Technical Notes and Correspondence

On Optimal Partial Broadcasting of Wireless Sensor Networks for Kalman Filtering

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***Abstract*—State estimation using wireless sensor networks (WSNs) is an important technique in many commercial and military applications, in which a group of (nonidentical) sensors take noisy observations of system state and send back to a fusion center through wireless broadcasting for state estimation. In order to minimize the terminal estimation error covariance at the fusion center, a partial broadcasting policy should tell which sensors to broadcast at each stage. The limited battery allows each sensor to broadcast only a few times. The limited wireless communication bandwidth allows only a few sensors to broadcast at the same time. Due to these couplings, the optimal partial broadcasting policy is not clear in general. Despite the abundant applications of partial broadcasting policies, theoretical analysis is rare. In this technical note, we provide a first study on the properties of optimal partial broadcasting policies. When there is no packet drop, a good-sensor-late-broadcast (GSLB) rule is shown to perform optimally for both the scalar system and the vector system. When packet drops with positive probability, situations in which the GSLB rule may or may not perform optimally are analyzed. Under different dropping rates, the GSLB rule is compared with several other policies through simulations.**

***Index Terms*—Kalman filtering, partial broadcasting, wireless sensor network (WSN).**

I. Introduction

State estimation using wireless sensor networks (WSNs) has become an important technique in many commercial and military applications. Usually a group of (nonidentical) sensors take noisy observations of the system state and send back to a fusion center through wireless broadcasting. The fusion center processes all the information from the sensors and outputs a state estimate. Due to the limited battery at each sensor and the limited wireless communication bandwidth, usually only part of the sensors can broadcast at a time. The policy that tells which sensors to broadcast at each time is called a partial broadcasting policy. The optimal partial broadcasting policy, which minimizes the estimated state error covariance at a terminal stage, is considered in this technical note.

There are three difficulties to find the optimal partial broadcasting policy. First, the limited battery capacity makes the decision making at different stages correlated. Second, the limited communication bandwidth renders the decision making at different sensors correlated. Third, the random packet drop not only degrades the amount of information that a sensor shares with the fusion center, but also makes the sequence of estimated state error covariance a stochastic one. This substantially complicates the theoretical analysis, as will be discussed in Section IV.

Due to the aforementioned difficulties, despite the abundant applications of partial broadcasting policies, theoretical analysis is rare. In this technical note, we focus on the finite-horizon discrete-time state estimation of a linear time-invariant system and provide a first study on the properties of optimal partial broadcasting policies. When there is no packet drop, a good-sensor-late-broadcast (GSLB) rule is shown to perform optimally, which means that sensors with small observation noise should broadcast as late as possible. An algorithm is then presented to generate the optimal policy. When there is a positive probability of packet drop, situations in which the GSLB rule may or may not perform optimally are analyzed. Under different packet dropping rates the GSLB rule is compared with the optimal policy, a round-robin policy, a random policy, and a greedy policy through simulation.

The rest of the technical note is organized as follows. A brief literature review is presented in Section II. The problem is mathematically formulated in Section III. The main results are shown in Section IV, where Section IV-A considers scalar system without packet drop; Section IV-B considers vector system without packet drop; Section IV-C considers the case of packet drop; Section IV-D contains the simulation results; and Section IV-E discusses extensions. A brief conclusion is presented in Section V.

II. Literature Review

Partial broadcasting policy optimization is related to the sensor selection problem, where a central node selects a group of sensors to perform certain tasks. The sensor selection problem in general is NP-complete [1]. Many heuristics have been developed to solve this problem approximately. Zhao *et al.* [2] suggested selecting the most informative sensors. Xiao *et al.* [3] developed an incremental selection heuristic to provide enough detection probability. Xu *et al.* [4] discussed different heuristics for prediction and wake-up mechanisms.

To consider the uncertainty in estimation and tracking, the sensor selection problem has been formulated as a partially observable Markov decision process (MDP) [5], [6] or a hierarchical MDP [7]. However, due to the huge state space approximate solutions were obtained instead.

Some researchers focus on linear Gaussian state-space models. Alriksson *et al.* [8] used experiments to show that a distributed approach where communication only takes place between neighbors performed almost as well as the centralized Kalman filter. Shi *et al.* [9], [10] systematically analyzed the tradeoff between the estimation quality and the communication and computation capacities of each node. Joshi and Boyd [11] used convex optimization to approximately solve the sensor measurement selection problem. Ambrosino *et al.* [12] considered the channel capacity constraint. Sinopoli *et al.* [13]

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considered the effect of independent and identically distributed (i.i.d.) packet drop on state estimation, and studied the statistical convergence properties of the estimation error covariance. They showed that there exists a critical value for the arrival rate of the observations, beyond which a transition to an unbounded state error covariance occurs. Huang and Dey [14] and Xie and Xie [15] considered the effect of Markovian packet drops. Hespanha *et al.* [16] surveyed recent progress in networked control systems.

Savage and La Scala [17] considered the optimal scheduling of scalar Gauss–Markov systems with a terminal cost function. Although both their paper and this technical note focus on Gauss–Markov system with terminal cost, the differences are clear. First, they used a single sensor to measure and track multiple targets, while in this technical note we consider state estimation of a single system through multiple sensors. As a result, the objective functions are different. They minimized the total estimated state error variance of multiple systems at a terminal stage, while we minimize the expectation of the estimated state error covariance of a single system at a terminal stage. Second, a limited total measure budget is considered in [17], and no constraint on the communication is imposed because a single sensor is used. However, in this technical note the constraints are caused by the limited communication power of each sensor and the limited wireless communication bandwidth among the sensors. Third, data packet drops are considered in this technical note as a natural consequence of wireless communications among sensors, but are not considered in [17]. Last but not the least, they considered scalar system but we consider both scalar system and vector system. Their proof technique does not apply to our analysis for the vector system. And our sample-path-based analysis technique for the packet drop case is also new to the literature in this research area to the best of our knowledge.

Li *et al.* [18] considered partial broadcasting of WSNs. They developed a good-estimates-first-broadcast policy to minimize the one-stage estimated error covariance, which is different from the terminal cost considered in this technical note.

Gupta *et al.* [19] considered a stochastic sensor selection algorithm. They provided upper and lower bounds for the expected error covariance for a given random schedule, and developed an algorithm to minimize the upper bound on the expected steady-state performance. This technical note is different from theirs because 1) we consider the terminal cost of finite stages and 2) we consider the expected error covariance directly, not an upper or lower bound, but their work points out interesting future work of this technical note as will be discussed in Section IV-E.

Note that in this technical note an optimal schedule is of interest which determines beforehand which sensors to broadcast at each stage. Such open-loop schedules are easy to implement and do not require much computing capabilities from each sensors. More generally, sensors could be scheduled in a closed-loop way, say based on the difference between the state estimate at the fusion center and the state estimate that could be obtained using full (or partial) sensor information. Feedback policies of this type have been examined in the literature on event-based sampling, say [20] and [21]. Imer and Basar [22] also considered a joint encoder (at the sensor) and decoder (at the fusion center) design problem for Gauss–Markov systems with average cost criteria. These feedback policies are useful when sensors have some computing capabilities.

III. Problem Formulation

Let $\mathcal{B} = \{0, 1\}$. Let $\mathbb{S}_+$ be the set of all n-by-n positive definite matrices. Consider a system evolving as follows:

$$x_{k+1} = Ax_k + w_k \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state at stage k with initial value $x_0 \sim N(\mathbf{0}, \Pi_0)$; $A \in \mathbb{R}^{n\times n}$; $w_k \sim N(\mathbf{0}, Q)$ is the process noise assumed white, $Q \geq 0$ (positive semi-definite). The system state is observed by M sensors

$$y_k^{(i)} = Cx_k + v_k^{(i)}, i = 1, \ldots, M \tag{2}$$

where $y_k^{(i)} \in \mathbb{R}^m$ is the observation; $C \in \mathbb{R}^{m\times n}$; $v_k^{(i)} \sim N(\mathbf{0}, R^{(i)})$ is the observation noise assumed white, $R^{(i)} \geq 0$. Assume x_0, w_k, and v_k are mutually uncorrelated, and $0 < R^{(1)} < R^{(2)} < \cdots < R^{(M)}$. At each time, some sensors are selected to broadcast their local observations $y_k^{(i)}$ back to the fusion center. Let $I_k = (I_k(1), \ldots, I_k(M))^T \in \mathcal{B}^M$ denote such a selection, where $I_k(i) = 1$ (or 0) means sensor i is (not) selected at time k. Let $\mathbf{I}_k = (I_1, \ldots, I_k)$ denote the selection from time 1 to k. Then $\mathbf{I}_N$ represents a partial broadcasting policy, where N is the length of the horizon of interest. A message will reach the fusion center with probability $0 < \lambda \leq 1$. Denote $b_k = (b_k(1), \ldots, b_k(M))^T \in \mathcal{B}^M$, where $b_k(i) = 0$ (or 1) means the packet from sensor i at time k is (not) dropped, and define $\mathbf{b}_N = (b_1, \ldots, b_N)$. Then the observations from sensors in $s(I_k \odot b_k)$ will reach the fusion center at time k, where $I_k \odot b_k = (I_k(1)b_k(1), \ldots, I_k(M)b_k(M))^T$ and $s(I) = \{i | I(i) = 1\}$. The optimal estimate $\hat{x}_k$ using a Kalman filter is [23]

$$\begin{aligned}
\hat{x}_{k|k-1} &= A\hat{x}_{k-1} \\
P_{k|k-1} &= AP_{k-1}A^T + Q \\
P_k^{-1} &= P_{k|k-1}^{-1} + \sum_{i\in s(I_k \odot b_k)} C^T (R^{(i)})^{-1} C \\
K_k &= P_k C^T [(R^{(i)})^{-1}, i \in s(I_k \odot b_k)] \\
\hat{x}_k &= \hat{x}_{k|k-1} + K_k([y_k^{(i)T}, i \in s(I_k \odot b_k)]^T \\
&\quad - [C^T, \ldots, C^T]^T \hat{x}_{k|k-1})
\end{aligned}$$

where the recursion starts from $\hat{x}_0 = \mathbf{0}$ and $P_0 = \Pi_0$. If $s(I_k \odot b_k) = \varnothing$, then we have [13] $\hat{x}_k = A\hat{x}_{k-1}$, $P_k = AP_{k-1}A^T + Q$. To simplify notations, introduce functions $h, \tilde{g}^I, g^I : \mathbb{S}_+ \to \mathbb{S}_+$ as

$$\begin{aligned}
h(X) &= AXA^T + Q \\
\tilde{g}^I(X) &= (X^{-1} + R_I)^{-1}, \ R_I = \sum_{i\in s(I)} C^T (R^{(i)})^{-1} C \\
g^I(X) &= \tilde{g}^I h(X)
\end{aligned}$$

where for functions f_1, f_2, $f_1 f_2(x) \equiv f_1(f_2(x))$. Then we have $P_k = g^{I_k \odot b_k}(P_{k-1})$. The partial broadcasting policy optimization problem can be cast as follows:

$$\begin{aligned}
\min_{\mathbf{I}_N} \quad & E_{\mathbf{b}_N}[P_N(\mathbf{I}_N, \mathbf{b}_N)] \\
\text{s.t.} \quad & \sum_{k=1}^{N} I_k(i) \leq C_i, i = 1, \ldots, M \\
& I_k^T I_k \leq B, k = 1, \ldots, N
\end{aligned} \tag{3}$$

where the objective function is the expected estimated state error covariance at time N; the two groups of constraints correspond to the battery and communication bandwidth constraints, respectively.

IV. Main Results

A. *Scalar System Without Packet Drop*

For scalar system (1) and (2) can be simplified to

$$\begin{aligned}
x_{k+1} &= ax_k + w_k \\
y_k^{(i)} &= x_k + v_k^{(i)}
\end{aligned}$$

where $w_k \sim N(0,q), q \geq 0$ and $v_k^{(i)} \sim N(0, r^{(i)})$. We start from this simple case and focus on the following rule.

The Good-Sensors-Late-Broadcast (GSLB) Rule: Sensors should broadcast as much as possible in the lifetime and as late as possible. Furthermore, if sensor i broadcasts at stage k and not at any future stages, then sensor $j > i$ should not broadcast at any stage $k' > k$.

We will show that

Theorem 1: When $|a| \geq 1$ and $\lambda = 1$, if a policy $\mathbf{I}_N$ violates the GSLB rule, there exists another policy $\mathbf{I}'_N$ that satisfies the GSLB rule and is no worse than $\mathbf{I}_N$, i.e., $P_N(\mathbf{I}'_N) \leq P_N(\mathbf{I}_N)$.

We will prove Theorem 1 through three steps. First, we will show that more broadcastings are always beneficial (Lemma 2). Second, sensors should broadcast as late as possible (Lemma 3). Third, exchanging an early broadcasting of a good sensor with a late broadcasting of a bad sensor is always beneficial (Lemma 4). We now follow the three steps to prove Theorem 1. Define functions $d, \tilde{f}^I, f^I : \mathbb{R} \rightarrow \mathbb{R}$ as

$$d(x) = \frac{x}{(qx+a)^2},$$
$$\tilde{f}^I(x) = x + R_I$$
$$f^I(x) = \tilde{f}^I d(x).$$

Then we have $P_k^{-1} = f^{I_k \odot b_k}(P_{k-1}^{-1})$.

Lemma 1: $f^{I+e_i}(x) > f^I(x), x > 0, I(i) = 0, e_i$ is the vector with only the ith component being 1 and the rest being 0.

Proof: Because $R_{e_i} > 0$, we have $f^{I+e_i}(x) = d(x) + R_{I+e_i} = d(x) + R_I + R_{e_i} = f^I(x) + R_{e_i} > f^I(x)$. ■

Lemma 1 means that $P_k^{-1}(I_k + e_i) > P_k^{-1}(I_k)$, if $I_k(i) = 0$. To show that more broadcastings of sensor i at time k are also beneficial to P_N, we need the following properties.

1) Property 1: If $x_1 > x_2 > 0, d(x_1) > d(x_2)$.

Proof: Note that $d(d(x))/dx = a^2/(qx+a^2)^2 > 0$. ■

2) Property 2: If $x_1 > x_2 > 0, f^I(x_1) > f^I(x_2), \forall I \in \mathcal{B}^M$.

Proof: By definition, $f^I(x) = d(x) + R_I$. From Property 1, we know that $d(x)$ is strictly increasing with respect to (w.r.t.) x. Thus $f^I(x)$ is strictly increasing w.r.t x. This completes the proof. ■

Combining Lemma 1 and Property 1, we can see that $P_N^{-1}(I_k + e_i) > P_N^{-1}(I_k)$, which implies that $P_N(I_k + e_i) < P_N(I_k)$. Then we have

Lemma 2: $P_N(I_k + e_i) < P_N(I_k), I(i) = 0$.

Lemma 2 implies that sensor i should broadcast exactly C_i times by time N. Next we have the following:

3) Property 3: $d(x) + \alpha \geq d(x+\alpha), x > 0, \alpha > 0$.

Proof: By definition, we have

$$d(x) + \alpha - d(x+\alpha) = \frac{x}{qx+a^2} + \alpha - \frac{x+\alpha}{q(x+\alpha)+a^2}. \quad (4)$$

The right-hand side of (4) equals to

$$\frac{\alpha q^2 x^2 + \alpha(q(q\alpha + a^2) + qa^2)x + \alpha a^2(q\alpha + a^2 - 1)}{(qx+a^2)(q(x+\alpha)+a^2)}. \quad (5)$$

Since $|a| \geq 1$, we have $\alpha a^2(q\alpha + a^2 - 1) \geq 0$. So (5) ≥ 0. ■

Lemma 3: $f^{I_2+e_i} f^{I_1}(x) \geq f^{I_2} f^{I_1+e_i}(x), x > 0, I_1(i) = I_2(i) = 0$.

Proof: By definition, we have $f^{I_2+e_i} f^{I_1}(x) - f^{I_2} f^{I_1+e_i}(x) = d(f^{I_1}(x)) + R_{e_i} - d(f^{I_1}(x) + R_{e_i}) \geq 0$, where the last inequality follows from Property 3. ■

Thus postponing the broadcasting always reduces P_N. For the third step of the proof of Theorem 1, we have the following:

4) Property 4: $d(x+\alpha) + \beta \geq d(x+\beta) + \alpha, \alpha < \beta, x > 0$.

Proof: By definition and after some deduction, we have

$$\begin{aligned}&(d(x+\alpha)+\beta) - (d(x+\beta)+\alpha) = \\ &\frac{(\beta-\alpha)(q^2(x+\alpha)(x+\beta) + qa^2(2x+\alpha+\beta) + a^4 - a^2)}{(q(x+\alpha)+a^2)(q(x+\beta)+a^2)} \\ &\leq 0\end{aligned}$$

where the last inequality is due to $|a| \geq 1$ and as a consequence $a^4 - a^2 \geq 0$. ■

Lemma 4: $f^{I_2+e_i} f^{I_1+e_j}(x) \geq f^{I_2+e_j} f^{I_1+e_i}(x), x > 0, i < j, I_1(i) = I_1(j) = I_2(i) = I_2(j) = 0$.

Proof: By definition, we have

$$f^{I_2+e_i} f^{I_1+e_j}(x) = d(d(x) + R_{I_1} + R_{e_j}) + R_{I_2} + R_{e_i}$$
$$f^{I_2+e_j} f^{I_1+e_i}(x) = d(d(x) + R_{I_1} + R_{e_i}) + R_{I_2} + R_{e_j}.$$

Note that $R_{e_i} > R_{e_j}$. Following Property 4, we have $f^{I_2+e_i} f^{I_1+e_j}(x) - f^{I_2+e_j} f^{I_1+e_i}(x) \geq 0$. ■

Lemma 4 implies that exchanging an early broadcasting of a good sensor with a late broadcasting of a bad sensor is always beneficial. Now we can prove Theorem 1.

Proof (of Theorem 1): If a policy $\mathbf{I}_N$ violates the GSLB rule, we can first add broadcasting of sensor i if it has not broadcast C_i times by time N. Second, we postpone the broadcastings of all the sensors as late as possible, while keeping the relative order among the broadcastings not changed. Third, starting from the earliest broadcasting of sensor 1, if any other sensor broadcasts later, exchange the two broadcastings. Repeat the exchange operation for sensors $2, \ldots, M$. When this is completed, we obtain a policy $\mathbf{I}'_N$, which satisfies the GSLB rule. Lemmas 2-4 ensure that the above modifications of $\mathbf{I}_N$ do not increase P_N, i.e., $P_N(\mathbf{I}'_N) \leq P_N(\mathbf{I}_N)$. ■

Note that following the modifications in the proof of Theorem 1, the resultant policy is optimal. This leads to Algorithm 1 that efficiently constructs the optimal policy after exactly $\min\{NB, \sum_{i=1}^M C_i\}$ steps.

In the special case when $B = 1$, each time only one sensor can broadcast. Then the optimal policy selects sensor $\bar{s}+1$ in the first $N - \sum_{i=1}^{\bar{s}} C_i$ stages, selects sensor $\bar{s}$ in the following $C_{\bar{s}}$ stages, and continues the selection similarly in the rest of the stages, where $\bar{s}$ is determined by $\sum_{i=1}^{\bar{s}} C_i \leq N$ and $\sum_{i=1}^{\bar{s}+1} C_i > N$.

Algorithm 1 Construct the Optimal Policy When $\lambda = 1$

$n_i = 0, i = 1, \ldots, M$. $\mathbf{I}_N = \mathbf{0}$.
for $i = 1$ to M **do**
  for $k = N$ down to 1 **do**
    if $(n_i < C_i)$ and $(\sum_{l=1}^{i-1} I_k(l) < B)$ **then**
      $I_k(i) = 1$. $n_i = n_i + 1$.
    end if
  end for
end for
Output $\mathbf{I}_N$.

B. Vector System Without Packet Drop

For vector system we follow three similar steps to show the optimality of the GSLB rule under the following assumption.

Assumption 1: $Q^{-1} + C^T(R^{(j)})^{-1}C < C^T(R^{(i)})^{-1}C, \forall i < j$.

Note that the left-hand side of the above inequality quantifies the information contained in the state estimation if the state at the last stage is accurately known and the observation of sensor j at this stage is received. The right-hand side quantifies the information contained in the state estimation if only the observation of sensor i at this stage is received. So Assumption 1 implies that sensor i is significantly better than sensor j. One can obtain more accurate state estimation simply using the observation of sensor i than jointly using the system state at the last stage and the observation of sensor j at this stage. Under this assumption, we have the following results.

Lemma 5: $g^{I+e_i}(X) < g^I(X), \forall X \geq 0, I(i) = 0.$

Proof: Note that

$$\begin{aligned} & g^{I+e_i}(X) < g^I(X) \\ & \Leftrightarrow \tilde{g}^{I+e_i}(X) < \tilde{g}^I(X) \\ & \Leftrightarrow \left(X^{-1} + \sum_{j\in s(I+e_i)} C^T (R^{(j)})^{-1} C\right)^{-1} \\ & \quad < \left(X^{-1} + \sum_{j\in s(I)} C^T (R^{(j)})^{-1} C\right)^{-1} \\ & \Leftrightarrow X^{-1} + \sum_{j\in s(I+e_i)} C^T (R^{(j)})^{-1} C \\ & \quad > X^{-1} + \sum_{j\in s(I)} C^T (R^{(j)})^{-1} C \\ & \Leftrightarrow C^T (R^{(i)})^{-1} C > 0 \end{aligned}$$

where the last line holds from Assumption 1. ■

Lemma 6: Under Assumption 1, $g^{I_2+e_i} g^{I_1+e_j}(X) < g^{I_2+e_j} g^{I_1+e_i}(X), \forall X \geq 0, I_1(i) = I_1(j) = I_2(i) = I_2(j) = 0.$

Proof: We have

$$\begin{aligned} & g^{I_2+e_i} g^{I_1+e_j}(X) < g^{I_2+e_j} g^{I_1+e_i}(X) \\ & \Leftrightarrow \tilde{g}^{I_2+e_i} h \tilde{g}^{I_1+e_j}(X) < \tilde{g}^{I_2+e_j} h \tilde{g}^{I_1+e_i}(X) \\ & \Leftrightarrow \left[\left(h\left(g^{I_1+e_j}(X)\right)\right)^{-1} + \sum_{l\in s(I_2+e_i)} C^T (R^{(l)})^{-1} C\right]^{-1} \\ & \quad < \left[\left(h\left(g^{I_1+e_i}(X)\right)\right)^{-1} + \sum_{l\in s(I_2+e_j)} C^T (R^{(l)})^{-1} C\right]^{-1} \\ & \Leftrightarrow \left(h\left(g^{I_1+e_j}(X)\right)\right)^{-1} + \sum_{l\in s(I_2+e_i)} C^T (R^{(l)})^{-1} C \\ & \quad > \left(h\left(g^{I_1+e_i}(X)\right)\right)^{-1} + \sum_{l\in s(I_2+e_j)} C^T (R^{(l)})^{-1} C \\ & \Leftrightarrow \left(h\left(g^{I_1+e_j}(X)\right)\right)^{-1} + C^T (R^{(i)})^{-1} C \\ & \quad > \left(h\left(g^{I_1+e_i}(X)\right)\right)^{-1} + C^T (R^{(j)})^{-1} C \\ & \Leftarrow C^T (R^{(i)})^{-1} C > \left(h\left(g^{I_1+e_i}(X)\right)\right)^{-1} + C^T (R^{(j)})^{-1} C. \end{aligned} \tag{6}$$

Note that under Assumption 1, we have

$$\begin{aligned} C^T (R^{(i)})^{-1} C &> Q^{-1} + C^T (R^{(j)})^{-1} C \\ &\geq \left(h\left(g^{I_1+e_i}(X)\right)\right)^{-1} + C^T (R^{(j)})^{-1} C \end{aligned} \tag{7}$$

where the last inequality is due to $h(X) \geq h(0) = Q, \forall X \geq 0$. Combining (6) and (7), this completes the proof. ■

Lemma 7: $g^{I_2+e_i} g^{I_1}(X) < g^{I_2} g^{I_1+e_i}(X), X \geq 0, I_1(i) = I_2(i) = 0.$

Proof: Consider a fictitious sensor j with $R^{(j)} \gg R^{(i)}$. Lemma 7 is a natural corollary of Lemma 6 for $R^{(j)} \to \infty$. ■

Combining Lemmas 5–7 we have the following:

Theorem 2: Under Assumption 1 and when $\lambda = 1$, if a policy $\mathbf{I}_N$ violates the GSLB rule, there exists another policy $\mathbf{I}'_N$ that satisfies the GSLB rule and is no-worse than $\mathbf{I}_N$.

Note that Algorithm 1 also outputs the optimal policy for vector system. Theorems 1 and 2 show the optimality of the GSLB rule under different conditions besides $\lambda = 1$. Theorem 1 uses $|a| \geq 1$ and Theorem 2 uses Assumption 1. These two sufficient conditions do not imply one the other. It remains open whether Theorems 1 and 2 hold under other conditions.

C. Packet Drop

When packet drops with probability, i.e., $\lambda < 1$, the analysis is complicated because P_N is random. We focus on scalar system and take a sample path view to compare the performances of different policies on each (pair) of sample paths. First, we show that more broadcastings are beneficial.

Lemma 8: $E_b[f^{(I+e_i)\odot b}(x)] > E_b[f^{I\odot b}(x)], x > 0, I(i) = 0.$

Proof: We have

$$\begin{aligned} E_b[f^{(I+e_i)\odot b}(x)] &= \frac{1}{|\mathcal{B}^M|} \sum_{b\in\mathcal{B}^M} f^{(I+e_i)\odot b}(x) \\ E_b[f^{I\odot b}(x)] &= \frac{1}{|\mathcal{B}^M|} \sum_{b\in\mathcal{B}^M} f^{I\odot b}(x). \end{aligned}$$

Note that $(I + e_i) \odot b = I \odot b + e_i \odot b$. Then from Lemma 1, we have $f^{(I+e_i)\odot b}(x) > f^{I\odot b}(x)$, if $b(i) = 1$; $f^{(I+e_i)\odot b}(x) = f^{I\odot b}(x)$, if $b(i) = 0$. Since $\Pr\{b(i) = 1\} = \lambda > 0$, we then have $\sum_{b\in\mathcal{B}^M} f^{(I+e_i)\odot b}(x) > \sum_{b\in\mathcal{B}^M} f^{I\odot b}(x)$. Hence, $E_b[f^{(I+e_i)\odot b}(x)] > E_b[f^{I\odot b}(x)]$. ■

Lemma 9: $E_b[g^{(I+e_i)\odot b}(x)] < E_b[g^{I\odot b}(x)], x > 0, I(i) = 0.$

Proof: Note that $g^{(I+e_i)\odot b}(x) = (f^{(I+e_i)\odot b}(1/x))^{-1}$ and $g^{I\odot b}(x) = (f^{I\odot b}(1/x))^{-1}$. Lemma 8 shows that $f^{(I+e_i)\odot b}(1/x) \geq f^{I\odot b}(1/x)$, where the inequality is strict if $b(i) = 1$. We have

$$g^{(I+e_i)\odot b}(x) \leq g^{I\odot b}(x). \tag{8}$$

Then $E_b[g^{(I+e_i)\odot b}(x)] < E_b[g^{I\odot b}(x)]$ as $\Pr\{b(i) = 1\} = \lambda > 0$. ■

Lemma 9 implies that the additional broadcasting of a sensor at time k reduces $E[P_k]$. It turns out that $E[P_N]$ is also reduced, but we need the following monotonicity of $g^I(x)$.

1) Property 5: If $x_1 > x_2 > 0$, then $g^I(x_1) > g^I(x_2), \forall I$.

Proof: Note that $g^I(x_1) = (f^I(1/x_1))^{-1}$ and $g^I(x_2) = (f^I(1/x_2))^{-1}$. Property 1 shows that $f^I(1/x_1) < f^I(1/x_2)$. Thus, we have $g^I(x_1) > g^I(x_2)$. ■

Theorem 3: $E_{\mathbf{b}_N}[g^{I_N\odot b_N} \cdots g^{I_{k+1}\odot b_{k+1}} g^{(I_k+e_i)\odot b_k}(x)] < E_{\mathbf{b}_N}[g^{I_N\odot b_N} \cdots g^{I_{k+1}\odot b_{k+1}} g^{I_k\odot b_k}(x)], x > 0, I_k(i) = 0.$

Proof: Combining (8) and Property 5 we have

$$\begin{aligned} & g^{I_N\odot b_N} \cdots g^{I_{k+1}\odot b_{k+1}} g^{(I_k+e_i)\odot b_i}(x) \\ & \qquad \leq g^{I_N\odot b_N} \cdots g^{I_{k+1}\odot b_{k+1}} g^{I_k\odot b_k}(x). \end{aligned}$$

Then $E_{\mathbf{b}_N}[P_N(I_k + e_i)] < E_{\mathbf{b}_N}[P_N(I_k)]$ as $\Pr\{b_k(i) = 1\} > 0$. ■

Second, we show that postponing the broadcastings of sensors are always beneficial.

Theorem 4: $E_{\mathbf{b}_N}[g^{I_N\odot b_N} \cdots g^{(I_{k+1}+e_i)\odot b_{k+1}} g^{I_k\odot b_k}(x)] \leq E_{\mathbf{b}_N}[g^{I_N\odot b_N} \cdots g^{I_{k+1}\odot b_{k+1}} g^{(I_k+e_i)\odot b_k}(x)], \ x > 0, \ I_k(i) = I_{k+1}(i) = 0.$

Proof: The idea is to construct a different sample path $\mathbf{b}'_N$. The differences between $\mathbf{b}_N$ and $\mathbf{b}'_N$ are that $b'_k(i) = b_{k+1}(i), b'_{k+1}(i) =$

$b_k(i)$. In other words, in $\mathbf{b}'_N$, we exchange the channel randomness at time k and $k+1$ of sensor i in $\mathbf{b}_N$. Note that

$$\begin{aligned} &g^{(I_{k+1}+e_i)\odot b'_{k+1}} g^{I_k\odot b'_k}(x) \\ &\quad= \left(f^{(I_k+e_i)\odot b'_{k+1}} f^{I_k\odot b'_k}\left(\frac{1}{x}\right)\right)^{-1}, \\ &g^{I_{k+1}\odot b_{k+1}} g^{(I_k+e_i)\odot b_k}(x) \\ &\quad= \left(f^{I_{k+1}\odot b_{k+1}} f^{(I_k+e_i)\odot b_k}\left(\frac{1}{x}\right)\right)^{-1}. \end{aligned}$$

Case 1: $b_k(i) = 1$. We have $(I_{k+1}+e_i)\odot b'_{k+1} = I_{k+1}\odot b_{k+1}+e_i, (I_k+e_i)\odot b_k = I_k\odot b_k + e_i$. Then from Lemma 3, we have $f^{(I_{k+1}+e_i)\odot b'_{k+1}} f^{I_k\odot b'_k}(1/x) \geq f^{I_{k+1}\odot b_{k+1}} f^{(I_k+e_i)\odot b_k}(1/x)$.

Case 2: $b_k(i) = 0$. We have $(I_{k+1}+e_i)\odot b'_{k+1} = I_{k+1}\odot b_{k+1}, (I_k+e_i)\odot b_k = I_k\odot b_k$. Then we have $f^{(I_{k+1}+e_i)\odot b'_{k+1}} f^{I_k\odot b'_k}(1/x) = f^{I_{k+1}\odot b_{k+1}} f^{(I_k+e_i)\odot b_k}(1/x)$.

Combining the above two cases, we have $f^{(I_{k+1}+e_i)\odot b'_{k+1}} f^{I_k\odot b'_k}(1/x) \geq f^{I_{k+1}\odot b_{k+1}} f^{(I_k+e_i)\odot b_k}(1/x)$. Combining the above equations together, we have

$$g^{(I_{k+1}+e_i)\odot b'_{k+1}} g^{I_k\odot b'_k}(x) \leq g^{I_{k+1}\odot b_{k+1}} g^{(I_k+e_i)\odot b_k}(x).$$

Note that for every $\mathbf{b}_N$, such a $\mathbf{b}'_N$ can be constructed as above. And all such $\mathbf{b}'_N$'s are different. Since $\Pr\{b_k(i)=1\} = \lambda > 0$, combining with Property 5, we have $E_{\mathbf{b}'_N}[g^{I_N\odot b'_N}\cdots g^{(I_{k+1}+e_i)\odot b'_{k+1}} g^{I_k\odot b_k}(x)] \leq E_{\mathbf{b}_N}[g^{I_N\odot b_N}\cdots g^{I_{k+1}\odot b_{k+1}} g^{(I_k+e_i)\odot b_k}(x)]$. ■

For the third step towards showing the good performance of the GSLB rule, we need to show that exchanging an early broadcasting of a good sensor with a late broadcasting of a bad sensor is beneficial. We have the following:

Theorem 5: $E[f^{(I_{k+1}+e_i)\odot b_{k+1}} f^{(I_k+e_j)\odot b_k}(x)] \geq E[f^{(I_{k+1}+e_j)\odot b_{k+1}} f^{(I_k+e_i)\odot b_k}(x)]$, $x > 0$, $i < j$, $I_k(i) = I_k(j) = I_{k+1}(i) = I_{k+1}(j) = 0$.

Proof: For any given $b_k, b_{k+1} \in \mathcal{B}^M$, construct b'_k and b'_{k+1}) s.t. $b'_k(i) = b_{k+1}(i), b'_k(j) = b_{k+1}(j), b'_{k+1}(i) = b_k(i), b'_{k+1}(j) = b_k(j), b'_t(l) = b_t(l), l \neq i,j, t = k, k+1$. In other words, in b'_k and b'_{k+1} the channel randomness of sensors i and j are exactly the randomness in b_{k+1} and b_k, respectively. It is easy to verify that when $(b_k(i), b_{k+1}(j)) = (0,0)$, we have

$$f^{(I_{k+1}+e_i)\odot b'_{k+1}} f^{(I_k+e_j)\odot b'_k}(x) = f^{(I_{k+1}+e_j)\odot b_{k+1}} f^{(I_k+e_i)\odot b_k}(x).$$

When $(b_k(i), b_{k+1}(j)) = (1,1)$, we have

$$f^{(I_{k+1}+e_i)\odot b'_{k+1}} f^{(I_k+e_j)\odot b'_k}(x) \geq f^{(I_{k+1}+e_j)\odot b_{k+1}} f^{(I_k+e_i)\odot b_k}(x).$$

When $(b_k(i), b_{k+1}(j)) \in \{(0,1),(1,0)\}$, we have

$$\begin{aligned} &\sum_{(b_k(i),b_{k+1}(j))\in\{(0,1),(1,0)\}} f^{(I_{k+1}+e_i)\odot b'_{k+1}} f^{(I_k+e_j)\odot b'_k}(x) \\ &- \sum_{(b_k(i),b_{k+1}(j))\in\{(0,1),(1,0)\}} f^{(I_{k+1}+e_j)\odot b_{k+1}} f^{(I_k+e_i)\odot b_k}(x) \\ &\geq 0. \end{aligned}$$

Note that $\Pr\{b_k(i) = 0, b_{k+1}(j) = 1\} = \Pr\{b_k(i) = 1, b_{k+1}(j) = 0\} = \lambda(1-\lambda)$. Combining the three cases, we have $E[f^{(I_{k+1}+e_i)\odot b_{k+1}} f^{(I_k+e_j)\odot b_k}(x)] \geq E[f^{(I_{k+1}+e_j)\odot b_{k+1}} f^{(I_k+e_i)\odot b_k}(x)]$. ■

Note that Theorem 5 means $E[P_{k+1}^{-1}(\mathbf{I}_N)] \leq E[P_{k+1}^{-1}(\mathbf{I}'_N)]$. One may naturally wish to show that $E[P_{k+1}(\mathbf{I}_N)] \geq E[P_{k+1}(\mathbf{I}'_N)]$. However, the following counter example shows that this is not true for some cases. Consider two sensors with $c = 1, r^{(1)} = 1, r^{(2)} = 2, N = 2, C_1 = C_2 = 1$. Consider two policies $\mathbf{I}_N$ and $\mathbf{I}'_N$, where $s(\mathbf{I}_N) = [1,2]$ and $s(\mathbf{I}'_N) = [2,1]$. In other words $\mathbf{I}_N$ violates the GSLB rule and $\mathbf{I}'_N$ satisfies the rule. When packets drop with a positive probability, we have

$$\begin{aligned} E\left[P_N(\mathbf{I}'_N)\right] &= (1-\lambda)^2 h^2(P_0) + \lambda^2 g^{e_1} g^{e_2}(P_0) \\ &\quad + \lambda(1-\lambda)\left[g^{e_1}h(P_0) + hg^{e_2}(P_0)\right] \\ E\left[P_N(\mathbf{I}_N)\right] &= (1-\lambda)^2 h^2(P_0) + \lambda^2 g^{e_2} g^{e_1}(P_0) \\ &\quad + \lambda(1-\lambda)\left[g^{e_2}h(P_0) + hg^{e_1}(P_0)\right]. \end{aligned}$$

Therefore, we have

$$E\left[P_N(\mathbf{I}'_N)\right] - E\left[P_N(\mathbf{I}_N)\right] = \left(F_1(P_0) - F_2(P_0)\right)\lambda^2 + F_2(P_0)\lambda \tag{9}$$

where

$$\begin{aligned} F_1(P_0) &= g^{e_1} g^{e_2}(P_0) - g^{e_2} g^{e_1}(P_0), \\ F_2(P_0) &= g^{e_1}h(P_0) + hg^{e_2}(P_0) - g^{e_2}h(P_0) - hg^{e_1}(P_0). \end{aligned}$$

Regarding λ as a variable, then there are two roots of $E\left[P_N(\mathbf{I}'_N)\right] - E\left[P_N(\mathbf{I}_N)\right] = 0$ which are $\lambda_1 = 0$ and $\lambda_2 = F_2(P_0)/\left(F_2(P_0) - F_1(P_0)\right)$.

Case 1: $F_2(P_0) > 0$. Then

$$E\left[P_N(\mathbf{I}'_N)\right] - E\left[P_N(\mathbf{I}_N)\right] \begin{cases} > 0, & 0 < \lambda < \lambda_2 \\ = 0, & \lambda = \lambda_2 \\ < 0, & \lambda_2 < \lambda \leq 1. \end{cases}$$

Case 2: $F_2(P_0) \leq 0$. $E\left[P_N(\mathbf{I}'_N)\right] - E\left[P_N(\mathbf{I}_N)\right] < 0, 0 < \lambda \leq 1$.

For example, when $a = 2, P_0 = 1, q = 1$, $F_2(P_0) > 0$ and

$$E\left[P_N(\mathbf{I}'_N)\right] - E\left[P_N(\mathbf{I}_N)\right] \begin{cases} > 0, & 0 < \lambda < 0.7519 \\ = 0, & \lambda = 0.7519 \\ < 0, & 0.7519 < \lambda \leq 1. \end{cases}$$

This means that when packets arrive with high probability ($\lambda > \lambda_2$) the GSLB rule ($\mathbf{I}'_N$) performs optimally, but when packets arrive with low probability ($0 < \lambda < \lambda_2$) the GSLB rule does not perform optimally. In the following, we present another example, in which the GSLB rule performs optimally for all the packet-arrival rates. When $a = 1, P_0 = 1, q = 1$, we have $F_2(P_0) < 0$ and $E\left[P_N(\mathbf{I}'_N)\right] - E\left[P_N(\mathbf{I}_N)\right] < 0, 0 < \lambda \leq 1$.

Note that Theorem 5 shows that exchanging an early broadcasting of a good sensor with a late broadcasting of a bad sensor improves the amount of information contained in the state estimation. This implies that the GSLB rule may give good performance, if not optimal. We will use numerical results to show this in the next subsection.

D. Simulation Results

Consider a scalar system with two sensors, where $a = 1, c = 1, q = 1, r^{(1)} = 1, r^{(2)} = 2, \Pi_0 = 1, N = 10, C_1 = C_2 = 5, B = 1$. Compare the following policies.

- GSLB rule ($\mathbf{I}_{\mathrm{GSLB}}$).
- The optimal policy ($\mathbf{I}^*_N$), which is obtained through enumeration.
- Round-robin policy ($\mathbf{I}_{\mathrm{RR1}}$ and $\mathbf{I}_{\mathrm{RR2}}$), which iteratively picks sensor 1 and 2. $\mathbf{I}_{\mathrm{RR1}}$ and $\mathbf{I}_{\mathrm{RR2}}$ pick sensor 1 and 2 first, respectively.

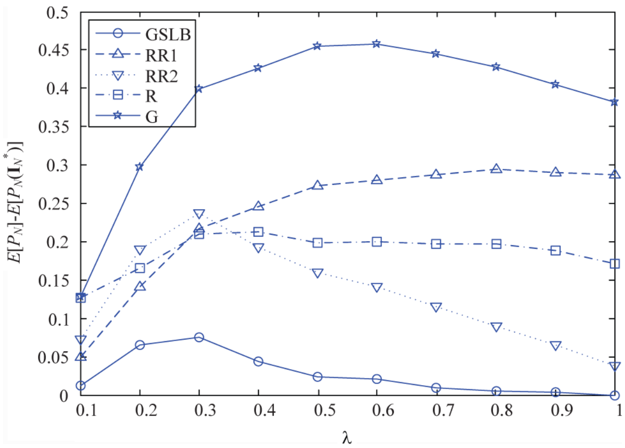

Fig. 1. $E[P_N] - E[P_N(\mathbf{I}_N^*)]$ for different policies and packet-arrival rates λ's.

- Random selection ($\mathbf{I}_{\rm R}$), which randomly picks a feasible schedule with equal probability.
- Greedy policy ($\mathbf{I}_{\rm G}$), which minimizes $E_k[P_k]$ for each k. Thus $\mathbf{I}_{\rm G}$ picks sensor 1 for stage $1 \sim 5$ and picks sensor 2 for the rest of the stages.

For $\lambda = 0.1, 0.2, \ldots, 1$, we estimate $E[P_N]$ of each policy by 10 000 iterations and show $E[P_N] - E[P_N(\mathbf{I}_N^*)]$ in Fig. 1. We can see that the GSLB rule performs optimally when $\lambda = 1$. In all the other cases, the GSLB rule performs slightly worse than the optimal policy but outperforms all the other policies.

E. Discussion

We discuss potential extensions of the GSLB rule to other cases. First, different objective functions. The optimality of the GSLB rule depends on the objective function in (3), which only considers the terminal error covariance. A more general objective function is

$$\sum_{k=1}^{N} \omega_k E_{\mathbf{b}_N}[P_k(\mathbf{I}_N, \mathbf{b}_N)], \omega_k \geq 0, k = 1, \ldots, N.$$

Unfortunately, the GSLB rule in general is not optimal for this objective function. To see this, consider a two-stage problem with only two sensors. Suppose only one sensor can broadcast at each time and each sensor can broadcast only once. Let $\omega_1 = 1$ and $\omega_2 = 0$. Then it is clear that selecting sensor 1 at stage 1 and sensor 2 at stage 2 is better than otherwise. Thus, the GSLB rule is not optimal in this case. Next, let $\omega_1 \gg \omega_2 > 0$. It is clear that the GSLB rule is not optimal in this case either. These two examples imply the following. Good sensors improve the estimation accuracy faster than bad sensors. Due to the limited battery, good sensors cannot broadcast at all stages. If the terminal error covariance is of interest, good sensors should broadcast as late as possible. Therefore, the GSLB rule holds. If the error covariances at early stages are of interest, good sensors should broadcast as early as possible. When a weighted sum of the error covariances at all the stages is of interest, the schedule for the broadcastings of good sensors becomes less clear and depends on the weights ω_k's.

Second, nonidentical channels. In some applications the broadcastings from sensors have different dropping rates due to the different geographic locations of the sensors. Then the GSLB rule is not optimal in general. To see this, consider two sensors. Each can broadcast only once in their lifetime. The limited bandwidth allows only one sensor to broadcast at each stage. Sensors 1 and 2 use different communication channels, with arrival rates λ_1 and λ_2, respectively. Consider two policies $\mathbf{I}_N$ and $\mathbf{I}'_N$, where $s(\mathbf{I}_N) = [1, 2]$ and $s(\mathbf{I}'_N) = [2, 1]$. In other words $\mathbf{I}_N$ violates the GSLB rule and $\mathbf{I}'_N$ satisfies the rule. We have

$$\begin{aligned}
E[P_N(\mathbf{I}_N)] &= (1-\lambda_1)(1-\lambda_2)h^2(P_0) \\
&\quad + (1-\lambda_1)\lambda_2 g^{e_2} h(P_0) \\
&\quad + \lambda_1(1-\lambda_2) h g^{e_1}(P_0) + \lambda_1\lambda_2 g^{e_2} g^{e_1}(P_0) \\
E[P_N(\mathbf{I}'_N)] &= (1-\lambda_1)(1-\lambda_2)h^2(P_0) \\
&\quad + (1-\lambda_2)\lambda_1 g^{e_1} h(P_0) \\
&\quad + \lambda_2(1-\lambda_1) h g^{e_2}(P_0) + \lambda_1\lambda_2 g^{e_1} g^{e_2}(P_0).
\end{aligned}$$

Consider the special case where $\lambda_2 = 0.5$. Then we have

$$\begin{aligned}
&E[P_N(\mathbf{I}'_N)] - E[P_N(\mathbf{I}_N)] \\
&\quad = 0.5\lambda_1 (F_3(P_0) - F_4(P_0)) + 0.5 F_4(P_0)
\end{aligned}$$

where

$$\begin{aligned}
F_3(P_0) &= g^{e_1} h(P_0) - h g^{e_1}(P_0) + g^{e_1} g^{e_2}(P_0) - g^{e_2} g^{e_1}(P_0) \\
F_4(P_0) &= h g^{e_2}(P_0) - g^{e_2} h(P_0).
\end{aligned}$$

Regarding λ_1 as a variable, then the root of $E[P_N(\mathbf{I}'_N)] - E[P_N(\mathbf{I}_N)] = 0$ is $\lambda_1 = \lambda^* = F_4(P_0)/(F_4(P_0) - F_3(P_0))$. Then we have

$$E[P_N(\mathbf{I}'_N)] - E[P_N(\mathbf{I}_N)] \begin{cases} > 0, & 0 < \lambda_1 < \lambda^* \\ = 0, & \lambda_1 = \lambda^* \\ < 0, & \lambda^* < \lambda_1 \leq 1. \end{cases} \tag{10}$$

This means when λ_1 is large, the GSLB rule is still optimal, but when λ_1 is small, the GSLB rule is not optimal.

Third, the selection of channels. In some other applications, sensors may choose which channel to use. Then a partial broadcasting policy should specify which sensors broadcast using which channels at each stage. Denote such a policy by $\mathbf{H}_N = (H_1, \ldots, H_N)$, where $H_k(i) \in \{1, \ldots, c\}$ specifies which channel is used by sensor i at stage k, or sensor i is not selected at stage k if $H_k(i) = 0$. An interesting question is whether good sensors should always choose good channels. We start from a simple example. Suppose there are two sensors with $r^{(1)} < r^{(2)}$ and there are two channels with packet-arrival rates $\lambda_1 > \lambda_2$. Consider two policies $H = (2, 1)^T$ and $H' = (1, 2)^T$. It is easy to verify that

$$E[P(H')] - E[P(H)] = [g^{e_1}(P_0) - g^{e_2}(P_0)](\lambda_1 - \lambda_2) < 0. \tag{11}$$

In other words, assigning good channel to good sensor is beneficial in this example. In general, if a policy $\mathbf{H}_N$ assigns a good channel to a bad sensor and assigns a bad channel to a good sensor at stage k, then one can construct $\mathbf{H}'_N$ that is almost the same as $\mathbf{H}_N$ except that at stage k the channel assignments for that two sensors are exchanged. Then one can follow the above idea to show that $E[P_N(\mathbf{H}'_N)] \leq E[P_N(\mathbf{H}_N)]$. The proof is omitted due to space limit.

Note that in the above more general cases we may consider an approximate problem of (3) that minimizes an upper bound of $E[P_N]$. The bounds in [19] would be very useful for such analysis. This would be an interesting future research topic.

V. Conclusion

In this technical note, we consider the discrete-time Kalman filtering of a linear time-invariant system using WSNs, where each sensor has limited communication budget and the WSN has a limited wireless communication bandwidth. First, when there is no packet drop, the good-sensor-late-broadcast (GSLB) rule is shown to perform optimally. An efficient algorithm is developed to construct this optimal policy. Second, when there is a positive probability of packet drop, we show that the GSLB rule improves $E[P_k^{-1}]$, and construct a counter example to show that the GSLB rule does not improve $E[P_k]$ in some cases. The combination of the two results suggests that the GSLB rule might have good performance, if not optimal. Third, the performance of the GSLB rule is compared with other policies using simulations. The results show that GSLB performs well if not optimal under all the packet dropping rates. Discussions show that the GSLB rule may not be optimal if a different objective function or nonidentical channels are considered. The important future research topics include relaxing Assumption 1 in Theorem 2 for the vector case, considering random delay in the wireless communication, and theoretically quantifying the performance loss of the GSLB rule when it is not optimal.

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Slow-Fast Controller Decomposition Bumpless Transfer for Adaptive Switching Control

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***Abstract*—A bumpless transfer technique with slow-fast controller decomposition is introduced. This bumpless transfer technique designated for adaptive switching control has complemented a simple bumpless transfer method that only ensures continuity on control signal using state-space model of controllers. Simulations demonstrate comparative advantages over a continuity assuring bumpless transfer method.**

***Index Terms*—Adaptive switching control, bumpless transfer, controller state reset, slow-fast decomposition.**

I. INTRODUCTION

Controller switching is frequently observed in various schemes of feedback control system. A switching from manual control to automatic control is one of the typical examples. A switching among multiple linearized controllers is also a switching strategy in nonlinear control [1]. One remarkable expansion of controller switching schemes is adaptive switching control. In adaptive switching control system described as in Fig. 1, controller output signal mismatch at switching instants can lead to discontinuities or abrupt changes called 'bumps' in controller output u. These bumpy transients are not desired in many cases. For example, a passenger aircraft that equips switching controller for intelligent adaptive control requires to avoiding bumpy transient. If controller switched and bumpy transient causes sudden change of aircraft's attitude, passengers would be scared and feel unstable.

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