Technical communique

Sensor scheduling over a packet-delaying network[☆]

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ABSTRACT

We consider sensor scheduling for state estimation of a scalar system over a packet-delaying network. The current measurement data can be sent over a delay-free channel if the sensor uses larger communication energy; the data will be delayed for one time step if the sensor uses less communication energy. We consider a cost function consisting of a weighted average estimation error and a weighted terminal estimation error and explicitly construct optimal power schedules to minimize this cost function subject to communication energy constraint. Simulations are provided to demonstrate the key ideas of the paper.

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1. Introduction

Networked control systems have gained much interest in the past decade (Hespanha, Naghshtabrizi, & Xu, 2007). In many networked control applications, available resources such as the communication energy is limited which brings new issues to networked controller and estimator design. In this paper, we consider state estimation over a network. The current sensor measurement data can be sent over a delay-free channel if the sensor uses more communication energy, and the data will be delayed for one time step if the sensor uses less communication energy.

Before we state the main contribution of this paper, we briefly go over some related works from literature. Quevedo and Ahlén (2008) studied state estimation using wireless sensor networks over fading channels. The packet loss probabilities were partially controlled by the transmission power levels used by the sensors. A predictive controller was developed which trades off sensor energy expenditure versus state estimation accuracy. Savage and Scala (2009) considered the problem of optimal measurement scheduling for scalar systems that minimizes the terminal error. Zhang, Basin, and Skliar (2007) considered optimal

state estimation with continuous, multirate, randomly sampled, and delayed measurements. Matveev and Savkin (2006, 2009) considered optimal state estimation in networked systems with asynchronous communication channels and switched sensors. More related topics and results can be found from the references in the aforementioned existing works.

One main difference between the current work and most of the above works is that we focus on the *design* aspect, i.e., the sensor decides whether to use the perfect channel using large communication energy or to use the packet-delaying channel using small communication energy, under the constraint of a total un-delayed transmission, while they focused on how to optimally estimate the system state under arbitrary data delay patterns. Another difference is that the cost function considered in this paper is rather general which consists of a weighted average error and terminal error. The weight can be chosen appropriately to reflect the requirement of different applications. The main contribution of this paper is an explicit construction of optimal sensor communication power schedules that minimize the cost function subject to the communication energy constraint.

The rest of the paper is organized as follows. In Section 2, we give the mathematical framework of the considered problem. The optimal estimation procedure is introduced in Section 3. The optimal schedules are then introduced in Section 4. Some concluding remarks are provided in the end.

Notations. $\mathbb{Z}$ is the set of non-negative integers. $\mathbb{N}$ is the set of natural numbers. $k \in \mathbb{Z}$ is the time index. $\mathbb{R}$ is the set of real numbers. $\mathbb{R}_+$ is the set of non-negative real numbers. For functions $f_1, f_2, f : \mathbb{R}_+ \to \mathbb{R}_+$ and $t \in \mathbb{Z}$, $f^0(x) \triangleq x$ and $f^t \triangleq \underbrace{f \cdots f}_{t\,\text{times}}(x)$.

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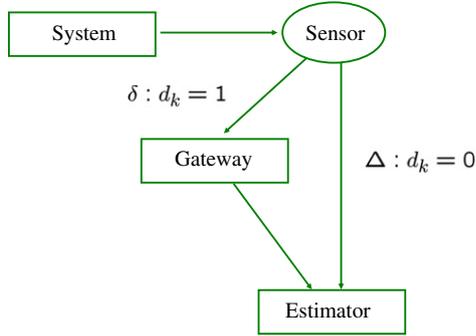
Fig. 1. Example of a packet-delaying network.

2. Problem setup

Consider the following scalar system

$$x_{k+1} = ax_k + w_k, \tag{1}$$
$$y_k = cx_k + v_k, \tag{2}$$

where $a, c \in \mathbb{R}$ and $a, c \neq 0$, $x_k \in \mathbb{R}$ is the system state at time k, y_k is the sensor measurement at k, w_k's, v_k's are zero-mean Gaussian noises with covariance $q > 0$ and $r > 0$. The initial state x_0 is also zero-mean Gaussian with covariance $\pi_0 > 0$. Assume w_k, v_k and x_0 are mutually uncorrelated for all k.

After y_k is obtained, the sensor sends y_k to a remote estimator which computes $\hat{x}_k$, the optimal estimate of x_k, based on all received data up to time k. A direct wireless communication with long distance in general consumes much more energy than a short one. In fact, the communication energy is roughly proportional to d^n where d is the distance between the sender and receiver and n, typically between 2 and 6, is the path loss component (Goldsmith, 2005). Therefore it is reasonable to assume that larger communication energy leads to larger communication radius and vice verse.

Inspired by this fact, we assume the sensor has two choices to send y_k at each k: if the sensor spends δ energy, y_k will be delayed for one step, e.g., via a gateway node (Fig. 1); and if the sensor spends $\Delta > \delta$ energy, y_k will arrive at the estimator without any delay, e.g., communicate with the estimator directly.

We do not consider packet drops in this paper, which complicate the problem and obtaining closed-form optimal power schedules is challenging. A complete analysis of optimal sensor power schedules involving both packet drops and delays is out of the scope of this paper and will be considered further in the future work.

Let $p_k = \mathbb{E}[(x_k - \hat{x}_k)^2|\text{all data received up to } k]$ be the estimation error covariance at the estimator. Consider the set of all deterministic schedules $\Theta = \{0, 1\}^T$. A schedule $\theta \in \Theta$ is represented as $\{\gamma_k(\theta) : k = 1, \ldots, T\}$ such that if $\gamma_k = 0$, δ energy is used at time k; and if $\gamma_k = 1$, Δ energy will be used. Clearly the estimation error covariance p_k depends on the underlying sensor data schedule θ, thus we write it as $p_k(\theta)$. Let $\alpha \in [0, 1]$ be given. Consider the following problem (Problem 2.1)

$$\min_{\theta \in \Theta} J(\theta) = \alpha \frac{1}{T} \sum_{k=1}^{T} (p_k(\theta)) + (1 - \alpha)(p_T(\theta))$$

$$\text{s.t.} \sum_{k=1}^{T} \gamma_k(\theta) = m.$$

Note that $J(\theta)$ consists of two components: when $\alpha = 0$, $J(\theta)$ is the terminal error covariance; and when $\alpha = 1$, $J(\theta)$ is the average error covariance.

3. Optimal state estimation

Recall that the optimal estimate $\hat{x}_k$ for x_k in (1) given y_k in (2), the previous optimal estimate $\hat{x}_{k-1}$, and the previous error covariance p_{k-1} is computed recursively from the Kalman filter (Anderson & Moore, 1979), which consists of a time update step and a measurement update step. If y_k is not available, then from Shi, Epstein, and Murray (2010), only the time update step is implemented. Define functions h and $g : \mathbb{R}_+ \to \mathbb{R}_+$ as

$$h(x) \triangleq a^2 x + q \quad \text{and} \quad g(x) \triangleq h(x) - \frac{c^2 h(x)^2}{c^2 h(x) + r}.$$

It is clear that g and h satisfy $g(x) < h(x)$, $\forall x \geq 0$. From Shi et al. (2010), the error covariance p_k evolves as $p_k = g(p_{k-1})$ when y_k is available and $p_k = h(p_{k-1})$ otherwise. Following the estimation scheme for delayed measurement in Shi, Xie, and Murray (2009), the estimation procedure at time k consists of the following two steps: first, if y_{k-1} arrives at k, recalculate $\hat{x}_{k-1}$ and p_{k-1} using y_{k-1} using the Kalman filter and go to the next step, and otherwise y_{k-1} must have arrived at time $k - 1$ and do nothing; second, if y_k arrives, calculate $\hat{x}_k$ and p_k using y_k using the Kalman filter, and otherwise calculate $\hat{x}_k$ and p_k only using the time update of the Kalman filter. Under this procedure, for a given schedule θ, one has: first, if $\gamma_k = 1$, then $y_1, \ldots, y_k$ have all arrived at the estimator. Thus the estimator is simply the Kalman filter and

$$p_k = g^k(p_0). \tag{3}$$

Second, if $\gamma_k = 0$, then $y_1, \ldots, y_{k-1}$ have all arrived at the estimator. Thus the estimator is the Kalman filter up to time $k - 1$ and

$$p_k = h(p_{k-1}) = hg^{k-1}(p_0). \tag{4}$$

4. Optimal schedules

In this section, we construct optimal power schedules to Problem 2.1. First consider $\alpha = 0$. For two feasible schedules θ_1 and θ_2, from (3), if $\gamma_T(\theta_1) = 1$ and $\gamma_T(\theta_2) = 1$, then $p_T(\theta_1) = p_T(\theta_2)$. If $\gamma_T(\theta_1) = 1$ and $\gamma_T(\theta_2) = 0$, then from (3) and (4), $p_T(\theta_1) = g^T(p_0) < h(g^{T-1}(p_0)) = p_T(\theta_2)$. Therefore we conclude that when $\alpha = 0$, any feasible schedule θ with $\gamma_T(\theta) = 1$ is an optimal schedule.

Now consider $\alpha = 1$. Define $\bar{p}$ as the unique solution to $\bar{p} = g(\bar{p})$. The next three lemmas, which are straightforward to verify from the definitions of g and h, are essential to derive the optimal power schedule in Theorem 4.4.

Lemma 4.1. *If $p_0 > \bar{p}$ then $g(p_0) < p_0$. And if $0 \leq p_0 < \bar{p}$ then $g(p_0) > p_0$.*

Lemma 4.2. *Let $i \in \mathbb{Z}$ and $j \in \mathbb{N}$. If $p_0 > \bar{p}$, then*

$$g^{i+1}(p_0) - hg^i(p_0) < g^{i+j+1}(p_0) - hg^{i+j}(p_0). \tag{5}$$

And if $p_0 \leq \bar{p}$, then

$$g^{i+1}(p_0) - hg^{i+1}(p_0) \geq g^{i+j+1}(p_0) - hg^{i+j+1}(p_0) \tag{6}$$

with equality iff $p_0 = \bar{p}$.

Lemma 4.3. *If $\alpha = 1$, the cost function $J(\theta)$ for a given schedule θ with $\gamma_{k_i}(\theta) = 1$ $(i = 1, \ldots, m, 1 \leq k_1 < k_2 < \cdots < k_m \leq T)$ is given as follows*

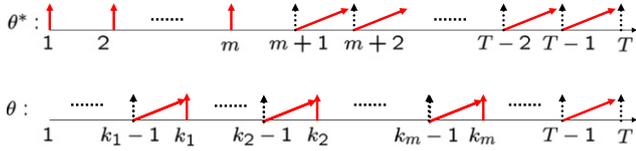$$J(\theta) = \sum_{k=1}^{T} hg^{k-1}(p_0) + \sum_{i=1}^{m} \left(g^{k_i}(p_0) - hg^{k_i-1}(p_0)\right). \tag{7}$$

Fig. 2. $p_0 > \overline{p}$: Optimal schedule θ^* and a general schedule θ.
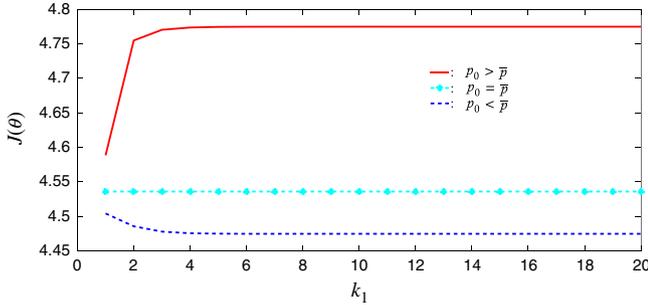

Fig. 3. $\alpha = 1 : J(\theta)$ as a function of θ for different initial conditions.

Theorem 4.4. *When $\alpha = 1$, an optimal schedule θ^* in terms of $\gamma_k(\theta^*)$ is given as follows:*

(1) $p_0 > \overline{p} : \gamma_k(\theta^*) = 1, \ 1 \le k \le m$. *Furthermore θ^* is the unique optimal schedule.*

(2) $p_0 = \overline{p}$: *any feasible schedule is optimal.*

(3) $p_0 < \overline{p} : \gamma_k(\theta^*) = 1, T - m + 1 \le k \le T$. *Furthermore θ^* is the unique optimal schedule.*

Proof. (1) $p_0 > \overline{p}$: Consider a general schedule θ. Fig. 2 shows the schedule θ^* and θ, where k_i is such that $\gamma_{k_i}(\theta) = 1$. One thing we immediately notice is that $k_i \ge i \ \forall \ i = 1, \ldots, m$. Furthermore $\theta = \theta^*$ iff $k_i = i$ for all $i = 1, \ldots, m$. From (5), (7) and using the fact that $k_i \ge i$, we easily arrive at $J(\theta^*) - J(\theta) \le 0$ and the equality holds iff $\theta = \theta^*$. (2) $p_0 = \overline{p}$: It is straightforward to show that any feasible schedule θ has cost $J(\theta) = m\overline{p} + (T - m)h(\overline{p})$. (3) $p_0 < \overline{p}$: Consider a general schedule θ which is the same as in the case $p_0 > \overline{p}$. First note that k_i now satisfies $k_i \le T - m + i, \ i = 1, \ldots, m$. Furthermore $\theta = \theta^*$ iff $k_i = T - m + i$ for all $i = 1, \ldots, m$. Therefore from (6) and (7), we arrive at $J(\theta) - J(\theta^*) \ge 0$ and the equality holds iff $\theta = \theta^*$. $\square$

Example 4.5. Consider system (1) and (2) with $a = 1.1, q = 2, c = 0.5, r = 1, m = 1$ and $T = 20$. Let k_1 be the time that the sensor uses the delay-free channel (i.e., $\gamma_{k_1}(\theta) = 1$). Fig. 3 plots $J(\theta)$ as a function of k_1. The red solid line is the plot for $p_0 = 5\overline{p} > \overline{p}$. The cyan dotted line is the plot for $p_0 = \overline{p}$. And the blue dashed line is the plot for $p_0 = 0.2\overline{p} < \overline{p}$. The result demonstrated from these plots agrees with Theorem 4.4.

The following result gives the optimal schedule for a general $\alpha \in (0, 1)$.

Theorem 4.6. *An optimal schedule is given as follows.*

(1) $p_0 > \overline{p}$: *If $\alpha > \alpha^* \triangleq \frac{T}{T - 1 + \frac{\epsilon_1}{\epsilon_2}}$ where*

$$\frac{\epsilon_1}{\epsilon_2} = \frac{\left(hg^{m-1}(p_0)\right)^2 \left[r + c^2 hg^{T-1}(p_0)\right]}{\left(hg^{T-1}(p_0)\right)^2 \left[r + c^2 hg^{m-1}(p_0)\right]},$$

then $\gamma_k = 1, \ k = 1, \ldots, m$. And if $\alpha \le \alpha^*$, then $\gamma_k = 1, k = 1, \ldots, m - 1$ and $\gamma_T = 1$.

(2) $p_0 = \overline{p}$: *any feasible schedule with $\gamma_T = 1$.*

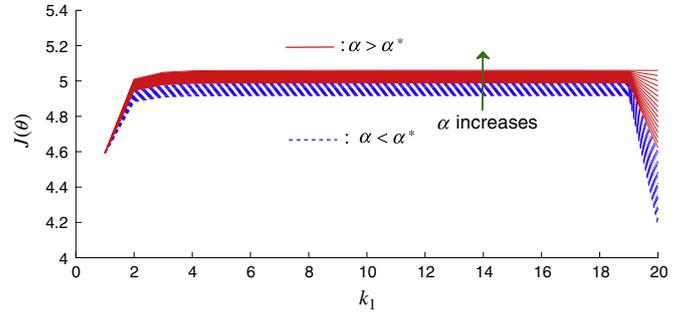(3) $p_0 < \overline{p} : \gamma_k = 1, T - m + 1 \le k \le T$.

Fig. 4. $p_0 > \overline{p} : J(\theta)$ as a function of θ for different α.

Proof. We only prove the first part as the second and third parts are direct results from the optimal power schedules for $\alpha = 0$ and $\alpha = 1$. Let θ_1 be the schedule with $\gamma_k(\theta_1) = 1, \ k = 1, \ldots, m$ and θ_2 be the schedule with $\gamma_k(\theta_2) = 1, 1 \le k \le m - 1$ and $\gamma_T(\theta_2) = 1$. Thus the first part is equivalent to say that if $\alpha > \alpha^*$ then θ_1 is optimal; and if $\alpha \le \alpha^*$, then θ_2 is optimal. First consider $\alpha > \alpha^*$: since

$$J(\theta_1) = \frac{\alpha}{T}\left(\sum_{k=1}^m g^k(p_0) + \sum_{k=m}^{T-1} hg^k(p_0)\right) + (1-\alpha)hg^{T-1}(p_0),$$

$$J(\theta_2) = \frac{\alpha}{T}\left(\sum_{k=1}^{m-1} g^k(p_0) + \sum_{k=m-1}^{T-2} hg^k(p_0) + g^T(p_0)\right)$$
$$+ (1-\alpha)g^T(p_0),$$

after some manipulation we get

$$J(\theta_1) - J(\theta_2) = \beta \frac{c^2\left(h(y)\right)^2}{r + c^2 h(y)} - \lambda \frac{c^2\left(h(x)\right)^2}{r + c^2 h(x)}, \quad (8)$$

where $x \triangleq g^{m-1}(p_0) \ge g^{T-1}(p_0) \triangleq y$ and $\lambda = \frac{\alpha}{T}, \ \beta = \frac{\alpha}{T} + 1 - \alpha$. Consequently $J(\theta_1) < J(\theta_2)$ iff the right hand side of (8) is less than 0, which is equivalent to $\alpha > \alpha^*$.

Consider any other schedule θ different from θ_1 and θ_2. If $\gamma_T(\theta) = 1$, then consider Problem 2.1 with T changed to $T - 1$, m changed to $m - 1$, and $\alpha = 1$. From Theorem 4.4, θ_2 restricted from $k = 1$ to $k = T - 1$ is optimal. Thus $J(\theta) > J(\theta_2) > J(\theta_1)$. If $\gamma_T(\theta) = 0$, then $p_T(\theta_1) = hg^{T-1}(p_0) = p_T(\theta)$. Again consider Problem 2.1 with T changed to $T - 1$ and $\alpha = 1$. From Theorem 4.4, θ_1 restricted from $k = 1$ to $k = T - 1$ is optimal. Again we have $J(\theta) > J(\theta_1)$. Therefore if $\alpha > \alpha^*$, θ_1 is optimal.

Now let us consider $\alpha \le \alpha^*$. First note that from the above proof, if $\alpha \le \alpha^*$, then $J(\theta_2) \le J(\theta_1)$. Now consider any other schedule θ different from θ_1 and θ_2. Similar to the previous analysis, if $\gamma_T(\theta) = 1$, then $J(\theta) \ge J(\theta_2)$. And if $\gamma_T(\theta) = 0$, then $J(\theta) \ge J(\theta_1) \ge J(\theta_2)$. Hence if $\alpha \le \alpha^*$, then θ_2 is optimal. $\square$

Example 4.7. Consider the same system parameters as in Example 4.5. One easily verifies that $\alpha^* = 0.8401$. Consider the initial condition $p_0 = 5\overline{p}$. According to Theorem 4.6, if $\alpha \le \alpha^*$, choosing $k_1 = T$ minimizes J; otherwise, choosing $k_1 = 1$ minimizes J. Fig. 4 plots $J(\theta)$ as a function of θ for different values of α in $\{0.7, 0.71, 0.72, \ldots, 0.98, 0.99, 1\}$. The red solid lines correspond to those α's such that $\alpha > \alpha^*$ and the blue dashed lines correspond to those α's such that $\alpha < \alpha^*$. The bottom line corresponds to $\alpha = 0.7$ and the top line corresponds to $\alpha = 1$. The result demonstrated from these plots agrees with Theorem 4.6.

5. Conclusion

In this paper, we consider sensor data scheduling over a packet-delaying network. Optimal schedules are explicitly constructed which satisfy the communication energy constraint and at the same time minimize a cost function consisting of a weighted

average error and a weighted terminal error. There are many interesting directions along the line of this work including generalizing the results to some higher-order systems, considering random delays and data packet drops, and scheduling of multi-sensors in a bandwidth limited scenario.

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