Optimal Periodic Transmission Power Schedules for Remote Estimation of ARMA Processes

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***Abstract*—We consider periodic sensor transmission power allocation with an average energy constraint. The sensor sends its Kalman filter-based state estimate to the remote estimator through an unreliable link. Dropout probabilities depend on the power level used. To encompass applications where the estimator needs to attend to multiple tasks, we allow for irregular sampling, following a periodic pattern. Using properties of an underlying Markov chain model, we derive an explicit expression for the estimation error covariance. The results are then used to study optimal sensor power scheduling which minimizes the average error covariance.**

***Index Terms*—Kalman filtering, multi-sampling, networked estimation, power scheduling.**

I. Introduction

Wireless sensor networks (WSNs) have been a hot research area in the last few years. Growing applications are found in various areas, for example, unmanned aerial vehicles, mobile sensor networks, remote surgery, automated highway systems, environment monitoring, smart grid and industrial automation. WSNs provide many advantages when compared to traditional wired sensors, such as low cost, easy installation, self-power and so on [1]. Not surprisingly, both research and applications of WSNs have been increasing tremendously. Despite the benefits the use of wireless sensors offers, there are still many shortcomings when designing networked sensing and control systems. In particular, wireless channels are subject to channel fading and interference, which may lead to data drops and performance degradation [2]. The underpinning theory reveals that there is a tradeoff relationship between transmission bit rate, bandwidth and transmission power [3], [4]. If higher transmission powers are used, then dropouts are less frequent. However, most wireless sensors use on-board batteries, which are difficult to replace. Since in many applications, wireless sensors are usually expected to work for several years without the replacement of batteries, energy conservation is of utmost importance, thus, power control becomes critical [5]–[9].

There are two important aspects in the study of WSNs, namely, the estimation problem and the communication problem. Traditionally, these two components have been studied separately. However, an integrated approach which takes into account communication constraints in the estimation process will, in general, give better performance versus energy use trade-offs. For the case of estimation of autoregressive moving average (ARMA) stochastic processes, those problems have gained much interest and have been studied extensively in recent years [10]–[12]. Quevedo *et al.* [13] studied stochastic stability of centralized Kalman filtering where data transmission is over parallel fading channels using transmission power control. Sinopoli *et al.* [14] investigated distributed control applications within sensor networks. They presented a hybrid model which consists of two components: continuous time-trigger components at the low level and discrete event-triggered components at the high level. Pantazis *et al.* [15] studied various power control schemes using different approaches in WSNs which focus on different performance metrics. Aziz *et al.* [16] investigated the energy efficiency issue and provided a comprehensive study extending the lifetime of battery powered WSNs by topology control. Hohlt *et al.* [17] presented a dynamic distributed time on-demand power-management protocol for data collection in sensor networks by enabling sensors to turn off their communication during idle time slots. More related works in the literature can be found from the references therein.

Among most of literatures studying power control problem, the sensor data are sampled following a regular schedule. However, one specifically designed irregular sampling scheme may yield better system performance or less energy consumption [18]–[20]. Yu *et al.* [19] proposed a sampling scheme for Gaussian hypothesis testing problems based on the class of Ali-Silvey distance measures. Comparisons with other sample schemes illustrate that the proposed sampling design leads a better detection performance. Niu *et al.* [20] studied the temporally staggered sensors in multi-sensor target tracking system. Based on the metric-average error variance, the comparison between temporally staggered sensors and synchronous sensors under different situations are provided to show the improved performance of staggered sensing.

Another motivation for the current work is that most existing literatures of sensor power scheduling considered finite-time horizon. In the case of finite-time horizon, the optimal state estimate and the corresponding error covariance can be computed recursively, which leads to solving the sensor scheduling

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or power control problem straightforwardly. As a result, many existing tools from optimization theory can be used. Compared to the finite-time horizon case, the problem of infinite-time horizon is much more difficult to handle [21]. Mcloughlin *et al.* [22] relaxed the general infinite-time horizon sensor scheduling problem for state estimation in linear time-invariant systems. Under the relaxation, they converted the original problem to a mixed integer quadratic programming problem which can be easily solved.

In the present work, we study infinite-time horizon periodic sensor transmission power allocation for remote state estimation under an average power constraint. The sampling is assumed to be irregular in order to incorporate a larger class of applications. When compared with previous literatures, the contribution of this work lies in integrating various theories such Kalman filtering, communication and Markov chain theories to provide a tractable solution for the challenging problem of infinite-time horizon power control for remote sensing, which are summarized as follows.

1) **Irregular sampling**: We focus on a situation where the sampling pattern is irregular, following a periodic schedule. This model is motivated by WSN architectures where the gateway, in addition to performing state estimation, needs to attend to other tasks. In other cases, though the state estimations are provided at every time step, the remote estimator may only sample and utilize the estimates only in some specific time steps. The model allows us to incorporate a large class of applications.
2) **Infinite-time horizon**: Under the infinite-time horizon, the proposed problem is non-linear and non-convex, which cannot be solved by the classic convex optimization methods. Instead, we propose a novel Markov chain model which can effectively deal with the infinite-time horizon scenario. Through this model, we convey the difficult problem into a relatively simple one which only relies on finding the minimal value of a scalar multi-variable function.
3) **Closed-form solution**: Using the Markov chain model, we show that the estimation error covariance have a stationary distribution which enables us to derive a closed-form solution of the estimation error covariance at each time step. This closed-form solution allows us to readily compare the performance of different power schedules. As mentioned previously, the problem cannot be solved via convex optimization techniques, but the optimal sensor power schedule can be found, via the Markov chain model, through the Lagrange multiplier method or any other numerical methods which can find the minimal value of a scalar multi-variable function.

The remainder of this manuscript is organized as follows. Section II presents the estimation architecture. Section III states the main problem of interest. Section IV develops a suitable model. In Section V, we derive optimal sensor power schedules. Numerical example are included in Section VI. Section VII draws conclusions.

Notation: $\mathbb{Z}$ denotes the set of all integers. $\mathbb{N}$ are the positive integers, $n \in \mathbb{N}$. $\mathbb{R}$ is the set of real numbers. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{S}_+^n$ (and $\mathbb{S}_{++}^n$) is the set of n by n positive semi-definite matrices (and positive definite matrices). When $X \in \mathbb{S}_+^n$ (and $\mathbb{S}_{++}^n$), we write $X \geq 0$ (and $X > 0$). $X \geq Y$ if $X - Y \in \mathbb{S}_+^n$. $\mathrm{Tr}(\cdot)$ is the trace of a matrix. The superscript $'$ stands for transposition. For functions f, f_1, f_2 with appropriate domains, $f_1 \circ f_2(x)$ stands for the function composition $f_1(f_2(x))$, and $f^n(x) \triangleq f(f^{n-1}(x))$ with $f^0(x) \triangleq x$. δ_{ij} is Dirac delta function, i.e., δ_{ij} equals to 1 when $i = j$ and 0 otherwise. The notation $\mathbb{P}[\cdot]$ refers to probability, $\mathbb{E}[\cdot]$ to expectation. For a matrix M, $M[j]$ denotes the j-th column of M while $M(k)$ denotes the k-th row of M.

II. Estimation Setup

We consider the problem of estimating the state of an ARMA process at a remote estimator where the sensor communicates with the estimator over a wireless channel. The process is described in state-space form via:

$$x_{k+1} = Ax_k + w_k, \tag{1}$$
$$y_k = Cx_k + v_k, \tag{2}$$

where $k \in \mathbb{N}$, $x_k \in \mathbb{R}^{n_x}$ is the process state vector at time k, $y_k \in \mathbb{R}^{n_y}$ is the measurement taken by the sensor, $w_k \in \mathbb{R}^{n_x}$ and $v_k \in \mathbb{R}^{n_y}$ are zero-mean i.i.d. Gaussian noises with $\mathbb{E}[w_k w_j'] = \delta_{kj} Q$ $(Q \geq 0)$, $\mathbb{E}[v_k (v_j)'] = \delta_{kj} R$ $(R > 0)$, $\mathbb{E}[w_k (v_j)'] = 0\ \forall j, k \in \mathbb{N}$. The initial state x_0 is a zero-mean Gaussian random vector with covariance $\Pi_0 \geq 0$ and is uncorrelated with w_k and v_k. The pair (A, C) is assumed to be observable and $(A, \sqrt{Q})$ is controllable.

A. Local State Estimate

In a networked estimation scenario, sensors are typically equipped with on-board processors [23] and utilization of their capabilities may improve the system performance significantly. For the situation at hand, at each time k, the sensor first locally runs a Kalman filter to estimate the state x_k based on all the measurements it collects up to time k and then transmit its local estimate to the remote estimator.[1]

In the sequel, we will denote $\hat{x}_k^s$ and P_k^s as the sensor's local state estimate and the corresponding error covariance, thus,

$$\hat{x}_k^s = \mathbb{E}[x_k | y_1, y_2, \ldots, y_k], \tag{3}$$
$$\hat{P}_k^s = \mathbb{E}\left[(x_k - \hat{x}_k^s)(x_k - \hat{x}_k^s)' | y_1, y_2, \ldots, y_k \right]. \tag{4}$$

These can be calculated by standard Kalman filter equations as follows [24]:

$$\hat{x}_{k|k-1}^s = A\hat{x}_{k-1}^s, \tag{5}$$
$$P_{k|k-1}^s = AP_{k-1}^s A' + Q, \tag{6}$$
$$K_k^s = P_{k|k-1}^s C' \left[CP_{k|k-1}^s C' + R \right]^{-1}, \tag{7}$$
$$\hat{x}_k^s = A\hat{x}_{k-1}^s + K_k^s \left(y_k - CA\hat{x}_{k-1}^s \right), \tag{8}$$
$$P_k^s = (I - K_k^s C) P_{k|k-1}^s, \tag{9}$$

where the recursion starts from $\hat{x}_0^s = 0$ and $P_0^s = \Pi_0 \geqslant 0$.

[1]Throughout this work, we assume that the bit-rate is large enough so that quantization effects can be ignored.

For notational ease, we introduce the functions $h, \tilde{g} : \mathbb{S}_+^n \to \mathbb{S}_+^n$ as

$$h(X) \triangleq AXA' + Q, \tag{10}$$
$$\tilde{g}(X) \triangleq X - XC'[CXC' + R]^{-1}CX. \tag{11}$$

It is well known (see, e.g.,[25]) that the estimation error covariance P_k^s in (9) will converge to a steady-state value exponentially fast. Without loss of generality, we assume that the Kalman filter at the sensor side has entered the steady state and simplify our subsequent discussion by setting:

$$P_k^s = \overline{P}, \quad k \geqslant 1, \tag{12}$$

where $\overline{P}$ is the steady-state error covariance, which is the unique positive semi-definite solution of $\tilde{g} \circ h(X) = X$. From [26], $\overline{P}$ has the following property.

Lemma 1: For $0 \leqslant t_1 \leqslant t_2$, the following inequality holds:

$$h^{t_1}(\overline{P}) \leqslant h^{t_2}(\overline{P}). \tag{13}$$

In addition, if $t_1 < t_2$, then

$$\mathrm{Tr}\left(h^{t_1}(\overline{P})\right) < \mathrm{Tr}\left(h^{t_2}(\overline{P})\right). \tag{14}$$

B. Wireless Communication Model

The sensor's local state estimate $\hat{x}_k^s$ is communicated to the remote estimator over an Additive White Gaussian Noise (AWGN) channel using Quadrature Amplitude Modulation (QAM).[2] Specifically, $\hat{x}_k^s$ is quantized into R bits and mapped to one of the 2^R available QAM symbols. The symbol error rate (SER) is given by

$$\mathrm{SER} = Q\left(\sqrt{\frac{2\beta M_k}{N_0 W}}\right), \tag{15}$$

where M_k is the transmit power for the QAM symbol at time k, N_0 is the AWGN noise power spectral density, W is the channel bandwidth, β is a constant which depends on R, and

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\eta^2/2)d\eta \tag{16}$$

is the Q-function. For sufficiently large SNR, we have (cf.,[8]),

$$\mathrm{SER} \approx \exp\left(-\beta \frac{M_k}{N_0 W}\right). \tag{17}$$

Based on the relationship in (15) and (17), it is clear that higher transmission power M_k leads to lower SER.

Throughout this work, we shall assume that the communication channel is time-invariant, i.e., β, N_0, W, are constants. Furthermore, we assume the controller can detect symbol errors.[3] Taking into account of the SER in the transmission of QAM symbols, the equivalent communication channel for $\hat{x}_k^s$ between the sensor and the remote estimator can be characterized by an independent binary random process (Bernoulli process) $\{\gamma_k\}, k \in \mathbb{N}$, where:

$$\gamma_k = \begin{cases} 1, & \text{if } \hat{x}_k^s \text{ arrives error-free at time } k, \\ 0, & \text{otherwise (regarded as dropout).} \end{cases} \tag{18}$$

Let $p_k = \mathbb{P}[\gamma_k = 1]$ as the probability of successful transmission of $\hat{x}_k^s$. From (17), we have

$$1 - p_k = (1 - \lambda)^{M_k}, \tag{19}$$

where λ is given by:

$$\lambda \triangleq 1 - \exp\left(-\beta/(N_0 W)\right) \in (0, 1). \tag{20}$$

To send the local state estimates to the remote estimator, the sensor node chooses from a continuum of available power levels $M_k \geqslant 0$. As noted above, different power levels lead to different dropout rates, and thereby affect estimation performance. From (19), the probability of successful transmission p_k will be higher as M_k increases. In the present work, we take into account energy constraints, and characterize optimal sensor power schedules, which minimize the error covariance at the remote estimator.

C. Remote State Estimation

We define I_k as the information available to the processor of the remote estimator from time 1 to time k, thus,

$$I_k = \{\gamma_1 \hat{x}_1^s, \gamma_2 \hat{x}_2^s, \ldots, \gamma_k \hat{x}_k^s\} \cup \{\gamma_1, \gamma_2, \ldots, \gamma_k\}, \tag{21}$$

cf.,[12]. Denote $\hat{x}_k$ and P_k as the remote estimator's state estimate and the corresponding error covariance based on I_k, i.e.,

$$\hat{x}_k = \mathbb{E}[x_k | I_k], \tag{22}$$
$$P_k = \mathbb{E}\left[(x_k - \hat{x}_k)(x_k - \hat{x}_k)' | I_k\right]. \tag{23}$$

At the estimator's side, similar to [23], [27], at instances where the sensor's local estimate arrives, the estimator synchronizes its own estimate with that of the sensor. Otherwise, the estimator just predicts x_k based on its previous estimate using the system model (1). In view of (18), the state estimate $\hat{x}_k$ thus obeys the recursion

$$\hat{x}_k = \begin{cases} \hat{x}_k^s, & \text{if } \gamma_k = 1, \\ A\hat{x}_{k-1}, & \text{if } \gamma_k = 0. \end{cases} \tag{24}$$

As a result, the state estimation error covariance P_k satisfies

$$P_k = \begin{cases} \overline{P}, & \text{if } \gamma_k = 1, \\ h(P_{k-1}), & \text{if} \gamma_k = 0. \end{cases} \tag{25}$$

In the sequel, we denote $d_k \in \{0, 1\}$ as the sample decision of the remote estimator at the k-th time. The remote estimator is only interested in the estimation performance at the sampling instances, i.e., it samples and utilizes the estimate provided by the processor at the k-th time when $d_k = 1$ and skips sampling when $d_k = 0$, see Fig. 1. We consider a pre-designed and given periodic sampling pattern with period $N \in \mathbb{N}$, i.e.,

$$d_{k+N} = d_k, \quad \forall k \in \mathbb{N}.$$

[2]QAM is a common modulation scheme widely used in IEEE 802.11g/n as well as 3G and LTE systems, due to its high bandwidth efficiency.

[3]In practice, symbol error can be detected via cyclic redundancy check (CRC).

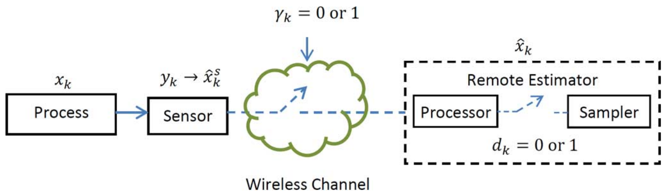

Fig. 1. System architecture.

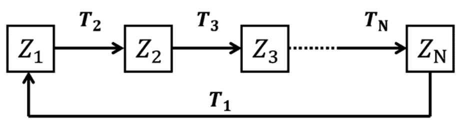

Fig. 2. Markov chain model of the estimation error covariance.

From Lemma 1 and (25), it is straightforward that for a given sampling instances d_k's, the trace of P_k becomes larger if the data packet is dropped. In other words, the estimation performance is enhanced with a larger probability of successful transmission p_k, which in turn can be achieved by increasing the transmission power M_k.

Remark 1: The present framework is motivated by applications where the remote estimator is in charge of multiple tasks, e.g., in a setup with multiple sensors, it samples and deals with the data from one sensor only at every time slot during a predefined period. This mechanism is quite common in practice, and reflects the fact that the collaborative nature of industrial wireless sensor networks brings several advantages over traditional wired industrial monitoring. For example, WirelessHART (Highway Addressable Remote Transducer Protocol) [28] networks utilize mesh networking, in which each device is able to transmit its own data as well as relay information from other devices in the network and control systems. Usually wireless sensor networks are self-configurable, to meet the requirements of wireless industrial applications. WirelessHART uses a central network manager to provide routing and communication schedules. The communication between devices is established by a single central controller: the PAN coordinator. The latter takes the role of the remote estimator in our current setting. □

III. Problem Statement

Based on the estimation setup, the system architecture is described as in Fig. 1. At every time step k, the sensor measures the process state x_k and sends its local state estimate $\hat{x}_k^s$ to the remote estimator through an unreliable wireless channel using power M_k. Then based on all the information it collects, i.e., I_k, the processor of the remote estimator updates the state estimation x_k and P_k. The remote estimator samples and utilizes the estimate when $d_k = 1$.

Our preceding discussion motivates us to consider periodic sensor power schedules with period N. Since we only care about the estimation quality at the sample points, we choose the trace of the average expected error covariance of all the estimator's sample points as the objective (loss) function, i.e.,

$$J(\theta) = \limsup_{T\to\infty} \frac{1}{T}\sum_{k=1}^{T} d_k \mathrm{Tr}\left\{\mathbb{E}[P_k]\right\}, \tag{26}$$

where $d_k \in \{0,1\}$ as defined in Section II-C is the given periodic sampling pattern with period N (i.e., it is not a design variable), and $\theta = \{M_1, M_2, \ldots, M_N\}$ is the periodic sensor power schedule with period N.

As discussed before, at each time step, higher transmission power M_k leads better estimation quality, i.e., lower $\mathbb{E}[P_k]$.[4] In practice, however, the total energy for the whole time horizon is limited. Thus we are interested in finding an optimal sensor power schedule, say $\theta^\star$, such that the system achieves the best performance under the energy constraint. To be more specific, we consider the following optimization problem:

Problem 1:

$$\begin{aligned} \min_{\theta} \quad & J(\theta), \\ \text{s.t.} \quad & \sum_{k=1}^{N} M_k \leqslant \overline{M}, \end{aligned} \tag{P1}$$

where $\overline{M}$ is the energy constraint within each period.

Intuitively since more energy is always beneficial for improving the system performance, the optimal power schedule remains the same if the constraint of Problem 1 is changed to $\sum_{k=1}^{N} M_k = \overline{M}$, which implies the necessary condition for the optimal solution to Problem 1. Lemma 2 supports this intuition.

Lemma 2: Let $\theta^\star = \{M_1^\star, M_2^\star, \ldots, M_N^\star\}$ be an optimal solution to Problem 1, then we have:

$$\sum_{k=1}^{N} M_k^\star = \overline{M}. \tag{27}$$

Proof: See the Appendix. ■

From Lemma 2, without loss of generality, we will focus on the following problem in this paper.

Problem 2:

$$\begin{aligned} \min_{\theta} \quad & J(\theta), \\ \text{s.t.} \quad & \sum_{k=1}^{N} M_k = \overline{M}, \end{aligned} \tag{P2}$$

IV. Markov Chain Model

As mentioned in the introduction, different from the case of finite-time horizon where the optimal state estimate and corresponding error covariance can be computed recursively, the problem of infinite-time horizon is much more difficult to handle as one typically cannot compute the objective function $J(\theta)$ easily.

In this section we will first derive a model for the periodic remote state estimation system with random dropouts and using a periodic power schedule. Then based on the Markov chain model, we will drive a closed-form expression for $J(\theta)$.

From the recursion of the error covariance P_k in (25), it is easy to see that at any time instant $k_2 \geqslant k_1$, the error covariance at the estimator side can be written as $P_{k_2} = h^{k_2-k_1}(\overline{P})$, where k_1 is the latest time when it successfully received sensor data.

[4]In Section V, we will further elucidate the dependence of J on M_k

Since P_k only takes value in the set of $\{\overline{P}, h(\overline{P}), h^2(\overline{P}), \ldots\}$, we have the following definition describing the state of the remote estimator.

Definition 1: Given the period N, if at the k-th time step, the state estimation error covariance $P_k = h^i(\overline{P})$, where i is a certain non-negative integer, then we say that the remote estimator is at state $S_k \triangleq Z_{i+1,j}$, where $k \bmod N = j$,

For example, suppose that the period $N = 3$. If at $k = 1$, we have $P_1 = h^2(\overline{P})$, i.e., $i = 2$, and $j = k \bmod N = 1$, then the state of the remote estimator, $S_1 = Z_{i+1,j} = Z_{3,1}$. As a comparison, if at $k = 4$, we have $P_4 = P_1 = h^2(\overline{P})$, since $4 \bmod 3 = 1$, we also have $S_4 = Z_{3,1}$, i.e., the remote estimator is in the same state at time steps $k = 1$ and $k = 4$. This is easy to understand because the period $N = 3$.

Thus for the estimator, the state space is given by

$$\mathbf{S} \triangleq Z_{i,j}, \quad i \in \mathbb{N},\ 1 \leqslant j \leqslant N,$$

which includes all possible states at any time.[5] Define the state sets for every time index within one period as:

$$\mathbb{Z}_j = \{Z_{i,j}, \forall i \in \mathbb{N}\},$$

where $j = 1, 2, \ldots, N$. Then, at any time step k,

$$S_k \in \mathbb{Z}_j, \quad j = k \bmod N.$$

Due to (24) and the fact that dropouts are independent, at any time $k+1$, the state S_{k+1} is only related to the previous state S_k, the stochastic process $\{S_k\}, k \in \mathbb{N}$ constitutes a Markov chain (with periodic state space) [29].

Consider the transition matrix from state set Z_{k-1} to Z_k, denoted as $\mathbb{T}_j$ where $j = k \bmod N$, then the entries of $\mathbb{T}_j$ can be expressed as:

$$\mathbb{T}_j(i_1, i_2) = \begin{cases} \mathbb{P}\left[Z_{i_2,1} | Z_{i_1,N}\right], & j = 1, \\ \mathbb{P}\left[Z_{i_2,j} | Z_{i_1,j-1}\right], & j > 1. \end{cases} \tag{28}$$

Thus, the system is described by N transition matrices $\{\mathbb{T}_j\}_{j=1,2,\ldots,N}$. Their entries can be easily computed as follows:

If no dropout occurs, as discussed before, then we have $P_k = \overline{P}$. Based on (28), this gives:

$$\begin{aligned} \mathbb{T}_j(i_1, 1) &= \mathbb{P}\left[Z_{1,j} | Z_{i_1,j-1}\right] \\ &= \mathbb{P}\left[P_k = \overline{P} | Z_{i_1,j-1}\right] \\ &= p_j, \forall i_1 \in \mathbb{N}. \end{aligned}$$

On the other hand, if a dropout occurs, then $P_k = h(P_{k-1})$, and we have:

$$\begin{aligned} \mathbb{T}_j(i_1, i_1+1) &= \mathbb{P}\left[Z_{i_1+1,j} | Z_{i_1,j-1}\right] \\ &= \mathbb{P}\left[P_k = h(P_{k-1}) | Z_{i_1,j-1}\right] \\ &= q_j, \forall i_1 \in \mathbb{N}, \end{aligned}$$

where $q_j = 1 - p_j$ is the corresponding dropout probability.

[5]Note that the number of consecutive dropouts is, in general, unbounded.

Other entries of $\mathbb{T}_j$ are 0, since the corresponding state transitions will never occur. This gives:

$$\mathbb{T}_j = \begin{bmatrix} p_j & q_j & & & \\ p_j & & q_j & & \\ p_j & & & q_j & \\ \vdots & & & & \ddots \end{bmatrix}, \quad j = 1, 2, \ldots, N, \tag{29}$$

where the missing entries are 0.

To elucidate the situation further, we will next construct the probability matrix

$$\Pi = (\pi_{i,j}), \quad (i,j) \in \mathbb{N} \times \{1, 2, \ldots, N\},$$

where $\pi_{i,j}$ is the limiting probability of state $Z_{i,j}$. More precisely,

$$\pi_{i,j} = \lim_{T \to \infty} \frac{t}{T}, \tag{30}$$

where t is the number of times that $Z_{i,j}$ occurred from time 1 to T.

It is easy to see that the limiting state probability of $h^{i-1}(\overline{P})$ is given by $\sum_{j=1}^{N} \pi_{i,j}$ for all $i \in \mathbb{N}$. Furthermore, since the schedule is periodic with period N, we have

$$\sum_{i=1}^{\infty} \pi_{i,j} = \frac{1}{N}, \quad j = 1, 2, \ldots, N. \tag{31}$$

Based on the definition of $\mathbb{T}_j$ in (28), since $\mathbb{T}_j$ is the transition matrix, the following relationship between $\mathbb{T}_j$ and Π can be established:

$$\Pi[1]^{\mathrm{T}} = \Pi[N]^{\mathrm{T}} \mathbb{T}_1, \quad \Pi[j]^{\mathrm{T}} = \Pi[j-1]^{\mathrm{T}} \mathbb{T}_j, \tag{32}$$

where $\Pi[j]$, $j = 1, \ldots, N$ is the j-th column of Π as defined before.

From (31) and (32), through some calculation, we can obtain the exact form of Π as:

$$\frac{1}{N} \begin{bmatrix} p_1 & p_2 & p_3 & \cdots & p_N \\ p_N q_1 & p_1 q_2 & p_2 q_3 & \cdots & p_{N-1} q_N \\ p_{N-1} q_N q_1 & p_N q_1 q_2 & p_1 q_2 q_3 & \cdots & p_{N-2} q_{N-1} q_N \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix}. \tag{33}$$

Example 1: Consider a simple case where $N = 3$. From (32), we have $\Pi[1]^{\mathrm{T}} = \Pi[3]^{\mathrm{T}} \mathbb{T}_1$, thus

$$\pi_{1,1} = \Pi[3]^{\mathrm{T}} \mathbb{T}_1[1] = p_1 \sum_{i=1}^{3} \pi_{i,3} = \frac{1}{N} p_1,$$

where $\mathbb{T}_1[1]$ denotes the first column of $\mathbb{T}_1$, i.e., $[p_1, p_1, p_1]^T$.

Similarly, we have

$$\pi_{1,2} = \frac{1}{N} p_2, \quad \pi_{1,3} = \frac{1}{N} p_3.$$

Next

$$\pi_{2,1} = \Pi[3]^{\mathrm{T}} \mathbb{T}_1[2] = q_1 \pi_{1,3} = \frac{1}{N} p_3 q_1,$$

and

$$\pi_{2,2} = \frac{1}{N} p_1 q_2, \quad \pi_{2,3} = \frac{1}{N} p_2 q_3.$$

Thus,

$$\pi_{3,1} = \Pi[3]^{\mathrm{T}} \mathbb{T}_1[3] = q_1 \pi_{2,3} = \frac{1}{N} p_2 q_3 q_1,$$

and

$$\pi_{3,2} = \frac{1}{N} p_3 q_1 q_2, \quad \pi_{3,3} = \frac{1}{N} p_1 q_2 q_3.$$

By repeating this procedure one obtains

$$\Pi = \frac{1}{3} \begin{bmatrix} p_1 & p_2 & p_3 \\ p_3 q_1 & p_1 q_2 & p_2 q_3 \\ p_2 q_3 q_1 & p_3 q_1 q_2 & p_1 q_2 q_3 \\ \vdots & \vdots & \vdots \end{bmatrix},$$

as per (33).

The following properties will be used in Section V to find optimal power schedules.

Proposition 1: 1. Suppose that the power schedule has period N and power constraint $\overline{M}$, as used in Problem 2. We then have

$$\prod_{k=1}^{N} q_k = M, \quad M \triangleq (1-\lambda)^{\overline{M}}, \tag{34}$$

where λ is as defined in (20).

2. Consider Π in (33). Then,

$$\forall b = iN + a, \quad i \in \mathbb{N}, \quad 1 \leqslant a \leqslant N,$$

we have $\pi_{b,j} = M^i \pi_{a,j}$ and $\Pi(b) = M^i \Pi(a)$, where $\Pi(k)$ is the k-th row of Π.

Proof: Expression (34) follows directly from the fact that $q_k = 1 - p_k$ and using (19).

To establish 2., similar to Example 1, and since

$$\Pi[j]^{\mathrm{T}} = \Pi[j-1]^{\mathrm{T}} \mathbb{T}_j,$$

when $i > N$, we have:

$$\begin{aligned} \pi_{i,j} &= \Pi[j-1]^{\mathrm{T}} \mathbb{T}_j[i] \\ &= q_j \pi_{i-1,j-1} \\ &= q_j q_{j-1} \pi_{i-2,j-2} \\ &\ \ \vdots \\ &= \prod_{k=1}^{N} q_k \pi_{i-N,j-N} \\ &= \prod_{k=1}^{N} q_k \pi_{i-N,j}. \end{aligned}$$

Since $\prod_{k=1}^{N} q_k = M$, we have $\pi_{i,j} = M \pi_{i-N,j}$. Using this property i times, proves the result. ∎

From Proposition 1, we can see that there exists a simple relationship between every N rows in Π. This property simplifies our subsequent analysis significantly. When considering the probability matrix Π, we just need to investigate its first N rows.

V. Optimal Sensor Scheduling

In this section, we show how to design optimal power schedules using the Markov chain model presented in Section IV.

A. Preliminaries

The following lemma is a modified matrix version of the classical Hardy-Littlewood-Polya rearrangement inequality.

Lemma 3: Consider a sequence of semi-definite matrices

$$0 \leqslant Y_1 \leqslant Y_2 \leqslant \cdots \leqslant Y_n$$

and a sequence of real numbers $a_1, a_2, \ldots, a_n$. Denote $b_1, b_2, \ldots, b_n$ as another sequence of real numbers such that

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i.$$

Consider $k \in \{1, 2, \ldots, n\}$ such that $b_i \leqslant a_i, \forall i \leqslant k$, and $b_i \geqslant a_i, \forall i > k$. We then have

$$\sum_{i=1}^{n} a_i Y_i \leqslant \sum_{i=1}^{n} b_i Y_i.$$

Proof: Direct calculations give

$$\begin{aligned} \sum_{i=1}^{n} a_i Y_i - \sum_{i=1}^{n} b_i Y_i &= \sum_{i=1}^{n} (a_i - b_i) Y_i \\ &= \sum_{i=1}^{k} (a_i - b_i) Y_i - \sum_{i=k+1}^{n} (b_i - a_i) Y_i \\ &\leqslant \sum_{i=1}^{k} (a_i - b_i) Y_k - \sum_{i=k+1}^{n} (b_i - a_i) Y_k \\ &= Y_k \left[\sum_{i=1}^{k} (a_i - b_i) - \sum_{i=k+1}^{n} (b_i - a_i) \right] \\ &= Y_k \left(\sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i \right) = 0. \end{aligned}$$

∎

The following lemmas follow directly from well-known results of mathematical analysis.

Lemma 4: [30] For a continuous multi-variable scalar function defined on a closed set, there always exists the maximum and minimum value of this function, and both of them can be achieved within this closed set.

Lemma 5: [30], [31] Consider a multi-variate function $f(x_1, x_2, \ldots, x_n)$ subject to the constraints

$$g(x_1, x_2, \ldots, x_n) = c,$$

where f and g are functions with continuous first partial derivatives. Lagrange multipliers can be used to find the critical point (local extremum) of function f.

B. Main Results

We first note that the objective function (26) can be written as

$$J(\theta)=\frac{1}{N}\sum_{k=1}^{N}d_k\mathcal{P}_k, \tag{35}$$

where

$$\mathcal{P}_k\triangleq\lim_{T\to\infty}\frac{1}{T}\sum_{i=0}^{T}\mathrm{Tr}\left\{\mathbb{E}[P_{k+iN}]\right\}. \tag{36}$$

Now using (30) and Proposition 1, it is easy to see that:

$$\begin{aligned}\mathcal{P}_k&=\sum_{i=1}^{\infty}\pi_{i,k}\mathrm{Tr}\left\{h^{i-1}(\overline{P})\right\}\\&=\sum_{i=1}^{N}(1+M+M^2+\cdots)\pi_{i,k}\mathrm{Tr}\left\{h^{i-1}(\overline{P})\right\}\\&=\frac{1}{1-M}\sum_{i=1}^{N}\pi_{i,k}\mathrm{Tr}\left\{h^{i-1}(\overline{P})\right\}.\end{aligned}$$

As a consequence of the above, in order to compare two power schedules under the same energy constraint, one only needs to compare:

$$\tilde{J}(\theta)=\frac{1}{N}\sum_{k=1}^{N}d_k\tilde{\mathcal{P}_k}, \tag{37}$$

where

$$\tilde{\mathcal{P}_k}\triangleq\sum_{i=1}^{N}\pi_{i,k}\mathrm{Tr}\left\{h^{i-1}(\overline{P})\right\}. \tag{38}$$

Consider the power schedule design Problem 2. Intuitively we should not allocate any energy when it is not at the estimator's sample time. The following lemma supports this intuition.

Lemma 6: Given the estimator's sample scheme $\{d_1,d_2,\dots,d_N\}$ and a sensor power schedule scheme $\theta:\{M_1,M_2,\dots,M_N\}$. If there exists a certain t such that $d_t=0$ and $M_t\neq 0$, then the following new schedule $\theta'=\{M'_1,M'_2,\dots,M'_N\}$ with[6]

$$M'_k=\begin{cases}0, & \text{if } k=t,\\ M_{t+1}+M_t, & \text{if } k=t+1,\\ M_k, & \text{otherwise,}\end{cases}$$

has the same energy cost as θ, however, achieves $J(\theta')<J(\theta)$.

Proof: Since $M_t+M_{t+1}=M'_t+M'_{t+1}$, we have

$$q_tq_{t+1}=q'_tq'_{t+1}.$$

Thus for any $i>t$ such that $d_i=1$, considering the elements of the i-th column of Π, we have

$$\pi_{i,i-t}=(1-q_{t+1})q_{t+2}q_{t+3}\cdots q_{i+1}q_i,$$

and

$$\pi_{i,i-t+1}=(1-q_t)q_{t+1}q_{t+1}\cdots q_{i+1}q_i.$$

[6]If $t=N$, then set $t+1=1$.

It is straightforward to see that $\pi'_{i,i-t}>\pi_{i,i-t}$, whereas $\pi'_{i,i-t+1}<\pi_{i,i-t+1}$ and other elements in this row remain unchanged. Since $\mathcal{P}_k=\sum_{i=1}^{\infty}\pi_{i,k}\mathrm{Tr}\{h^{i-1}(\overline{P})\}$, Lemma 1 and Lemma 3 give $\mathcal{P}'_k<\mathcal{P}_k$. Consequently, $J(\theta')<J(\theta)$, which completes the proof. ∎

Lemma 6 gives a necessary condition on a sensor power scheme to be optimal, namely, $M_k=0$ if $d_k=0$.

By using our preceding analysis, the following result characterizes the optimal sensor power schedule.

Theorem 1 (Optimal Sensor Power Schedule): Consider the sampling instants $\{k_1,k_2,\dots,k_m\}\subset\{1,2,\dots,N\}$. We then have

$$\tilde{J}(\theta)=\frac{1}{N}\sum_{j=1}^{m}\left(\sum_{i=1}^{m}\pi_{k_i,k_j}\mathrm{Tr}\left\{h^{k_i-1}(\overline{P})\right\}\right). \tag{39}$$

The optimal solution to Problem 2 is given by the solution to the following set of equations:

$$\begin{cases}\partial f\left(q_{k_1},q_{k_2},\dots,q_{k_m},\lambda_L\right)/\partial q_{k_1}=0,\\ \partial f\left(q_{k_1},q_{k_2},\dots,q_{k_m},\lambda_L\right)/\partial q_{k_2}=0,\\ \cdots\\ \partial f\left(q_{k_1},q_{k_2},\dots,q_{k_m},\lambda_L\right)/\partial q_{k_m}=0,\\ \partial f\left(q_{k_1},q_{k_2},\dots,q_{k_m},\lambda_L\right)/\partial \lambda_L=0,\end{cases} \tag{40}$$

where

$$f\left(q_{k_1},q_{k_2},\dots,q_{k_m},\lambda_L\right)\triangleq\tilde{J}(\theta)-\lambda_L\left(\prod_{i=1}^{m}q_{k_i}-M\right). \tag{41}$$

Proof: Given the estimator's sample scheme $\{d_1,d_2,\dots,d_n\}$ with $d_{k_1}=d_{k_2}=\cdots=d_{k_m}=1$, for any $d_i=0$, Lemma 6 gives that $M_i=0$, $q_i=1$. Then from (33), we have

$$\pi_{i,k_j}=(1-q_i)q_{i+1}q_{i+2}\dots q_{k_j}=0,\quad \forall j=1,2,\dots,m.$$

Thus

$$\tilde{\mathcal{P}_k}=\sum_{i=1}^{N}\pi_{i,k}\mathrm{Tr}\left\{h^{i-1}(\overline{P})\right\}=\sum_{i=1}^{m}\pi_{k_i,k}\mathrm{Tr}\left\{h^{k_i-1}(\overline{P})\right\},$$

and

$$\begin{aligned}\tilde{J}(\theta)&=\frac{1}{N}\sum_{k=1}^{N}d_k\tilde{\mathcal{P}_k}=\frac{1}{N}\sum_{j=1}^{m}\tilde{\mathcal{P}}_{k_j}\\&=\frac{1}{N}\sum_{j=1}^{m}\left(\sum_{i=1}^{m}\pi_{k_i,k_j}\mathrm{Tr}\left\{h^{k_i-1}(\overline{P})\right\}\right),\end{aligned}$$

which establishes (39).

Since $\pi_{i,j}$ is function of $q_{k_1},q_{k_2},\dots,q_{k_m}$, we can regard $\tilde{J}(\theta)$ as a multi-variable scalar function in terms of $q_{k_1},q_{k_2},\dots,q_{k_m}$, with constraints

$$q_{k_i}=(1-\lambda)^{M_{k_i}}\in[\overline{M},1]$$

and

$$\prod_{i=1}^{m}q_{k_i}=(1-\lambda)^{\overline{M}}=M.$$

Now, $\tilde{J}(\theta)$ is a continuous scalar function on a closed set. From Lemma 4, there always exists a minimal value of $\tilde{J}(\theta)$ under these constraints.

From Lemma 5, the use of Lagrange multipliers can convey our problem into solving a multi-variable equation series. By solving (40), we can calculate the critical points of $\tilde{J}(\theta)$. Through comparison the critical points with the m boundary points, i.e., $q_{k_i} = M, q_{k_j} = 1, \forall j \neq i$, we can obtain the minimal point within the entire set, which is the optimal solution to Problem 2. It minimizes $\tilde{J}(\theta)$, and thereby $J(\theta)$. ■

The above result shows how to compute optimal periodic power schedules for the remote estimation problem at hand.

C. Uniform Sampling

As a special case, the present framework can also be used to investigate estimators which sample at all time instants $k \in \mathbb{N}$. Such systems were studied in previous works, including [32]–[34], where only two discrete power levels were taken into account. In those works one of the critical assumptions is that using high power will lead to error-free transmission. This simplifies the analysis significantly, since perfect reception amounts to a resetting of the estimator error covariance to the stationary value $\overline{P}$, see (12). In contrast, in the present work we do not rely upon perfect transmissions. This allows us to study more practical schemes, which are, however, more difficult to analyze. A byproduct of our results in Sections IV and V is that, when applied to uniform sampling, the optimal solution can be obtained systematically. In addition, we are able to prove that, when the period $N \leqslant 3$, the optimal sensor power schedule follows the principle "as uniform as possible" [34], i.e., to distribute the energy evenly during the entire time-horizon.

For $N = 2$, the first two rows of Π are given as:

$$\frac{1}{2}\begin{bmatrix} p_1 & p_2 \\ p_2 q_1 & p_1 q_2 \end{bmatrix}.$$

Thus, we have

$$\begin{aligned}\tilde{J}(\theta) &= (p_1 + p_2)\mathrm{Tr}\{\overline{P}\} + (p_1 q_2 + p_2 q_1)\mathrm{Tr}\left\{h(\overline{P})\right\}, \\ p_1 + p_2 &= 2 - \left[(1-\lambda)^{M_1} + (1-\lambda)^{M_2}\right], \\ p_1 q_2 + p_2 q_1 &= 2(1-\lambda)^m + \left[(1-\lambda)^{M_1} + (1-\lambda)^{M_2}\right].\end{aligned}$$

Since $M_1 + M_2 = \overline{M}$ is constant, it is easy to see that

$$(1-\lambda)^{M_1} + (1-\lambda)^{M_2}$$

is minimal when $M_1 = M_2$. Since

$$(p_1 + p_2) + (p_1 q_2 + p_2 q_1) = 2(1-\lambda)^{\overline{M}}$$

is constant, the result of Lemma 3, gives that $\tilde{J}$ is minimal when $M_1 = M_2$, yielding the optimal sensor power scheduling for case $N = 2$ as $\left\{\overline{M}^{\frac{1}{2}}, \overline{M}^{\frac{1}{2}}\right\}$. Similarly, it can be shown that with sampling at all instants $k \in \mathbb{N}$, with $N = 3$, the optimal sensor power scheduling is given by $\left\{\overline{M}^{\frac{1}{3}}, \overline{M}^{\frac{1}{3}}, \overline{M}^{\frac{1}{3}}\right\}$. For the case $N > 3$, simulations also support that the optimal solution is to distribute the energy uniformly. The analysis of more general cases, however, lies beyond the scope of the current work. The optimal sensor power schedule can be computed, nevertheless, from the procedure of Theorem 1.

VI. Examples

Consider a scalar system with parameters $A = 1.2, C = 0.7$, $R = Q = 0.8$, $\lambda = 0.8$. For this system, we have $\overline{P} = 0.9245$, $h(\overline{P}) = 2.1312$, $h^2(\overline{P}) = 3.8689$, $h^3(\overline{P}) = 6.3712$.

A. Optimal Power Schedule

Suppose that the estimator's sample scheme $\{d_k\}$ is {1, 0, 1, 0, 0}, and $\overline{M} = 2$. Because the sensor power schedule scheme θ is in the form of $\{M_1, 0, M_2, 0, 0\}$, which implies that $q_1 = (1-\lambda)^{M_1}, q_3 = (1-\lambda)^{M_3}, q_2 = q_4 = q_5 = (1-\lambda)^0 = 1$, we can write the first $N = 5$ rows of Π as follows:

$$\frac{1}{5}\begin{bmatrix} p_1 & 0 & p_3 & 0 & 0 \\ 0 & p_1 & 0 & p_3 & 0 \\ 0 & 0 & p_1 q_3 & 0 & p_3 \\ p_3 q_1 & 0 & 0 & p_1 q_3 & 0 \\ 0 & p_3 q_1 & 0 & 0 & p_1 q_3 \end{bmatrix},$$

where $p_1 = 1 - q_1$ and $p_3 = 1 - q_3$.

Thus we have

$$\begin{aligned}\tilde{J}(\theta) &= \frac{1}{5}(\tilde{\mathcal{P}}_1 + \tilde{\mathcal{P}}_2) \\ &= \frac{1}{5}\left[p_1\overline{P} + p_3 q_1 h^3(\overline{P}) + p_3\overline{P} + p_1 q_3 h^2(\overline{P})\right] \\ &= \frac{1}{5}\left[(2 - q_1 - q_3)\overline{P} + (1 - q_3) q_1 h^3(\overline{P}) \right. \\ &\qquad \left. + (1 - q_1) q_3 h^2(\overline{P})\right].\end{aligned}$$

The constraints are given by

$$q_1 q_3 = (1-\lambda)^{M_1}(1-\lambda)^{M_2} = (1-\lambda)^{\overline{M}} = (1-0.8)^2 = 0.04.$$

Based on Theorem 1, applying the Lagrange Multiplier method, (40) yields $M_1 = 1.1911, q_1 = 0.1471, M_3 = 0.8089$, $q_3 = 0.2720$. It is easy to verify that the obtained solution is better than the boundary points. Thus the optimal sensor power schedule uses a periodic power level sequence $\{1.1911, 0, 0.8089, 0, 0, 1.1911, 0, 0.8089, 0, 0, \ldots\}$.

Remark 2: From the example we can see that the optimal schedule does not amount to distributing the energy as uniformly as possible to the sample points, i.e., {1, 0, 1, 0, 0}. This stands in contrast to the case where the estimator operates at all instants $k \in \mathbb{N}$, as studied in [34].

B. Simulation Study

Consider the same system as in Section VI-A but with sample period $N = 7$. Suppose that the estimator's sample scheme $\{d_k\}$ is {1, 0, 1, 0, 0, 0, 0}, and $\overline{M} = 3.5$. Based on Theorem 1, we can obtain the optimal power schedule:

$$\theta^\star = \{2.2389, 0, 1.2611, 0, 0, 0, 0\}.$$

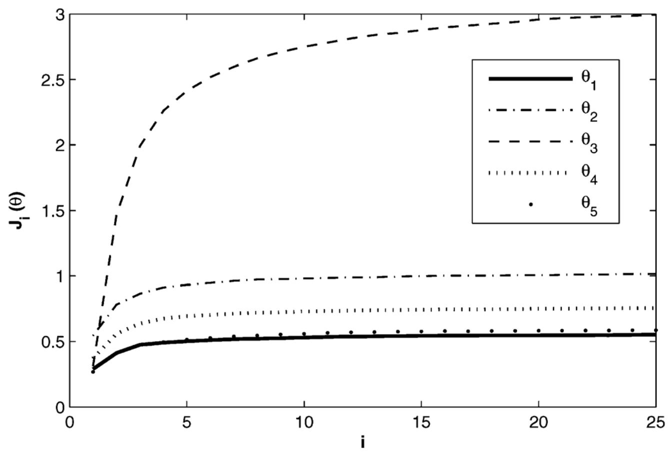

Fig. 3. Comparison of schedules $\theta_1, \theta_2, \theta_3, \theta_4$ and θ_5 when $\overline{M} = 3.5$.

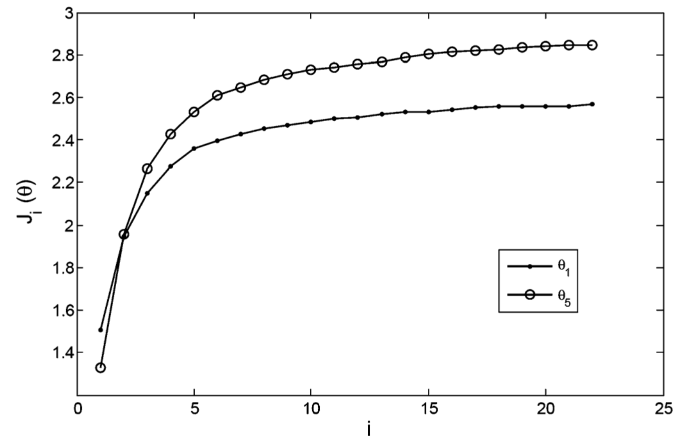

Fig. 4. Comparison of schedules θ_1 and θ_5 when $\overline{M} = 2.5$.

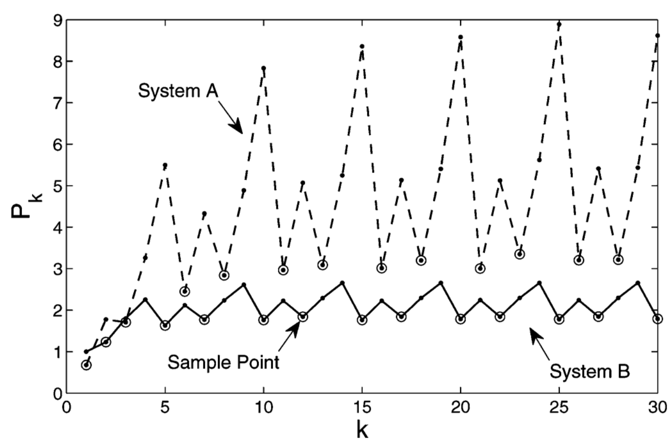

Fig. 5. Comparison of estimation performance for two systems.

To compare the optimal design with others, we consider following 5 intuitive schemes:

$$\theta_1 : \{2.2389, 0, 1.2611, 0, 0, 0, 0\}.$$
$$\theta_2 : \{3.5, 0, 0, 0, 0, 0, 0\},$$
$$\theta_3 : \{0, 0, 3.5, 0, 0, 0, 0\},$$
$$\theta_4 : \{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5\},$$
$$\theta_5 : \{1.75, 0, 1.75, 0, 0, 0, 0\}.$$

θ_1 is the optimal scheme calculated based on our main theorem, θ_2 and θ_3 represent the power scheme that distribute all the energy on a single sample point, θ_4 denotes the uniform energy distribution on every point within a period and θ_5 corresponds to the uniform energy distribution on every sample point within a period.

Define

$$J_i(\theta) = \frac{1}{iN} \sum_{k=1}^{iN} d_k \mathrm{Tr}\left\{\mathbb{E}[P_k]\right\}, \tag{42}$$

as the empirical approximation (via 10000 Monte Carlo simulations) of $J(\theta)$ in (26), evaluated at the end of the i-th period. Fig. 3 shows the comparison between $\theta_1, \theta_2, \theta_3, \theta_4$ and θ_5. The simulations support that θ_1 gives the best performance, which is consistent with the results in Theorem 1.

One may note that the performance gap between θ_1 and θ_5 is close. It is because that an energy constraint $\overline{M} = 3.5$ is chosen in order to ensure the stability of the system under all five schedules. Under this constraint, the average data packet arrival rate becomes sufficiently high to provide an accurate estimation for both θ_1 and θ_5. When $\overline{M}$ becomes smaller, e.g., $\overline{M} = 2.5$, the performance gap between θ_1 and θ_5 increases as shown in Fig. 4.

C. Multi-Systems Sampling

In this example we consider the proposed situation in Section I that the remote estimator samples the data from two sensors, each measuring two different ARMA processes. Suppose the parameters of the two systems are given by:

System A: $A_A = 1.2$, $C_A = 0.7$, $R_A = Q_A = 0.8$, $\lambda_A = 0.8$, $\overline{M}_A = 2$;

System B: $A_B = 0.9$, $C_B = 0.7$, $R_B = Q_B = 0.8$, $\lambda_B = 0.8$, $\overline{M}_B = 1$.

System A is the unstable system we studied in Section VI-A. System B is a stable system with same parameters as system A except $A_B = 0.9$. Also since it is a stable system, we have less energy for system B: $\overline{M}_B = 1$.

Suppose that the remote estimator's sample period is 5, i.e., $N = 5$, and the sample scheme for system A is {1, 0, 1, 0, 0} while for system B is {0, 1, 0, 0, 1}, thus the optimal power schedules for system A and B are {1.1911, 0, 0.8089, 0, 0} and {0, 0.4040, 0, 0, 0.5960}, respectively.

Our simulation compares the estimation performance for the two systems and Fig. 5 shows the error covariances of the state estimates at the remote estimator for both systems including the time slots which are not sampled.

As we can see from the Fig. 5, system B has smaller state estimate error of the sample points than system A since system B is a stable system. This inspires us to investigate how to design the sampling patterns when multiple processes are to be estimated, which is an interesting problem but out of the scope of this paper and we will leave it for the future work.

VII. Conclusion

We have studied the problem of periodic sensor transmission power schedule with an energy constraint. We derived the explicit expression of the estimator's error covariance at each time slot and showed how to compute the optimal sensor power scheduling to minimize the average error covariance. Future work includes finding the optimal schedule when transmission loss is governed by a Markov chain and investigating sensor power scheduling in a multi-sensor architecture with possibly time-varying system dynamics. It is also of interest to investigate how to design the sampling patterns when multiple processes are to be estimated.

APPENDIX

Proof of Lemma 2: We prove this lemma by contradiction. Suppose that $\theta = \{M_1, M_2, \ldots, M_N\}$ is an optimal power schedule to Problem 1 with $\sum_{k=1}^{N} M_k < \overline{M}$. Note that

$$\overline{M} - \sum_{k=1}^{N} M_k \triangleq \delta > 0,$$

then we can construct a new power schedule $\hat{\theta} = \{\hat{M}_1 \hat{M}_2, \ldots, \hat{M}_N\}$ based on θ according to the follow step:

$$\hat{M}_k = \begin{cases} M_k + \delta, & \text{if } k = 1, \\ M_k, & \text{otherwise.} \end{cases}$$

From (19) we have $\hat{p}_1 > p_1$ and $\hat{q}_1 < q_1$. Thus from (33) and (38), the definition of $\pi_{i,j}$ and $\tilde{\mathcal{P}}_k$, it is straightforward to show $\hat{\tilde{\mathcal{P}}}_k < \tilde{\mathcal{P}}_k$. Consequently, $J(\hat{\theta}) < J(\theta)$, i.e., θ cannot be optimal. Therefore, any optimal solution to Problem 1 needs to satisfy (27). ■

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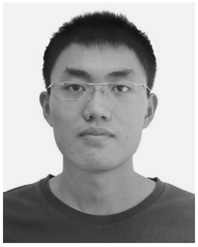

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