote the sets of nonnegative and positive integers, respectively. For $a,\ b \in \mathbb{N}$ and $a \le b$, $u_{a:b}$ denotes $\{u(a), u(a+1), \ldots, u(b)\}$. $\mathbb{R}$ denotes the set of real numbers. For $m,\ n \in \mathbb{N}^+$, $\mathbb{R}^{m\times n}$ denotes the set of m by n real-valued matrices, whereas $\mathbb{R}^m$ is short for $\mathbb{R}^{m\times 1}$. For $Z \in \mathbb{R}^{m\times n}$, $Z^\top$ denotes the transpose of Z, whereas $Z^{-\top}$ denotes $(Z^\top)^{-1}$ if Z is square and nonsingular. For a random variable x, $\mathbf{E}(\mathrm{x})$ denotes its expectation, and x denotes its realization.

2. Problem formulation

Consider the system in Fig. 1. The process is Linear Time-Invariant (LTI) and evolves in discrete time driven by white noise:

$$\mathrm{x}_{k+1} = A\mathrm{x}_k + \mathrm{w}_k, \tag{1}$$

where $\mathrm{x}_k \in \mathbb{R}^n$ is the state, and $\mathrm{w}_k \in \mathbb{R}^n$ is the noise input, which is zero-mean Gaussian with covariance $Q > 0$.

The initial state x_0 is Gaussian with $\mathbf{E}(\mathrm{x}_0) = \mu_0$ and covariance $P_0 > 0$. Assume that A is nonsingular. Note that this assumption is not restrictive as (1) is typically a model that comes from discretizing a stochastic differential equation $\mathrm{d}x = A_1 x\mathrm{d}t + B_1\mathrm{d}w$, in which case $A = e^{A_1 h}$, for a sampling period h, is clearly invertible. The state information is measured by a battery-powered smart sensor, which communicates with a remote state estimator through a wireless channel, and the measurement equation is

$$\mathrm{y}_k = C\mathrm{x}_k + \mathrm{v}_k, \tag{2}$$

where $\mathrm{v}_k \in \mathbb{R}^m$ is zero-mean Gaussian with covariance $R > 0$. In addition, x_0, w_k and v_k are uncorrelated with each other. We assume that (C, A) is detectable. For consideration of the limited sensor battery capacity and the communication cost, an event-based data scheduler is integrated in the sensor. At each time instant k, the measurement information y_k is sent directly to the event-based scheduler; the estimator provides a prediction $\hat{x}_{k|k-1}$ of the current state x_k and sends the prediction $\hat{x}_{k|k-1}$ to the scheduler via the wireless channel. Based on y_k and $\hat{x}_{k|k-1}$, the scheduler computes γ_k according to the following event-triggered condition:

$$\gamma_k = \begin{cases} 0, & \text{if } \|y_k - C\hat{x}_{k|k-1}\|_\infty \le \delta \\ 1, & \text{otherwise} \end{cases} \tag{3}$$

and decides whether to allow a data transmission, where δ is a tuning parameter that determines the sensitivity of the event-based scheduler. Only when $\gamma_k = 1$, the sensor transmits y_k to the estimator. As a result, if $\gamma_k = 1$, the estimator knows the exact value of y_k; otherwise it only knows that the value of y_k lies in a known region. The ultimate goal of the estimator is to provide an estimate $\hat{x}_{k|k}$ of x_k based on the known information. Notice that this type of feedback communication strategy is not energy-saving itself and an alternative strategy is to include a copy of the estimator in the scheduler, which instead adds to the computational burden of the scheduler. We will show that the obtained result in this paper in fact does not require the feedback communication except when an event occurs, and only a simple prediction step is needed for the scheduler during a non-event time instant.

In this paper, the first objective is to determine, at time k, the optimal estimate $\hat{x}_{k|k}$ of x_k that maximizes the joint probability distribution function of $\mathrm{x}_{0:k}$ and $\mathrm{y}_{1:k}$:

$$f_{\mathrm{x}_{0:k},\mathrm{y}_{1:k}}(\hat{x}_{0|0}, x_1, \ldots, x_k, \hat{y}_1, \ldots, \hat{y}_k) \tag{4}$$

where $x_{0:k}$ and $\hat{y}_{1:k}$ are the optimization parameters. If $\gamma_t = 1$, $\hat{y}_t = y_t$; otherwise the value of $\hat{y}_t$ lies in $[\underline{y}_t,\ \bar{y}_t]$ at time instant t, where

$$\underline{y}_t = C\hat{x}_{t|t-1} - \delta\mathbf{1}_m,$$
$$\bar{y}_t = C\hat{x}_{t|t-1} + \delta\mathbf{1}_m,$$

$\mathbf{1}_m = [\underbrace{1\ 1 \ldots 1}_{m \text{ times}}]^\top$, $t = 1, 2, \ldots, k$. Consequently, at time instant k, the estimator solves the following optimization problem:

$$\begin{aligned}
\max_{x_{1:k}, \hat{y}_{1:k}} \quad & f_{\mathrm{x}_{0:k}, \mathrm{y}_{1:k}}(\hat{x}_{0|0}, x_1, \ldots, x_k, \hat{y}_1, \ldots, \hat{y}_k) \\
s.t. \quad & x_t = Ax_{t-1} + w_{t-1}, \\
& y_t = Cx_t + v_t. \\
& \hat{y}_t = y_t, \qquad \textit{if } \gamma_t = 1; \\
& \hat{y}_t \in [\underline{y}_t, \bar{y}_t], \quad \textit{if } \gamma_t = 0. \\
& t \in \{1, 2, \ldots, k\}.
\end{aligned} \tag{5}$$

The objective function in (5) is the joint probability distribution function of $\mathrm{x}_{0:k}$ and $\mathrm{y}_{1:k}$, which is always Gaussian regardless of the event-driven communication. Therefore the additional information introduced by the event-based scheduler is not reflected in the objective function and is only exploited in the constraints in (5).

Based on the solution to (5), we further look into a simpler yet more interesting one-step event-based ML estimate problem by taking the determined values of x_t and $\hat{y}_t$ (at time instant $t < k$) into account, namely, by fixing the values of x_t and $\hat{y}_t$ to the one determined at time instant t for $t = 1, 2, \ldots, k-1$ and only considering x_k and $\hat{y}_k$ as optimization variables:

$$\begin{aligned}
\max_{x_k, \hat{y}_k} \quad & f_{\mathrm{x}_{0:k}, \mathrm{y}_{1:k}}(\hat{x}_{0|0}, \ldots, \hat{x}_{k-1|k-1}, x_k, \hat{y}_1, \ldots, \hat{y}_k) \\
s.t. \quad & x_k = A\hat{x}_{k-1|k-1} + w_{k-1}, \\
& y_k = Cx_k + v_k. \\
& \hat{y}_k = y_k, \qquad \text{if } \gamma_k = 1; \\
& \hat{y}_k \in [\underline{y}_k, \bar{y}_k], \quad \text{if } \gamma_k = 0.
\end{aligned} \tag{6}$$

For this problem, we show that the solution has a simple recursive form, and the communication rate is possible to be analyzed in terms of upper and lower bounds from the process side.

3. Solution to the event-based ML estimation problem

In this section, the solution to problem (5) is derived. From Lemma 9.3.1 of Goodwin, Seron, and De Dona (2005), we have

$$\begin{aligned}
f_{\mathrm{x}_{0:k}, \mathrm{y}_{1:k}}(x_{0:k}, y_{1:k}) = \alpha \cdot \exp\left\{-\frac{1}{2}\sum_{t=0}^{k-1} w_t^\top Q^{-1} w_t\right\} \\
\cdot \exp\left\{-\frac{1}{2}\sum_{t=1}^{k} v_t^\top R^{-1} v_t\right\} \\
\cdot \exp\left\{-\frac{1}{2}(x_0 - \mu_0)^\top P_0^{-1}(x_0 - \mu_0)\right\}
\end{aligned} \tag{7}$$

where α is a positive constant, and w_t and v_t satisfy $w_t = x_{t+1} - Ax_t$ and $v_t = y_t - Cx_t$, respectively. As a result, the estimation problem that needs to be solved at time k is equivalent to

$$\begin{aligned}
\min_{w_{0:k-1}, v_{1:k}} \quad & \sum_{t=0}^{k-1} w_t^\top Q^{-1} w_t + \sum_{t=1}^{k} v_t^\top R^{-1} v_t \\
& + (x_0 - \mu_0)^\top P_0^{-1}(x_0 - \mu_0) \\
s.t. \quad & x_t = Ax_{t-1} + w_{t-1}, \\
& Cx_t + v_t = y_t, \qquad \text{if } \gamma_t = 1; \\
& Cx_t + v_t \le \bar{y}_t, \qquad \text{if } \gamma_t = 0; \\
& -Cx_t - v_t \le -\underline{y}_t, \quad \text{if } \gamma_t = 0. \\
& t \in \{1, 2, \ldots, k\}.
\end{aligned} \tag{8}$$

Before continuing, let us define the value function $V(w_{0:k-1}, v_{1:k})$ as

$$\begin{aligned}
V(w_{0:k-1}, v_{1:k}) := \sum_{t=0}^{k-1} w_t^\top Q^{-1} w_t + \sum_{t=1}^{k} v_t^\top R^{-1} v_t \\
+ (x_0 - \mu_0)^\top P_0^{-1}(x_0 - \mu_0).
\end{aligned} \tag{9}$$

For brevity, we use V_k^* to denote the optimal value function at time k, namely, $V_k^* := V(w_{0:k-1}^*, v_{1:k}^*)$. In the following, an active-set approach will be utilized to characterize the structure of the optimal solution to (8). To maintain the simplicity in the description and derivation of the results, we assume that C and v_t can be decomposed as[2]

$$C = \begin{bmatrix} \tilde{C}_t^\top & \hat{C}_t^\top \end{bmatrix}^\top \tag{10}$$

and $v_t = [\tilde{v}_t^\top\ \hat{v}_t^\top]^\top$, where $\tilde{C}_t$ and $\tilde{v}_t$ correspond to the set of active constraints

$$\tilde{v}_t + \tilde{C}_t \tilde{x}_t = \tilde{y}_t \tag{11}$$

at time t that lead to the optimal solution to (8). Correspondingly the covariance matrix R is decomposed as

$$R = \begin{bmatrix} \tilde{R}_t & \hat{R}_t \\ \hat{R}_t^\top & \bar{R}_t \end{bmatrix}^{-1}. \tag{12}$$

Define $R_t^* := (\tilde{R}_t - \hat{R}_t \bar{R}_t^{-1} \hat{R}_t^\top)^{-1}$. Utilizing these notations, for the problem in (8), we have the following results.

Theorem 1. *The optimal solution to problem (8) has the following properties:*

1. *The optimal prediction satisfies*

$$\hat{x}_{t+1|t} = A\hat{x}_{t|t};$$

the optimal estimation $\hat{x}_{t|t}$ satisfies

$$\hat{x}_{t|t} = \begin{cases} \hat{x}_{t|t-1} + P_{t|t-1}C^\top(R + CP_{t|t-1}C^\top)^{-1} \\ (y_t - C\hat{x}_{t|t-1}), \quad \text{if } \gamma_t = 1; \\ \hat{x}_{t|t-1} + P_{t|t-1}\tilde{C}_t^\top(R_t^* + \tilde{C}_t P_{t|t-1}\tilde{C}_t^\top)^{-1} \\ (\tilde{y}_t - \tilde{C}_t\hat{x}_{t|t-1}), \quad \text{if } \gamma_t = 0. \end{cases} \tag{13}$$

with $\hat{x}_{0|0} = \mu_0$, $P_{t|t-1} = AP_{t-1|t-1}A^\top + Q$, $P_{0|0} = P_0$; if $\gamma_t = 1$,

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}C^\top(R + CP_{t|t-1}C^\top)^{-1}CP_{t|t-1};$$

if $\gamma_t = 0$,

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}\tilde{C}_t^\top(R_t^* + \tilde{C}_t P_{t|t-1}\tilde{C}_t^\top)^{-1}\tilde{C}_t P_{t|t-1}.$$

2. *The optimal value function V_t^* satisfies*

$$V_t^* = (x_t - \hat{x}_{t|t})^\top P_{t|t}^{-1}(x_t - \hat{x}_{t|t}) + \Upsilon_t, \tag{14}$$

with

$$V_0^* = (x_0 - \hat{x}_{0|0})^\top P_{0|0}^{-1}(x_0 - \hat{x}_{0|0}) + \Upsilon_0,$$

$\Upsilon_0 = 0$; if $\gamma_t = 1$,

$$\Upsilon_t = \Upsilon_{t-1} + (y_t - C\hat{x}_{t|t-1})^\top(R + CP_{t|t-1}C^\top)^{-1}(y_t - C\hat{x}_{t|t-1});$$

if $\gamma_t = 0$,

$$\begin{aligned}
\Upsilon_t &= \Upsilon_{t-1} + (\tilde{y}_t - \tilde{C}_t\hat{x}_{t|t-1})^\top \\
&\quad \times (R_t^* + \tilde{C}_t P_{t|t-1}\tilde{C}_t^\top)^{-1}(\tilde{y}_t - \tilde{C}_t\hat{x}_{t|t-1}).
\end{aligned}$$

Proof. The problem in (8) is a quadratic optimization problem with linear equality and inequality constraints. According to the first-order Karush–Kuhn–Tucker conditions (which is necessary and sufficient for local optimality in this case) (Bazaraa, Sherali, & Shetty, 2006), the global optimizer of this problem can be obtained by enumerating all sets of active constraints and testing the feasibility with respect to problem (8) of the solution to the

[2] Notice that when this decomposition assumption does not hold, the results can be proved following the same argument but at the cost of more complicated notations.

corresponding quadratic optimization problem with equality constraints. Therefore, to characterize the structure of the optimal solution, it suffices to consider the set of optimal active constraints given in (11).

Without loss of generality, we first claim that the optimal value function V_{t-1}^* at time instant $k-1$ has the following form:

$$V_{t-1}^* = \Upsilon_{t-1} + (x_{t-1} - \hat{x}_{t-1|t-1})^\top P_{t-1|t-1}^{-1}(x_{t-1} - \hat{x}_{t-1|t-1}), \quad (15)$$

and then we provide an inductive proof for it. Note that this is satisfied at $k=1$ with $\Upsilon_0 = 0$ and $V_0^* = (x_0 - \hat{x}_{0|0})^\top P_{0|0}^{-1}(x_0 - \hat{x}_{0|0}) + \Upsilon_0$, where $\hat{x}_{0|0} = \mu_0$, $P_{0|0} = P_0$.

If $\gamma_t \neq 0$, following a similar argument as that in the proof of Lemma 9.6.1 of Goodwin et al. (2005), we have

$$\begin{aligned}
&V_t^* = (x_t - \hat{x}_{t|t})^\top P_{t|t}^{-1}(x_t - \hat{x}_{t|t}) + \Upsilon_t,\\
&\Upsilon_t = \Upsilon_{t-1} + (y_t - C\hat{x}_{t|t-1})^\top (R + CP_{t|t-1}C^\top)^{-1}(y_t - C\hat{x}_{t|t-1}),\\
&\hat{x}_{t|t-1} = A\hat{x}_{t-1|t-1},\\
&\hat{x}_{t|t} = \hat{x}_{t|t-1} + P_{t|t-1}C^\top (R + CP_{t|t-1}C^\top)^{-1}(y_t - C\hat{x}_{t|t-1}),\\
&P_{t|t-1} = AP_{t-1|t-1}A^\top + Q,\\
&P_{t|t} = P_{t|t-1} - P_{t|t-1}C^\top (R + CP_{t|t-1}C^\top)^{-1}CP_{t|t-1}.
\end{aligned} \quad (16)$$

If $\gamma_t = 0$, at time instant t, we solve

$$\begin{aligned}
\min_{w_{t-1}, v_t} \quad & w_{t-1}^\top Q^{-1} w_{t-1} + v_t^\top R^{-1} v_t\\
& + (A^{-1}x_t - A^{-1}Bw_{t-1})^\top P_{t-1}^{-1}\\
& (A^{-1}x_t - A^{-1}Bw_{t-1})\\
s.t. \quad & \tilde{v}_t + \tilde{C}_t \tilde{x}_t = \tilde{y}_t,
\end{aligned} \quad (17)$$

where Υ_{t-1} is independent of w_{t-1} and v_t, and the relationship $x_{t-1} = A^{-1}x_t - A^{-1}w_{t-1}$ is used.

From (12), the resulting optimization problem with equality constraints can be written as

$$\begin{aligned}
\min_{w_{t-1}, \tilde{v}_t, \hat{v}_t} \quad & w_{t-1}^\top Q^{-1} w_{t-1} + \tilde{v}_t^\top \tilde{R}_t \tilde{v}_t + \tilde{v}_t^\top \hat{R}_t \hat{v}_t\\
& + \hat{v}_t^\top \hat{R}_t^\top \tilde{v}_t + \hat{v}_t^\top \bar{R}_t \hat{v}_t + \Upsilon_{t-1}\\
& + (A^{-1}x_t - A^{-1}Bw_{t-1})^\top P_{t-1}^{-1}\\
& (A^{-1}x_t - A^{-1}Bw_{t-1})\\
s.t. \quad & \tilde{v}_t + \tilde{C}_t \tilde{x}_t = \tilde{y}_t.
\end{aligned} \quad (18)$$

This problem can be solved in two steps:

1. Optimal prediction. In this step, we identify the optimal w_{t-1}. Due to the structure of the problem, we obtain the same result as that in the case $\gamma_t = 1$. The optimizer is

$$\begin{aligned}
w_{t-1}^* = {} & (A^{-\top}P_{t-1|t-1}^{-1}A^{-1} + Q^{-1})^{-1}\\
& A^{-\top}P_{t-1|t-1}^{-1}(A^{-1}x_t - \hat{x}_{t-1|t-1}),
\end{aligned} \quad (19)$$

and the optimal prediction and the corresponding value function are $\hat{x}_{t|t-1} = A\hat{x}_{t-1|t-1}$ and

$$\begin{aligned}
V_t^\star = {} & (x_t - \hat{x}_{t|t-1})^\top P_{t|t-1}^{-1}(x_t - \hat{x}_{t|t-1}) + \tilde{v}_t^\top \tilde{R}_t \tilde{v}_t\\
& + \tilde{v}_t^\top \hat{R}_t \hat{v}_t + \hat{v}_t^\top \hat{R}_t^\top \tilde{v}_t + \hat{v}_t^\top \bar{R}_t \hat{v}_t + \Upsilon_{t-1},
\end{aligned} \quad (20)$$

respectively, where $P_{t|t-1} = AP_{t-1|t-1}A^\top + Q$.

2. Measurement update. In this step, we optimize $V_t^\star$ with respect to $\tilde{v}_t$ and $\hat{v}_t$ subject to the active constraints. To do this, we include the constraints into the objective function and differentiate $V_t^\star$ with respect to x_t and $\hat{v}_t$, respectively, which leads to

$$\begin{aligned}
&P_{t|t-1}^{-1}(x_t - \hat{x}_{t|t-1}) - \tilde{C}_t^\top \tilde{R}_t(\tilde{y}_t - \tilde{C}_t x_t) - \tilde{C}_t \hat{R}_t \hat{v}_t = 0,\\
&\hat{R}_t^\top (\tilde{y}_t - \tilde{C}_t x_t) + \bar{R}_t \hat{v}_t = 0.
\end{aligned} \quad (21)$$

Some further matrix manipulations lead to

$$\begin{aligned}
&V_t^* = (x_t - \hat{x}_{t|t})^\top P_{t|t}^{-1}(x_t - \hat{x}_{t|t}) + \Upsilon_t,\\
&\Upsilon_t = \Upsilon_{t-1} + (\tilde{y}_t - \tilde{C}_t\hat{x}_{t|t-1})^\top\\
&\qquad \times (R_t^* + \tilde{C}_t P_{t|t-1}\tilde{C}_t^\top)^{-1}(\tilde{y}_t - \tilde{C}_t\hat{x}_{t|t-1}),\\
&\hat{x}_{t|t} = \hat{x}_{t|t-1} + P_{t|t-1}\tilde{C}_t^\top (R_t^* + \tilde{C}_t P_{t|t-1}\tilde{C}_t^\top)^{-1}(\tilde{y}_t - \tilde{C}_t\hat{x}_{t|t-1}),\\
&P_{t|t} = P_{t|t-1} - P_{t|t-1}\tilde{C}_t^\top (R_t^* + \tilde{C}_t P_{t|t-1}\tilde{C}_t^\top)^{-1}\tilde{C}_t P_{t|t-1},\\
&R_t^* = (\tilde{R}_t - \hat{R}_t \bar{R}_t^{-1}\hat{R}_t^\top)^{-1}.
\end{aligned} \quad (22)$$

This completes the proof. □

Remark 2. The above result provides insights into the structure of the optimal solution to (5). However, to find the optimal solution to (5) at time k, we need to consider all possible (3^{mk}) combinations of active constraint sets considering $t = 1, 2, \ldots, k$ and compare the corresponding value functions according to (16) and (22). As a result, the computation burden will increase exponentially with respect to the time horizon. Alternatively, since the problem is a Quadratic Programming (QP) problem subject to linear constraints, standard QP solvers can be applied to find the optimal solution as well. However, the issue is that the dimension of the optimization parameters in the QP problem increases linearly with respect to k, due to the lack of a recursive structure of the optimal solution from time k to $k+1$ (this follows from the observation that the set of optimal active constraints for problem (5) at time k may not be part of the set of optimal active constraints for the problem at time $k+1$).

4. One-step event-based ML state estimation

Motivated by the implementation issues of the optimal solution to problem (5) discussed in Remark 2, we further look into the one-step event-based ML estimation problem in (6) in this section. As will be shown later, this formulation allows us to obtain a recursive solution and is a consequence of the compromise between optimality and implementability.

4.1. Solution to the problem

In this case, the problem that needs to be solved at time k is equivalent to

$$\begin{aligned}
V(w_{k-1}^\dagger, v_k^\dagger) := {} & \min_{w_{k-1}, v_k} \sum_{t=0}^{k-1} w_t^\top Q^{-1} w_t + \sum_{t=1}^{k} v_t^\top R^{-1} v_t\\
& + (x_0 - \mu_0)^\top P_0^{-1}(x_0 - \mu_0)\\
s.t.\ & x_k = Ax_{k-1} + w_{k-1},\\
& Cx_k + v_k = y_k, \quad \text{if } \gamma_k = 1;\\
& Cx_k + v_k \le \bar{y}_k, \quad \text{if } \gamma_k = 0;\\
& -Cx_k - v_k \le -\underline{y}_k, \quad \text{if } \gamma_k = 0.\\
& w_{t-1} = w_{t-1}^\dagger,\ v_t = v_t^\dagger,\\
& t \in \{1, 2, \ldots, k-1\}.
\end{aligned} \quad (23)$$

For notational simplicity, define $V_k^\dagger := V(w_{k-1}^\dagger, v_k^\dagger)$. For this problem, we have the following result.

Theorem 3. *The optimal solution to problem (23) has the following properties:*

1. *The optimal prediction is unbiased and satisfies $\hat{x}_{k+1|k} = A\hat{x}_{k|k}$; the optimal estimation $\hat{x}_{k|k}$ is also unbiased and satisfies*

$$\hat{x}_{k|k} = \begin{cases} A\hat{x}_{k-1|k-1} + P_{k|k-1}C^\top \\ (R + CP_{k|k-1}C^\top)^{-1}(y_k - CA\hat{x}_{k-1|k-1}), \\ \quad \text{if } \gamma_k = 1; \\ A\hat{x}_{k-1|k-1}, \quad \text{if } \gamma_k = 0. \end{cases} \quad (24)$$

with $\hat{x}_{0|0} = \mu_0$, $P_{k|k-1} = AP_{k-1|k-1}A^\top + Q$, $P_{0|0} = P_0$*; if* $\gamma_k = 1$,
$P_{k|k} = P_{k|k-1} - P_{k|k-1}C^\top(R + CP_{k|k-1}C^\top)^{-1}CP_{k|k-1}$;
if $\gamma_k = 0$,
$P_{k|k} = AP_{k-1|k-1}A^\top + Q$.

2. *The optimal value function* $V_k^\dagger$ *satisfies*

$$V_k^\dagger = (x_k - \hat{x}_{k|k})^\top P_{k|k}^{-1}(x_k - \hat{x}_{k|k}) + \Upsilon_k, \tag{25}$$

with

$$V_0^\dagger = (x_0 - \hat{x}_{0|0})^\top P_{0|0}^{-1}(x_0 - \hat{x}_{0|0}) + \Upsilon_0,$$
$\Upsilon_0 = 0$*; if* $\gamma_k = 1$,
$$\Upsilon_k = \Upsilon_{k-1} + (y_k - C\hat{x}_{k|k-1})^\top(R + CP_{k|k-1}C^\top)^{-1}(y_k - C\hat{x}_{k|k-1});$$
if $\gamma_k = 0$, $\quad \Upsilon_k = \Upsilon_{k-1}$.

Proof. The proof of this result follows from a similar argument as that in Theorem 1. In particular, when $\gamma_k = 0$ and no constraint is active, the counter-part of results in (22) reduces to

$$\begin{aligned} &V_k^\dagger = (x_k - \hat{x}_{k|k})^\top P_{k|k}^{-1}(x_k - \hat{x}_{k|k}) + \Upsilon_k,\\ &\Upsilon_k = \Upsilon_{k-1},\\ &\hat{x}_{k|k} = \hat{x}_{k|k-1},\\ &P_{k|k} = P_{k|k-1}. \end{aligned} \tag{26}$$

The optimizer for this unconstrained case satisfies all constraints in (23). In addition, since $R > 0$, by Schur complement, we have $R_k^* > 0$ in (22), which further implies $\Upsilon_k \geq \Upsilon_{k-1}$ and thus the solution to the constrained case leads to a cost larger or equal than the solution to the unconstrained case. Therefore, when $\gamma_k = 0$, the optimization problem in (23) is solved by (26).

Finally, the unbiasedness of the optimal prediction and estimation follows directly from their structure and the fact that $\hat{x}_{0|0} = \mathbf{E}(\mathrm{x}_0)$, which completes the proof. □

The above result indicates that when the exact value of the measurement is unavailable and the information of the set-valued measurement $\hat{y}_t \in [\underline{y}_t, \bar{y}_t]$ is exploited instead, the one-step optimal state prediction also serves the optimal estimation in the sense of one-step maximum likelihood. Note that this does not hold in general for the event-based ML estimation problem in (5). Since the conditional distribution $f_{\mathrm{x}_{1:k}|\mathrm{y}_{1:k}}(x_{1:k}|y_{1:k})$ is no longer Gaussian due to the additional information of the set-valued measurement, the ML estimate does not necessarily coincide with the MMSE estimate for event-based estimation, which is different from the case of periodic state estimation of linear Gaussian systems.

Remark 4. In Wu et al. (2013), when $\gamma_k = 1$, $P_{k|k}$ has the same update equation, but when $\gamma_k = 0$, $P_{k|k}$ evolves rather differently. The resultant estimate here has a much simpler form, which does not require to solve the integrations in Wu et al. (2013). Notice that for original ML estimation problems in (5), $P_{k|k}$ is not the estimation error covariance matrix for the estimate $\hat{x}_{k|k}$, but rather a time-varying parameter that helps to generate the ML estimate subject to the event-triggering rule. As a result, the obtained update equations have essentially different meanings compared with those in Sinopoli et al. (2004).

4.2. Discussions on the communication rate

We now briefly elaborate on the average communication rate. Viewed from the process side, the resultant state estimator behaves exactly like the standard Kalman filter with intermittent observations (Mo & Sinopoli, 2012; Sinopoli et al., 2004): when $\gamma_k = 1$, the optimal estimator considers both time and measurement updates of the Kalman filter; when $\gamma_k = 0$, the optimal estimator only performs the time update. Therefore on the process side the resultant prediction error $\hat{e}_{k|k-1} := \mathrm{x}_k - \hat{x}_{k|k-1}$ is zero-mean Gaussian with covariance $P_{k|k-1}$.[3] Denote $\mathrm{z}_k := \mathrm{y}_k - C\hat{x}_{k|k-1}$. Since $\mathrm{y}_k - C\hat{x}_{k|k-1} = C\hat{e}_{k|k-1} + \mathrm{v}_k$, we have $\mathbf{E}(\mathrm{z}_k) = 0$ and $\mathbf{E}(\mathrm{z}_k\mathrm{z}_k^\top) := \Phi_k = CP_{k|k-1}C^\top + R$. Define $\Omega := \{z \in \mathbb{R}^m | \, \|z\|_\infty \leq \delta\}$, and we have

$$\mathbf{E}(\gamma_k) = 1 - \int_\Omega f_{\mathrm{z}_k}(z)\mathrm{d}z, \tag{27}$$

where $f_{\mathrm{z}_k}(z) = (2\pi)^{-m/2}(\det \Phi_k)^{-1/2}\exp(-\frac{1}{2}z^\top\Phi_k^{-1}z)$. Although the analytical calculation of the integration in (27) is not possible, we show that lower and upper bounds for $\mathbf{E}(\gamma_k)$ can be developed.

To do this, consider the calculation of $\int_{\Omega_0} f_{\mathrm{z}_k}(z)\mathrm{d}z$, where $\Omega_0 := \{z | \, z^\top\Phi_k^{-1}z \leq r^2\}$. Define $\Omega_0^\perp := \{z | \, z^\top\Phi_k^{-1}z > r^2\}$. Since $\Omega_0 \cup \Omega_0^\perp = \mathbb{R}^m$, $\int_{\Omega_0} f_{\mathrm{z}_k}(z)\mathrm{d}z = 1 - \int_{\Omega_0^\perp} f_{\mathrm{z}_k}(z)\mathrm{d}z$. For the integration over $\Omega_0^\perp$, we have the following result.

Lemma 5. $\int_{\Omega_0^\perp} f_{\mathrm{z}_k}(z)\mathrm{d}z = \Gamma(m/2, r^2/2)/\Gamma(m/2)$.

Proof. See Appendix A. □

From this result, the upper and lower bounds of $\mathbf{E}(\gamma_k)$ can be evaluated by considering inner and outer ellipsoidal approximations Ω_0 of Ω, the main effort of which lies in finding appropriate values for r.

In addition, due to the structure of the optimal estimate, the estimator does not need to send the optimal prediction to the remote scheduler when no event occurs, since in this case the same prediction can be generated by the scheduler based on the previous prediction (which is also the optimal estimation) according to $\hat{x}_{t+1|t} = A\hat{x}_{t|t-1} = A\hat{x}_{t|t}$ with a little additional computation cost. In this way, the communication cost is further reduced by the proposed state estimation method.

5. Application to sensorless event-based estimation of a DC motor system

In this section, we illustrate the proposed results with a sensorless remote estimation problem involving a DC motor. The mechanical and electrical dynamics of the DC motor system are given by (Franklin, Powell, & Emami-Naeini, 2006)

$$J_m\frac{\mathrm{d}^2\theta_m}{\mathrm{d}t^2} + b\frac{\mathrm{d}\theta_m}{\mathrm{d}t} + T_L = K_t i_a,$$
$$L_a\frac{\mathrm{d}i_a}{\mathrm{d}t} + R_a i_a = v_a - K_e\frac{\mathrm{d}\theta_m}{\mathrm{d}t},$$

where J_m is the rotor inertia, θ_m is the shaft rotational position, T_L is the load torque, b is the viscous friction coefficient, K_t is the torque constant, K_e is the electric constant, L_a is the armature inductance, R_a is the armature resistance, and v_a is the DC voltage input. The motor parameters are summarized in Table 1, which are obtained based on experimental measurements of a 500 W permanent magnet DC motor with rated speed, current and voltage equal to 314.16 Rad/s, 3.5 A and 180 V, respectively (Chevrel & Siala, 1997).

The objective here is to estimate the shaft rotational position θ_m, shaft rotational speed $\dot{\theta}_m$ and armature current i_a with a current sensor (e.g., a Hall effect sensor). This is called the sensorless control/estimation technique[4] in the industrial electronics community (Holtz, 2002, 2005; Su & McKeever, 2004). The estimation

[3] Notice that according to the standard Kalman filter theory (Anderson & Moore, 1979), this covariance is unconditional.

[4] Here 'sensorless' means the elimination of the speed sensor.

Table 1
Motor parameters.

Parameter	Value	Unit
L_a	20.25	H
R_a	16.4	Ω
K_e	0.0233	V/(Rad/s)
K_t	0.0183	N m/A
J_m	9	g cm^2
b	0.0064	N m/(Rad/s)

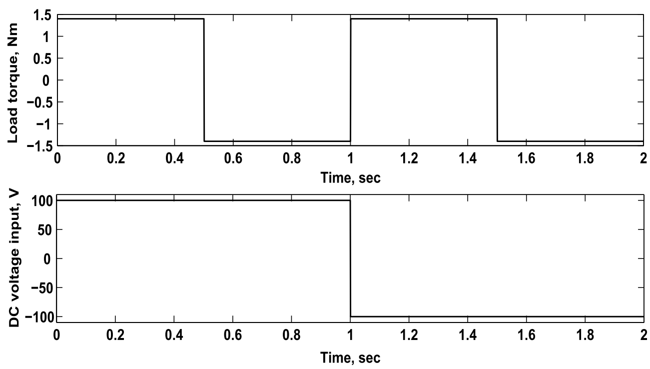

Fig. 2. Plot of the input signals.

is performed by a remote estimator collecting the measurement information through a battery-powered wireless channel. In this work, we consider the load type to be piecewise constant, which can be provided by a synchronous machine. Since both the load torque and DC voltage are only subject to step changes, it is reasonable to assume that these signals are known/generated by the remote estimator.

To implement the estimator, a state-space model is first derived. Since the direct consideration of shaft rotational position will introduce an undetectable mode (in fact the corresponding eigenvalue equals 1) to the system, we choose the state vector as $x := [\dot{\theta}_m \; i_a]^\top$, the input vector as $u := [T_L \; v_a]^\top$, and the measurement output as $y := i_a$, which lead to the state-space model:

$$\dot{x}(t) = \begin{bmatrix} -\dfrac{b}{J_m} & \dfrac{K_t}{J_m} \\ -\dfrac{K_e}{L_a} & -\dfrac{R_a}{L_a} \end{bmatrix} x(t) + \begin{bmatrix} -\dfrac{1}{J_m} & 0 \\ 0 & \dfrac{1}{L_a} \end{bmatrix} u(t), \tag{28}$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t). \tag{29}$$

Notice that based on the estimation of the rotational speed, the shaft rotational position can be estimated as well. With these parameter settings, a discrete-time model is obtained with sampling time chosen as $T_s = 0.001$ s:

$$\mathrm{x}_{k+1} = \begin{bmatrix} 0.9951 & 0.2289 \\ -0.0177 & 0.8672 \end{bmatrix} \mathrm{x}_k + \begin{bmatrix} -0.4158 & 0.0038 \\ 0.0038 & 0.0301 \end{bmatrix} u_k + \mathrm{w}_k, \tag{30}$$

$$\mathrm{y}_k = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathrm{x}_k + \mathrm{v}_k, \tag{31}$$

where w_k and v_k are further introduced to model the noisy operating environment. Specifically, $\mathrm{w}_k := [\mathrm{w}_k^1 \; \mathrm{w}_k^2]^\top$ with w_k^1 characterizing the mechanical noise that couples into the speed-loop and w_k^2 modeling the electrical noise that couples into the voltage input, and v_k models the measurement noise. The covariance matrices of w_k and v_k are assumed to be $Q = \begin{bmatrix} 0.2013 & 0.0430 \\ 0.0430 & 0.0363 \end{bmatrix}$ and $R = 0.03$, respectively. Since both inputs are known to the remote estimator, the proposed results can be applied by only modifying

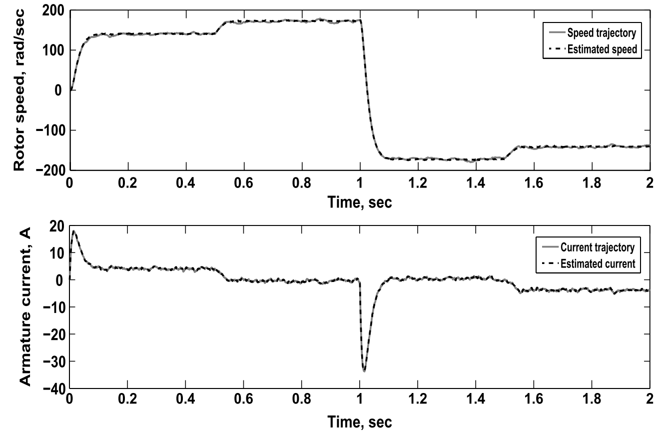

Fig. 3. Performance of the state estimation strategy.

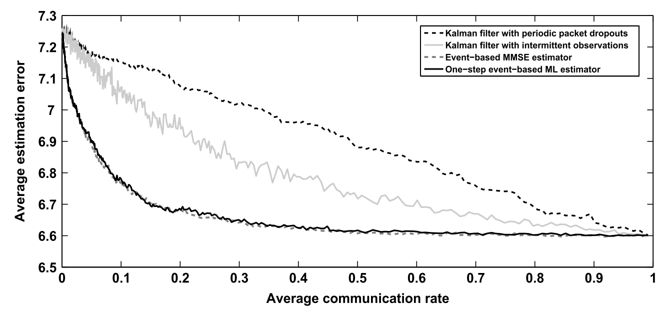

Fig. 4. Tradeoff between the estimation performance and the communication rate.

the prediction as $\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_k$, B being the discretized input matrix.

First, the event-triggering level is set to $\delta = 0.4$ A. The input signals utilized are plotted in Fig. 2. The proposed event-based sensor data scheduling and state estimation strategy are applied and the estimation performance is shown in Fig. 3.

Second, by varying the event-triggering threshold δ, the relationship between estimation performance and average communication rate is further analyzed and compared with the other three methods, namely, the Kalman filter with periodic packet dropouts[5], Kalman filter with intermittent observations (Sinopoli et al., 2004), and the event-based MMSE estimator (Wu et al., 2013). The results are shown in Fig. 4, where the average communication rate is defined by

$$\tilde{\gamma} := \frac{1}{N}\sum_{k=1}^{N} \gamma_k, \tag{32}$$

N being the simulation horizon, and the average estimation error is defined by

$$\epsilon := \frac{1}{N}\sum_{k=1}^{N} \|x_k - \hat{x}_{k|k}\|^2. \tag{33}$$

It is shown that at the same average communication rate, the performance of the Kalman filter with intermittent observations is better than that with periodic packet dropouts, and the performance of the one-step event-based ML estimator is very close to that of the event-based MMSE estimator, at a much decreased

5 To implement this filter, we assume that the first L measurement are lost in each period T; when no measurement is available, only prediction is performed.

communication burden (due to the fact that no feedback communication is required for predicted state when $\gamma_k = 0$ and no normalizing matrix needs to be transmitted to the scheduler at all time instants). In this sense, the ML estimator has greater applicability in this wireless communication scenario with satisfactory estimation performance and potentially prolonged battery life.

6. Conclusion

In this work, an event-based state estimation problem is studied in the framework of ML estimation. We show that the optimal estimator is parameterized by a time-varying Riccati equation associated with an exponential computational complexity. A one-step event-based estimation problem with reduced computation burden is also studied and a recursive solution similar to the Kalman filter with intermittent observations is obtained. Discussions on the communication rate are also presented for this problem. An alternative approach to reduce the computational burden of the general event-based ML estimation problems is to consider the formulation of receding horizon estimation (Alessandri, Baglietto, & Battistelli, 2003; Goodwin et al., 2005; Muske, Rawlings, & Lee, 1993; Rao, Rawlings, & Lee, 2001), which points out the topic for future research. Furthermore, if data is dropped with probability p, then the estimator can infer with probability $1-p$ that y_k is within a threshold and with probability p that y_k is sent but lost, i.e., with probability p that y_k is out of the two bounds, which is also some information. Our framework should be rich enough to deal with this situation.

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Appendix. Proof of Lemma 5

From the definition of $\Omega_0^{\perp}$, we have

$$\begin{aligned}
&\int_{\Omega_0^{\perp}} f_{z_k}(z)\mathrm{d}z \\
&= \int_{z^\top \Phi_k^{-1} z > r^2} (2\pi)^{-\frac{m}{2}} (\det \Phi_k)^{-\frac{1}{2}} \exp\left(-\frac{1}{2} z^\top \Phi_k^{-1} z\right) \mathrm{d}z \\
&= (2\pi)^{-\frac{m}{2}} \int_{p^\top p > r^2} \exp\left(-\frac{1}{2} p^\top p\right) \mathrm{d}p \\
&= (2\pi)^{-\frac{m}{2}} \frac{2\pi^{m/2}}{\Gamma(m/2)} \int_r^\infty v^{m-1} \exp(-v^2/2)\mathrm{d}v \\
&= \frac{1}{\Gamma(m/2)} \int_{r^2/2}^\infty t^{m/2-1} \exp(-t/2)\mathrm{d}t \\
&= \frac{\Gamma(m/2, r^2/2)}{\Gamma(m/2)},
\end{aligned}$$

where the second equality is obtained by using $p = \Phi_k^{-1/2} z$ and $\mathrm{d}p = (\det \Phi_k)^{-1/2}\mathrm{d}z$, the third equality is obtained by converting the Cartesian coordinates $p = [p_1, p_2, \ldots, p_m]^\top$ to polar coordinates $[v, \theta_1, \theta_2, \ldots, \theta_{m-1}]^\top$ and $\mathrm{d}p = v^{m-1} \sin^{m-2}\theta_1 \sin^{m-3}\theta_2 \cdots \sin\theta_{m-2}\mathrm{d}v\mathrm{d}\theta_1\mathrm{d}\theta_2 \cdots \mathrm{d}\theta_{m-1}$, the fourth one is obtained by using $t = p^2/2$ and the surface area formula for an $(m-1)$-dimensional unit sphere $S_{m-1} = 2\pi^{m/2}/\Gamma(m/2)$, where $\Gamma(m/2) := \int_0^\infty t^{m/2-1} \exp(-t)\mathrm{d}t$, and the fifth one follows from the definition of the incomplete Gamma function $\Gamma(a, b) := \int_b^\infty t^{a-1}\exp(-t)\mathrm{d}t$.

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