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Optimal two-sensor scheduling under duty cycle constraint[☆]

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ABSTRACT

We consider periodic sensor scheduling in this paper. A system is observed by two sensors. The two sensors communicate their data with a remote state estimator via a bandwidth-limited network which allows only one sensor to send its data at each time. We derive the optimal duty cycle pair and a corresponding sensor data schedule to minimize the trace of the average estimation error covariance. Simulations are provided to demonstrate the results.

1. Introduction

Wireless sensor networks (WSNs) have found a wealth of applications and have attracted attention from both industrial and academic communities [1]. In real applications, network bandwidth and sensor energy are often limited, which makes it difficult to collect data at every time step. Thus, one should properly choose a subset of sensors to use at each time step, which gives rise to the sensor scheduling problem.

Generally speaking, sensor scheduling is often non-convex and has integer constraints and thus is a difficult problem. Many strategies have been proposed recently. A single-step sensor scheduling problem was studied in [2] using the convex relaxation method. The corresponding multiple-step correlated problem was solved in [3] using reweighed L_1 approximation to relax the non-convex problem to a convex one. The stochastic sensor selection problem for a network with the star topology was considered in [4]. The work [5] introduced a time-varying opportunistic protocol to maximize the sensor network lifetime which depends on the channel-state information and the residual energy information. Similar problem has been considered in [6], where the sensor scheduling problem was formulated as a stochastic shortest path Markov decision process. More related works can be obtained from the references therein.

This work is mostly based on [7] where the authors considered sensor scheduling under the constraints of limited sensor energy and network bandwidth. Under some mild assumptions, they presented an optimal periodic sensor schedule which minimizes the average estimation error. There is great practical and theoretical utility in considering a periodic scheme. The theoretical utility rests in the fact that it explicitly reveals the optimal scheduling scheme. The practical utility is that it allows an efficient implementation. Though simple and easy to implement, their result requires that the sum of the upper bounds of the sensor duty cycles equals 100% exactly, which is quite restrictive. Bound of a duty cycle can be considered as the energy constraint for each sensor or can reflect the cost associated with each use of the sensor. For example, consider estimating the state of a process (e.g., temperature, humidity, etc.) using two sensors. The first one is an expensive device and it can only be used for no more than 30% of the whole time horizon. The second one is cheap and it can be used whenever needed. Therefore the upper bounds of the two sensors add up to 130%. Since different duty cycle pairs may lead to different optimal estimation performances, it is then important to seek the optimal duty cycle pair.

In this paper we extend the periodic scheme in [7] to the case when the sum of the upper bounds of the sensor duty cycles is greater than one. The main contributions of our work consist of the following.

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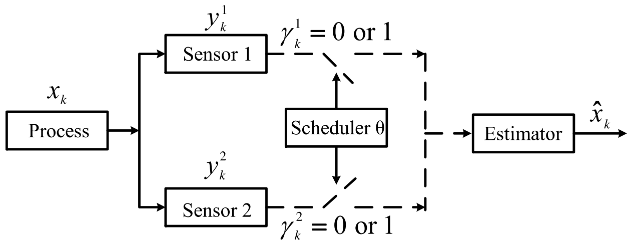

Fig. 1. System block diagram.

(1) We derive the optimal sensor duty cycle pair for a general higher-order system, which enables us to use the scheme proposed in [7] to construct the optimal periodic sensor schedule.
(2) We establish the relationship between the scheduling rule and the system parameters, which is novel and not shown in [7] as their optimal schedule only depends on the upper bounds of the sensor duty cycle.

The remainder of the paper is organized as follows. In Section 2, the problem is formulated. Section 3 presents some preliminaries. Section 4 introduces the main result. Section 5 provides two illustrative examples. Some concluding remarks are drawn in Section 6.

Notations. $\mathbb{E}$ is the mathematical expectation. $\mathbb{Z}^+$ is the set of positive integers. $\mathbb{S}_+^n$ is the set of positive semi-definite matrices with dimension n by n. Let $X, Y \in \mathbb{S}_+^n$, we say $X \leq Y$ if $Y - X \geq 0$. $\mathrm{Tr}(X)$ is the trace of X. Surely $Y - X \geq 0$ implies $\mathrm{Tr}(Y - X) \geq 0$. $(S_1)^p(S_2)^q$ means S_1 is scheduled p times followed by S_2 that is scheduled q times. For functions $f, g : \mathbb{S}_+^n \to \mathbb{S}_+^n$ and $\forall t \in \mathbb{Z}$, define $f \circ g(X) \triangleq f(g(X))$, $g^{[0]}(X) \triangleq X$ and $g^{[t]}(X) \triangleq \underbrace{g \circ \cdots \circ g}_{t}(X)$.

2. Problem setup

2.1. System models

Consider the following system (Fig. 1):

$$
\begin{aligned}
x_{k+1} &= Ax_k + w_k, \\
y_k^1 &= C_1 x_k + v_k^1, \\
y_k^2 &= C_2 x_k + v_k^2,
\end{aligned} \tag{1}
$$

where $x_k \in \mathbb{R}^n$ is the state vector, noise $\omega_k \in \mathbb{R}^n$, $v_k^i \in \mathbb{R}^{m_i}$, $(i = 1, 2)$ and initial state x_0 are zero-mean, Gaussian vectors with covariances $Q \geq 0$, $R_i > 0$ and $\Pi_0 > 0$, respectively. We assume x_0, $\{\omega_k\}$, $\{v_k^i\}$ are uncorrelated, the pair $(A, \sqrt{Q})$ is stabilizable and $(A, [C_1; C_2])$ is detectable.

Following [7], denote $y_{k_1:k_2}^i \triangleq \{y_{k_1}^i, \ldots, y_{k_2}^i\}$ as the measurements collected by sensor i (abbreviated as S_i) from time k_1 to k_2. Suppose at time k, S_i is scheduled to communicate with the remote estimator and it sends $z_k^i(D) \triangleq y_{k-D+1:k}^i$, where D is a given number (to be specified later). In other words, S_i sends the most recent D measurements in a single packet to the remote estimator.[1] Based on the received measurements, the estimator calculates the minimum mean-squared error estimate $\hat{x}_k(D)$ and the corresponding error covariance $P_k(D)$ as follows:

$$
\begin{aligned}
\hat{x}_k(D) &\triangleq \mathbb{E}[x_k | \text{all measurements received up to } k], \\
P_k(D) &\triangleq \mathbb{E}[(x_k - \hat{x}_k(D))(x_k - \hat{x}_k(D))'].
\end{aligned}
$$

At each time step, due to the limited network bandwidth, at most one sensor is able to communicate with the remote estimator. Let $\gamma_k^i = 1$ indicate that S_i is used at time k and $\gamma_k^i = 0$ otherwise.

2.2. Problem of interest

As illustrated in the introduction, in this paper we consider a periodic schedule $\theta \in \Theta$ with period $N(\theta)$. Define the duty cycle of each sensor under θ as

$$
J_i(\theta) \triangleq \frac{1}{N(\theta)} \sum_{k=1}^{N(\theta)} \gamma_k^i(\theta), \quad (i = 1, 2). \tag{2}
$$

We assume each sensor is scheduled at least once in each period; thus $J_i(\theta) > 0$, $i = 1, 2$. The trace of the average estimation error covariance at the estimator side is defined as

$$
P_{ave}(D, \theta) \triangleq \mathrm{Tr}\left(\lim_{T \to \infty} \frac{1}{T} \sum_{k=1}^{T} P_k(D, \theta) \right). \tag{3}
$$

In this paper, we are interested in the following problem in the case of $\Psi_1 + \Psi_2 \geq 1$.

Problem 2.1.

$$
\begin{aligned}
&\min_{D, \theta \in \Theta} P_{ave}(D, \theta) \\
&\text{s.t.} \quad \gamma_k^1(\theta) + \gamma_k^2(\theta) \leq 1, \\
&J_1(\theta) \leq \Psi_1, \\
&J_2(\theta) \leq \Psi_2,
\end{aligned} \tag{4}
$$

where Ψ_i is the upper bound of the duty cycle of S_i, which can be imposed by, for example, the energy constraint or the operation cost for S_i. We assume Ψ_i is a rational number in this paper for simplicity.[2] The problem when $\Psi_1 + \Psi_2 = 1$ and D is sufficiently large has been fully solved in [7]. In this paper we consider a more general case when $\Psi_1 + \Psi_2 \geq 1$. The case $\Psi_1 + \Psi_2 < 1$ is also interesting, but is much more challenging to solve and will be pursued in our future work.

Intuitively, there are numerous combinations of (J_1, J_2) that meet the constraint. The task is to find the optimal pair $(J_1^\star, J_2^\star)$ for minimizing $P_{ave}(D, \theta)$. It will be shown in Section 4 that the optimal duty cycle pair and hence the optimal scheduling rule depend on the system parameters. This relationship is not shown in [7] for the case of $\Psi_1 + \Psi_2 = 1$.

Before proceeding, we introduce the recursive estimation algorithm, based on which is our periodic scheduling scheme.

3. Preliminaries

Calculating $\hat{x}_k(D, \theta)$ and $P_k(D, \theta)$ is standard using the Kalman filter and can be found in [7]. In the sequel, we will focus on $P_k(D, \theta)$ and the average cost $P_{ave}(D, \theta)$. Let

$$
\begin{aligned}
h(X) &\triangleq AXA' + Q, \\
\tilde{g}_i(X) &\triangleq X - XC_i'(C_i X C_i' + R_i)^{-1} C_i X, \quad (i = 1, 2), \\
\tilde{g}(X) &\triangleq X - XC'(CXC' + R)^{-1} CX,
\end{aligned}
$$

where $C = [C_1; C_2]$ and $R = \mathrm{diag}(R_1, R_2)$. Denote

$$
g_1 \triangleq \tilde{g}_1 \circ h, \qquad g_2 \triangleq \tilde{g}_2 \circ h, \qquad g \triangleq \tilde{g} \circ h.
$$

Then $h(X)$ corresponds to the time update of estimation error covariance and $g_i(X)$ stands for the measurement update when the data of S_i are used. Similarly, $g(X)$ is adopted when the information of both sensors is available.

[1] In the WSNs used nowadays, sensors are equipped with some capability of memory. Specifically, as mentioned in [8], the minimum size of an Ethernet packet is 72 bytes, while a typical data point will only consume 2 bytes.

[2] When Ψ_i is irrational, one can easily obtain an approximate solution that is arbitrarily close to the true optimal one by replacing Ψ_i with a rational number that is sufficiently close to Ψ_i.

Since $(A, \sqrt{Q})$ is stabilizable and (A, C) is detectable, the steady-state estimation error covariance when both sensors are used is given by the unique solution of the equation $X = g(X)$, i.e., $\overline{P} = g(\overline{P})$, $(\overline{P} \geq 0)$ [9]. For any $0 \leq X \leq Y$ and $1 \leq t_1 \leq t_2$, we have the following inequalities whose proofs are direct and thus omitted:

$$h(X) \leq h(Y), \qquad g(X) \leq g_i(X), \quad (i = 1, 2),$$
$$g(X) \leq h(X), \qquad g_i^{[t_1]}(\overline{P}) \leq g_i^{[t_2]}(\overline{P}), \quad (i = 1, 2).$$

As in [7], we begin by considering infinite D and simplify $P_{ave}(\infty, \theta)$ as $P_{ave}(\theta)$. In other words, once S_i is used at time k, $y^i_{1:k}$ will be sent to the estimator.[3]

Recall that at most one sensor is allowed to send its data over the network at every time instant, i.e., either $\gamma_k^1 = 1$ or $\gamma_k^2 = 1$. When $\gamma_k^1 = 1$ or $\gamma_k^2 = 1$, we define τ_k^1 and τ_k^2 as follows:

$$\tau_k^1 \triangleq k - \max\{s : s < k, \gamma_s^2 = 1, \gamma_k^1 = 1\},$$
$$\tau_k^2 \triangleq k - \max\{s : s < k, \gamma_s^1 = 1, \gamma_k^2 = 1\}.$$

Here, τ_k^1 represents the time gap between the time when S_1 is used (which is assumed to be k) and the most recent time when S_2 is used. τ_k^2 is defined similarly. Then, if $\gamma_k^i = 1$, we have

$$P_k = g_i^{[\tau_k^i]}\left(g^{[k-\tau_k^i]}(P_0)\right), \quad (i = 1, 2). \tag{5}$$

As each sensor is used at least once during a period with length $N(\theta)$, one immediately has $\tau_k^i \leq N(\theta)$, $(i = 1, 2)$. Since $P_{k+1} = g(P_k)$ converges to $\overline{P}$ exponentially fast for any $P_0 \geq 0$ and we consider infinite time-horizon, without loss of generality, we have $g^{[k-\tau_k^i]}(P_0) = \overline{P}$, $(i = 1, 2)$. Thus, the error covariance at the estimator side is simply obtained as

$$P_k = \begin{cases} g_1^{[\tau_k^1]}(\overline{P}), & \text{if } \gamma_k^1 = 1, \\ g_2^{[\tau_k^2]}(\overline{P}), & \text{if } \gamma_k^2 = 1, \\ h(P_{k-1}), & \text{otherwise.} \end{cases} \tag{6}$$

Assume in one period N, S_1 is used p times and S_2 is used q times, r times are left for open-loop prediction, i.e., $p + q + r = N$. In the special case $\Psi_1 + \Psi_2 = 1$, one necessary condition for the optimal solution of Problem 2.1 is $r = 0$. Then an optimal periodic schedule proposed in [7] is as follows.

Lemma 3.1 ([7]). *Assume $p \leq \frac{1}{2}N$ and there exists an integer s such that $sp < q \leq (s+1)p$, $(s = 0, 1, \ldots)$, then an optimal periodic schedule $\theta^\star$ (over one period) can be constructed as*

$$\theta^\star : \{S_1(S_2)^{s+1}\}^{q-sp}\,\{S_1(S_2)^s\}^{(s+1)p-q}.$$

When $q = (s+1)p$, $\theta^\star$ reduces to $S_1(S_2)^{s+1}$. Moreover the optimality holds true for a finite D as long as $D \geq s + 2$.

Lemma 3.1 reveals that when $\Psi_1 + \Psi_2 = 1$, the duty cycle of each sensor is required to be fully used and the two sensors are scheduled "as uniformly as possible". The constraint $\Psi_1 + \Psi_2 = 1$, however, is clearly quite restrictive. In the next section, we will consider a much more general case when $\Psi_1 + \Psi_2 > 1$.

4. Optimal periodic sensor schedule

In this section we derive the optimal schedule for $\mathrm{Tr}[g_1(\overline{P})] > \mathrm{Tr}[g_2(\overline{P})]$ and $\mathrm{Tr}[g_1(\overline{P})] = \mathrm{Tr}[g_2(\overline{P})]$, respectively. The case $\mathrm{Tr}[g_1(\overline{P})] < \mathrm{Tr}[g_2(\overline{P})]$ is similar. Consider two classes of periodic schedules $\theta_m^a : S_1(S_2)^m$ and $\theta_n^b : S_2(S_1)^n$ with period $m + 1$ and $n + 1$, respectively. Then it is straightforward to verify that

$$P_{ave}(\theta_m^a) = \frac{1}{m+1}\mathrm{Tr}\left[g_1(\overline{P}) + \sum_{t=1}^{m} g_2^{[t]}(\overline{P})\right], \tag{7}$$

$$P_{ave}(\theta_n^b) = \frac{1}{n+1}\mathrm{Tr}\left[g_2(\overline{P}) + \sum_{t=1}^{n} g_1^{[t]}(\overline{P})\right]. \tag{8}$$

Using the increasing property of $\mathrm{Tr}[g_1^{[t]}(\overline{P})]$ with respect to t, we obtain the following result.

Lemma 4.1. *If $\mathrm{Tr}[g_1(\overline{P})] > \mathrm{Tr}[g_2(\overline{P})]$, then $P_{ave}(\theta_1^a) \leq P_{ave}(\theta_n^b)$, $n = 1, 2, \ldots$, with equality iff $n = 1$.*

This lemma tells us that when S_2 is more accurate than S_1, among all the schedules in the form of $\{\theta_m^a\}$ and $\{\theta_n^b\}$, the optimal one must fall into the class of θ_m^a. Then one question is naturally raised: what is the optimal m?

Let us denote

$$G^a(s) \triangleq s g_2^{[s]}(\overline{P}) - \sum_{t=1}^{s-1} g_2^{[t]}(\overline{P}). \tag{9}$$

The superscript "a" indicates that the expression above applies to the schedule θ_m^a. Then we have $\mathrm{Tr}[G^a(s)] \geq 0$. Moreover,

$$\mathrm{Tr}[G^a(s+1)] - \mathrm{Tr}[G^a(s)] = (s+1)\{\mathrm{Tr}[g_2^{[s+1]}(\overline{P})] - \mathrm{Tr}[g_2^{[s]}(\overline{P})]\} \geq 0,$$

which means $\mathrm{Tr}[G^a(s)]$ is an increasing function with respect to s and $\mathrm{Tr}[G^a(1)] = \mathrm{Tr}[g_2(\overline{P})]$.

Now consider the following two cases.

(1) There exists $m_0 \in \mathbb{Z}^+$ such that

$$\mathrm{Tr}[G^a(m_0)] < \mathrm{Tr}[g_1(\overline{P})] \leq \mathrm{Tr}[G^a(m_0+1)]. \tag{10}$$

(2) There does not exist $m_0 \in \mathbb{Z}^+$ such that (10) holds. In other words,

$$\lim_{s\to\infty} \mathrm{Tr}[G^a(s)] = \mathrm{Tr}[G^a(\infty)] < \mathrm{Tr}[g_1(\overline{P})]. \tag{11}$$

Let us first consider case one and we have the following result.

Theorem 4.2. *Assume there exists $m_0 \in \mathbb{Z}^+$ such that (10) holds. Then among all the schemes in $\{\theta_m^a\}$, the optimal one which minimizes $P_{ave}(\theta)$ is $\theta_{m_0}^a$.*

Proof. For two schemes θ_l^a and θ_{l+1}^a $(\forall l \geq 1)$, direct computation shows that

$$\begin{aligned} P_{ave}(\theta_{l+1}^a) - P_{ave}(\theta_l^a) &= \frac{1}{(l+1)(l+2)}\mathrm{Tr}\left[(l+1)\sum_{t=1}^{l+1} g_2^{[t]}(\overline{P}) \right. \\ &\qquad \left. - (l+2)\sum_{t=1}^{l} g_2^{[t]}(\overline{P}) - g_1(\overline{P})\right] \\ &= \frac{\mathrm{Tr}[G^a(l+1)] - \mathrm{Tr}[g_1(\overline{P})]}{(l+1)(l+2)}. \end{aligned}$$

Using the bounds of $\mathrm{Tr}[g_1(\overline{P})]$ given in Theorem 4.2, together with the increasing property of $\mathrm{Tr}[G^a(s)]$, we can verify that

(1) $P_{ave}(\theta_l^a) < P_{ave}(\theta_{l+1}^a)$, $(\forall l = m_0 + 1, m_0 + 2, \ldots)$,
(2) $P_{ave}(\theta_l^a) > P_{ave}(\theta_{l+1}^a)$, $(\forall l = m_0 - 1, m_0 - 2, \ldots)$,
(3) $P_{ave}(\theta_{m_0}^a) \leq P_{ave}(\theta_{m_0+1}^a)$,

with equality iff $\mathrm{Tr}[g_1(\overline{P})] = \mathrm{Tr}[G^a(m_0 + 1)]$. The proof is thus completed. □

[3] As we will see later in Lemma 3.1, finite D is enough which benefits from the periodic scheduling.

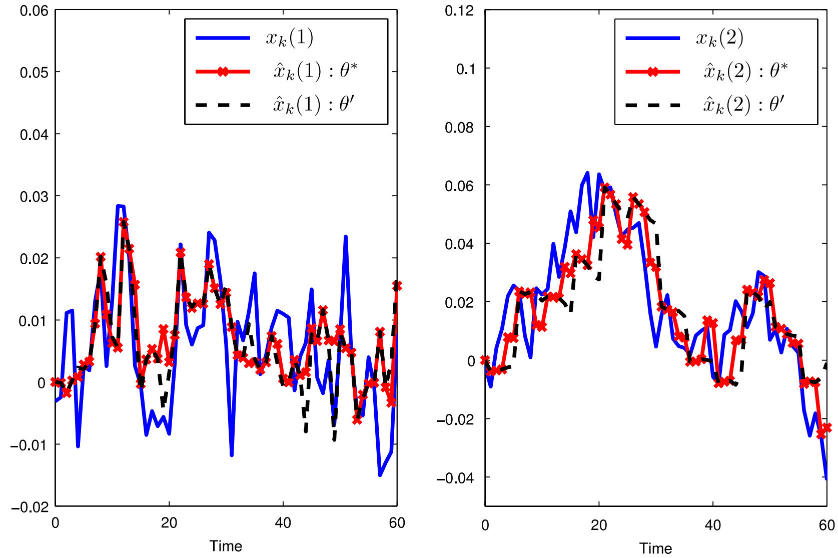

Fig. 2. State and estimate for each component when $(\Psi_1, \Psi_2) = (1, \frac{2}{5})$.

Based upon Theorem 4.2, we have the following main result.

Theorem 4.3. *Assume there exists $m_0 \in \mathbb{Z}^+$ such that (10) holds. Then the optimal duty cycle pair $(J_1^\star, J_2^\star)$ for the two sensors which minimizes $P_{ave}(\theta)$ is given by*

$$(J_1^\star, J_2^\star) = \begin{cases} \left(\dfrac{1}{m_0+1}, \dfrac{m_0}{m_0+1}\right), \\ \quad \text{if } \dfrac{1}{m_0+1} \le \Psi_1, \dfrac{m_0}{m_0+1} \le \Psi_2, \\ (1-\Psi_2, \Psi_2), \quad \text{if } \dfrac{1}{m_0+1} < \Psi_1, \dfrac{m_0}{m_0+1} > \Psi_2, \\ (\Psi_1, 1-\Psi_1), \quad \text{if } \dfrac{1}{m_0+1} > \Psi_1, \dfrac{m_0}{m_0+1} < \Psi_2. \end{cases}$$

Proof. Consider the following three sets of schedules ($\forall x, y = 1, 2, \ldots$):

$$\begin{aligned} &\theta_A : \{S_1(S_2)^{l+1}\}^x \, \{S_1(S_2)^l\}^{y+1}, \\ &\theta_B : \{S_1(S_2)^{l+1}\}^x \, \{S_1(S_2)^l\}^y, \\ &\theta_C : \{S_1(S_2)^{l+1}\}^{x+1} \, \{S_1(S_2)^l\}^y. \end{aligned} \tag{12}$$

From Lemma 3.1, θ_B gives an optimal periodic schedule in which the duty cycle of S_1 is $\frac{x+y}{(l+2)x+(l+1)y}$. Similar assertion holds for θ_A and θ_C for different duty cycles. Moreover, direct computation leads to

$$P_{ave}(\theta_A) - P_{ave}(\theta_B) = \frac{x\{\mathrm{Tr}[g_1(\overline{P})] - \mathrm{Tr}[G^a(l+1)]\}}{[(l+2)x+(l+1)y][(l+2)(x+1)+(l+1)(y+1)]}, \tag{13}$$

$$P_{ave}(\theta_B) - P_{ave}(\theta_C) = \frac{y\{\mathrm{Tr}[g_1(\overline{P})] - \mathrm{Tr}[G^a(l+1)]\}}{[(l+2)x+(l+1)y][(l+2)(x+1)+(l+1)y]}. \tag{14}$$

Following the same line of the proof of Theorem 4.2, we can verify the following:

(1) when $l = m_0+1, m_0+2, \ldots$
$P_{ave}(\theta_l^a) < P_{ave}(\theta_A) < P_{ave}(\theta_B) < P_{ave}(\theta_C) < P_{ave}(\theta_{l+1}^a)$.
(2) when $l = m_0-1, m_0-2, \ldots$
$P_{ave}(\theta_l^a) > P_{ave}(\theta_A) > P_{ave}(\theta_B) > P_{ave}(\theta_C) > P_{ave}(\theta_{l+1}^a)$.
(3) when $l = m_0$
- if $\mathrm{Tr}[g_1(\overline{P})] = \mathrm{Tr}[G^a(m_0+1)]$
$P_{ave}(\theta_{m_0}^a) = P_{ave}(\theta_A) = P_{ave}(\theta_B) = P_{ave}(\theta_C) = P_{ave}(\theta_{m_0+1}^a)$.
- otherwise
$P_{ave}(\theta_{m_0}^a) < P_{ave}(\theta_A) < P_{ave}(\theta_B) < P_{ave}(\theta_C) < P_{ave}(\theta_{m_0+1}^a)$.

For the special case when $y = 0$, θ_B is equivalent to θ_{l+1}^a; when $x = 1$ and $y \to \infty$, θ_B approaches to θ_l^a. In other words, any optimal periodic schedule under which J_1, the duty cycle of S_1 satisfies $\frac{1}{l+2} \le J_1 < \frac{1}{l+1}$, can be represented by one schedule in θ_B (the equality holds when $y = 0$).

Let us first consider $\mathrm{Tr}[g_1(\overline{P})] \ne \mathrm{Tr}[G^a(m_0+1)]$. From Theorem 4.2, among $\{\theta_m^a\}$, the optimal choice is $\theta_{m_0}^a$; thus when $\frac{1}{m_0+1} \le \Psi_1$, $\frac{m_0}{m_0+1} \le \Psi_2$, the result is clearly seen. When $\frac{1}{m_0+1} > \Psi_1$, for any optimal periodic schedule θ, due to the monotonicity of $P_{ave}(\theta)$ with respect to the ratio of S_1, the optimal duty cycles which minimize $P_{ave}(\theta)$ are given by $J_1^\star = \Psi_1$ and $J_2^\star = 1 - \Psi_1$.

In the special case when $\mathrm{Tr}[g_1(\overline{P})] = \mathrm{Tr}[G^a(m_0+1)]$ and $\Psi_1 \in [\frac{1}{m_0+2}, \frac{1}{m_0+1}]$, the costs of any optimal periodic schedule in this case are equal; thus without loss of generality, we choose $J_1^\star = \Psi_1$. □

Now let us consider case two, i.e., when (11) holds. In this case, for $\theta_A, \theta_B, \theta_C$ given by (12), we always have ($\forall x, y = 1, 2 \ldots$):

$$P_{ave}(\theta_l^a) > P_{ave}(\theta_A) > P_{ave}(\theta_B) > P_{ave}(\theta_C) > P_{ave}(\theta_{l+1}^a).$$

Hence we have the following result.

Proposition 4.4. *Assume (11) holds. Then the optimal duty cycle pair $(J_1^\star, J_2^\star)$ for the two sensors which minimizes $P_{ave}(\theta)$ is given by*

$$(J_1^\star, J_2^\star) = (1-\Psi_2, \Psi_2).$$

In the particular case when $\mathrm{Tr}[g_1(\overline{P})] = \mathrm{Tr}[g_2(\overline{P})]$, the optimal pair $(J_1^\star, J_2^\star)$ for the two sensors which minimizes $P_{ave}(\theta)$ is the same as those in Theorem 4.3 with $m_0 = 1$.

Once we have obtained the optimal duty cycle pair $(J_1^\star, J_2^\star)$ for the two sensors, we can use Lemma 3.1 to construct the optimal periodic schedule. For example, consider Theorem 4.3. In the first case $(J_1^\star, J_2^\star) = (\frac{1}{m_0+1}, \frac{m_0}{m_0+1})$, one can simply choose $p = 1, q = m_0$, $r = 0$, which implies $q = m_0 p$ and the length of the period $N = m_0 + 1$. Then from Lemma 3.1, an optimal periodic schedule (over one period) can be constructed as $\theta^\star : S_1(S_2)^{m_0}$. In the second case $(J_1^\star, J_2^\star) = (1-\Psi_2, \Psi_2)$. Write the rational number Ψ_2 as $\Psi_2 = \frac{m}{n}$, $(m, n \in \mathbb{Z}^+)$. Then by letting $N = n, q = m$ and $p = n - m$, an optimal periodic schedule can be obtained from Lemma 3.1. Similar procedure applies to the third case of Theorem 4.3 as well as Proposition 4.4.

Our work shows that when $\Psi_1 + \Psi_2 > 1$, the optimal scheduling rule relies on system dynamics. While [7] showed that, when

$\Psi_1 + \Psi_2 = 1$, the optimal schedule only relies on Ψ_1. Furthermore, Lemma 3.1 ensures the optimality of the periodic schedule for $\Psi_1 + \Psi_2 > 1$ as long as $D \geq s + 2$.

5. Examples

Consider system (1) with $A = [0.55\ 0.1;\ 0\ 0.95]$, $C_1 = [1\ 0]$, $C_2 = [0\ 1]$, $Q = \mathrm{diag}(1, 1)$, $R_1 = 1.7$, $R_2 = 1$. It means that S_2 is more accurate than S_1 and hence $\mathrm{Tr}[g_1(\overline{P})] > \mathrm{Tr}[g_2(\overline{P})]$. The steady-state error covariance $\overline{P} = [0.7106\ 0.0150;\ 0.0150\ 0.6073]$ and $m_0 = 2$ from Theorem 4.2, i.e., $(\frac{1}{m_0+1}, \frac{m_0}{m_0+1}) = (\frac{1}{3}, \frac{2}{3})$.

Consider the second case of Theorem 4.3, for example, the bounds of duty cycles $(\Psi_1, \Psi_2) = (1, \frac{2}{5})$. The optimal duty cycle pair $(J_1^\star, J_2^\star) = (\frac{3}{5}, \frac{2}{5})$ using Theorem 4.3 and corresponding optimal periodic schedule $\theta^\star$ is given by $S_2S_1S_1S_2S_1$ using Lemma 3.1. Consider another duty cycle pair $(J_1', J_2') = (\frac{4}{5}, \frac{1}{5})$. Using Lemma 3.1 again, we have $\theta' : S_2S_1S_1S_1S_1$. The state x_k and its estimate $\hat{x}_k$ for each component under the two schemes are shown in Fig. 2. Note that θ' performs better than $\theta^\star$ in terms of the first component of the state, i.e., $x_k(1)$. The reason is that S_1 just accounts for 60% in $\theta^\star$, while 80% in θ'. The advantage of $\theta^\star$ is shown in the second component of the state, i.e., $x_k(2)$. Moreover, $\mathrm{Tr}[P_k(\theta^\star)]$ versus $\mathrm{Tr}[P_k(\theta')]$ is given in Fig. 3, which shows that the former performs better than the latter.

To verify the first case of Theorem 4.3, let us take $(\Psi_1, \Psi_2) = (1, \frac{4}{5})$ as an example. Note that $(J_1^\star, J_2^\star) = (\frac{1}{3}, \frac{2}{3})$ using Theorem 4.3 and $\theta^\star$ is given by $S_1S_2S_2$ using Lemma 3.1. Consider another pair $(J_1', J_2') = (\frac{2}{3}, \frac{1}{3})$. Using Lemma 3.1 again, we have $\theta' : S_2S_1S_1$. When choosing $(J_1'', J_2'') = (\frac{1}{5}, \frac{4}{5})$, we have $\theta'' : S_1S_2S_2S_2S_2$. The performance comparison is shown in Fig. 4. Note that from Fig. 4, $P_{ave}(\theta^\star)$ is very close to $P_{ave}(\theta'')$. Using (7) as well as some direct computation, however, one has

$$
\begin{aligned}
&P_{ave}(\theta^\star) - P_{ave}(\theta'') \\
&= \frac{1}{15}\mathrm{Tr}\left\{2g_1(\overline{P}) + 2\sum_{t=1}^{2} g_2^{[t]}(\overline{P}) - 3[g_2^{[3]}(\overline{P}) + g_2^{[4]}(\overline{P})]\right\} \\
&< \frac{1}{15}\mathrm{Tr}\left\{2g_1(\overline{P}) + 2\sum_{t=1}^{2} g_2^{[t]}(\overline{P}) - 6g_2^{[3]}(\overline{P})\right\} \\
&= \frac{2}{15}\mathrm{Tr}[g_1(\overline{P}) - G^a(3)] \leq 0.
\end{aligned}
$$

The last inequality is due to $m_0 = 2$. The illustrative examples thus verify the theory we developed.

6. Conclusion

In this paper, we consider sensor scheduling over a bandwidth-limited network. We explicitly establish the relationship between the optimal duty cycle pair for the two sensors and the system parameters and obtain the optimal periodic sensor schedule. Future work includes extending the results to multi-sensor scenario and relaxing some of the constraints (e.g., sending D measurement data) considered in the paper.

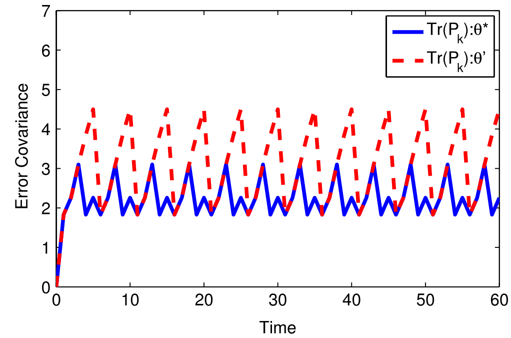

Fig. 3. Trace of error covariance of the two schemes when $(\Psi_1, \Psi_2) = (1, \frac{2}{5})$.

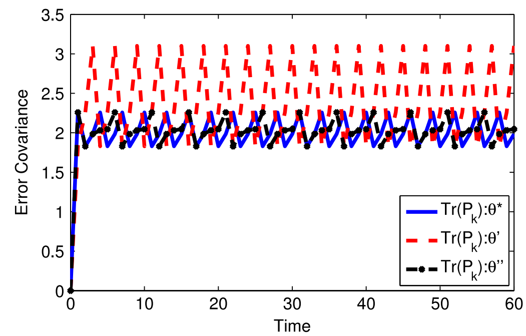

Fig. 4. Trace of error covariance of the three schemes when $(\Psi_1, \Psi_2) = (1, \frac{4}{5})$.

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